

Project Planning & Control
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Lecture – 38

Lesson - 07

Minimum Moment Concept

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Minimum Moment Concept

- Based on attempting to achieve a rectangular profile. (ideal ?)
- Moment of the resource histogram about the X axis is minimum for a rectangular profile.

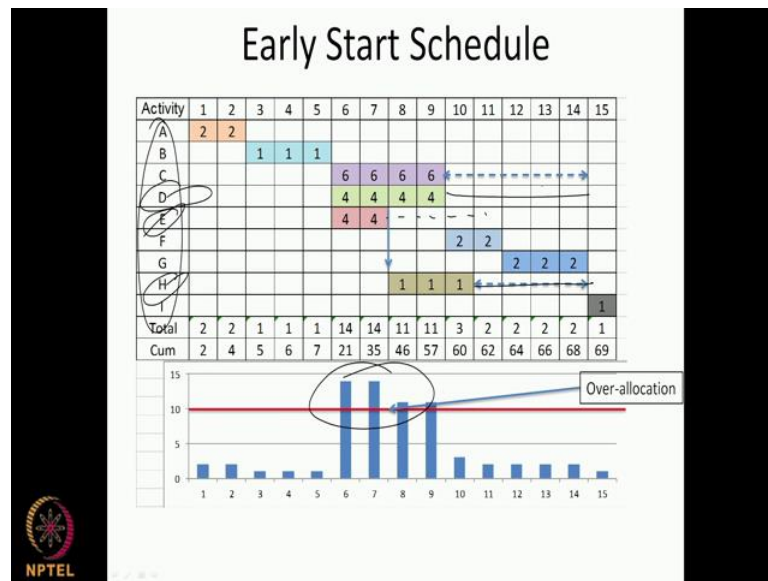
$M = \sum(Y*Y/2)$

Profile	Bar 1 Height	Bar 2 Height	Moment Calculation	Result
Profile 1	4	12	$M = 4*4/2 + 12*12/2$	80
Profile 2	6	10	$M = 6*6/2 + 10*10/2$	68
Profile 3	8	8	$M = 8*8/2 + 8*8/2$	64 MIN VALUE

What is the profile? What profile is the best for me? So, this is where the concept of minimum moment comes out. So, remember what we are trying to do is to get as flat the profile as possible of the resource, you know and this is ideal, but we want to get it as close to this as possible and if we go back and look, one way of expressing this so. So, while I can visually look at different profiles and then say oh this is the best profile for me to kind of use a mathematical approach to say to kind of, you know zero in on the best profile, we use the minimum moments.

So, we say the profile, the moment of the shape about this axis is minimum only when the profile of the resources are rectangle.

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So, to put it in another way if I had a profile, I am going back of you here, if I had a profile like this and I am going to start leveling it. There is; obviously, I cannot get perfectly a rectangular profile, but given my total sum of resources, the once I level it in a way where my floats are used appropriately, the best leveled profile will have the minimum moment for the whole histogram. So, that is the basis of the minimum moment algorithm and or the minimum moment metric which we are going to use and when we look at, if we take an example here.

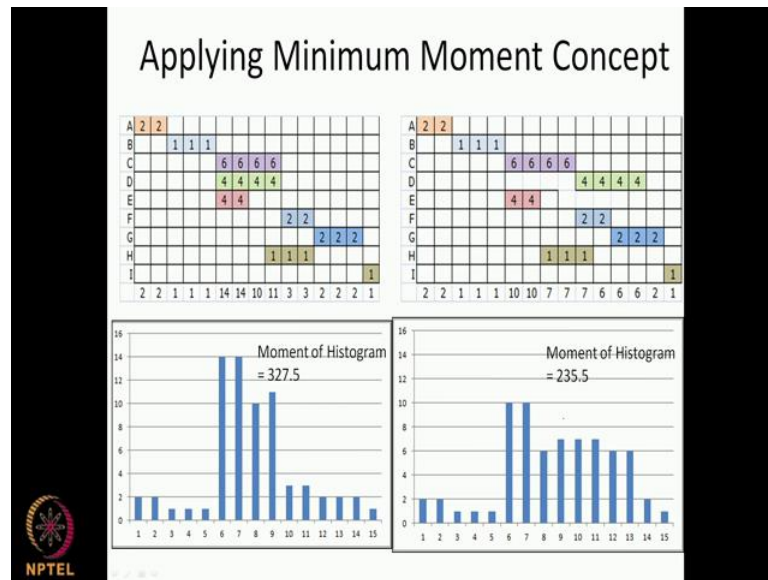
So, here we have just two resources, you know one is I mean I am sorry two resource profiles for 2 days. One is 4, the other is 12, the total is 16 and if I am going to take the moment of this about the x axis, it is $(4 \times 4)/2$; that is the moment of this, that is the height into the height divided by 2. So, this is the moment above of this rectangle. Similarly, the moment of this rectangle is 12; that is the $(\text{height} \times 12) / 2$. So, this is the moment of this rectangle, if I sum this up I get the moment of this histogram which is 80.

Now, for the same resource requirement which is 16, 10 plus 6; I have an alternate histogram here. So, here you have, you know the moment of this is $(6 \times 6) / 2 + 10 \times (10/2)$ and this is the moment of this histogram and you can see as my peak reduced, my moment also reduced and; obviously, because this is square of the height of the bar. You are going to as I reduce it, it reduces by the square. Now, I have an 8 and 8, which is again 16 total resource units and I have this so; obviously, $(8 \times 8) / 2$; that is 8 / 2 here.

This is the moment and 64 is the value I get and this will we be, for 16 units of resources

distributed between 2 days, this will be the minimum value of the moment about of of this histogram. So, based on, this is the concept we would like to use to explore, how resources can be leveled on our projects or at least, if it is not on our projects on the kind of idealized projects which we would like to do.

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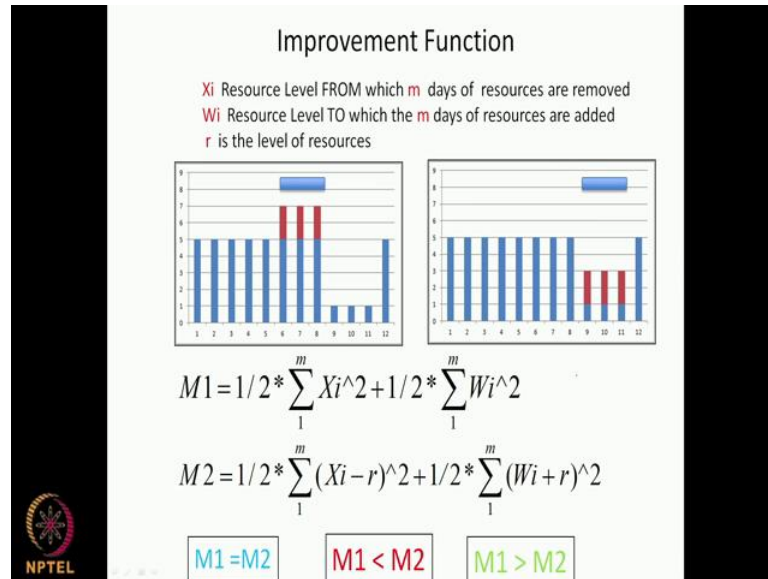
So, when we look at doing the minimum, when we look at the moment applying the moment again, so here is the earlier example. Now, this is the slightly different histogram than the early start, you will see that you know we have activity H, that is moved by 1 here. So, if you look at the histogram you can see here, you know if I calculated the moment of this this histogram for for activity D starting from here to here, which is loading this set of of days and the moment is 327.5.

Now, what we have done here is shift for all the way here, it is moved from here to here; move shift there and you can see the resource profile changes and the moment of the whole histogram has reduced. So, when we are looking from a leveling perspective, especially for a rectangular kind of a shape, we are saying this level this histogram is better than this, because it has a lower moment and it is more towards the rectangular profile than this and this metric gives us that justification, that this is more profiled towards rectangular than this.

So, using this as a basis, we now want to be able to go through for example, you know apply this in a very more stepwise systematic way. So, I am just again repeating that. The concept is to reduce the moment of the histogram in a systematic way and once we get to

the minimum moment, we say that is the best profile or that is the profile as closest to the rectangular profile which we would like to get.

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So, as we move, so when we move from here to here ((Refer Time: 06:08)) I can certainly calculate my moment as it is, but what we want to do is work towards a procedure, that can kind of give us you know a methodical way in being able to calculate whether a shift is improves or does not improve the rectangularity of the resource loaded histogram. So, when we move to the procedure of what we want to develop a procedure, we need a metric.

So, if we go back and look at the earlier function ((Refer Time: 06:45)) where we said there are 50 options, one we is to actually calculate the moment of the 50 different alternatives and then certainly take the minimum of that, that will certainly give you an answer, but the computational effort of calculating for 50 alternatives might be too high. Of course, 50 alternatives is not high in this case, but if you had, you know several 100 activities with several different days of float, it could certainly you know blow up into a very large figure, which is not easily computable.

So, in to be able to find out, how we go about whether something should be improved or not, we have developed something called an improvement function. So, what an improvement function tries to do is to rather than calculate the moment every time for if there is an activity here, for it to be moved to across it is float, each time it tells you whether the movement you are doing is causing improvement in, by improvement we

mean by reducing the moment of the histogram or is it causing an increase in the moment of a histogram.

So, this improvement function we will get a little bit into the, into what, how the improvement function is formulated. So, you have here a histogram and you can see that as we move this across to this side, there are resources that are taken from a certain set of days and move to another set of days. So, these resources actually moved here and that what it is showing up here. Now, when we are going up, when we have defining this there are few variables X_i is a resource level from.

So, X_i is the resource level from, so X_i is the resource level form which resources where moved, so this is X_i . So, this level is X_i and m days of resources are moved, so here in this case m is 3, so 3 days of resources were moved and W_i is the resource level to which the m days of resources. So, W_i is these days, this is the original, before moving it was X_i , W_i . So, this is I am going to write it here, this is X_i this is W_i .

I can put it as W_1, W_2, W_3 and X_1, X_2, X_3 if that makes it more specific. Now, r is the level of the resource, it is the level of the resources for that activity it had a certain resource load. Remember, we said they have certain, so that is r ; the resource load of that activity and so based on this you can see that if we are going to calculate the moment of the function of this histogram, I have just put it in an expanded form.

So, it is $\sum X_i$; that is for each of the three bars in this particular case $\sum X_i^2$, so it will be $(X_i \times X_i) / 2$,

$$\text{So } \frac{1}{2} \sum_1^m X_i^2 + \frac{1}{2} \sum_1^m W_i^2$$

$$\text{So, again } \frac{X_i^2}{2} + \frac{W_i^2}{2}$$

and this is shown in this formula here. This is the moment before we shifted the activity; now after shifting the activity what has happen to X_i is, here we go this is 6. So, this was at 7, so r amount of resources got taken away from this and that r amount of resources got added to this.

$$\frac{1}{2} (\sum_1^m (X_i - r)^2 + \sum_1^m (W_i + r)^2)$$

So, this is a fairly simple, it is basically this gives you moment 1 and the moment 2. Now, we have these three options, so after the shift either M_1 can be equal to M_2 which means, there was no benefit in my, no reduction in the moment by shifting or M_1 is less

than M_2 ; that means, when I shifted the moment of M_2 became greater; that means, this profile had a greater moment, if it had a greater moment M_1 would be less than M_2 or M_2 would be greater than M_1 or we could have that, before the shift the moment was more than after the shift.

So, which of these would you want to happen if you have going to actually do the shift? So, if my moment before the shift was greater and after the shift it is less, this is kind of a of sign I am looking for in order to able to be make the shift up to the activity. So, before I leave the slide just kind of reviewing, because this forms the basis of the minimum moment algorithm that we have a resource profile before shifting, we have a resource profile after shifting here and we have only one activity for which we are shifting across, we are not going to shift multiple activities.

The algorithm we are dealing with and the procedure we were dealing with, it does not involve at this stage shifting activities simultaneously. It involve shifting of a single activity, the resource load of that activity is r and that is r , before shifting the resource level on those particular days is X_i and before shifting, the resource level or the days to which you are going to shift T_o is W_i . You are shifting by m days, you have to be careful we will later look at, we know why we are to be careful with m .

m is the number of days of resources that are removed, please look at this. This is the number of days of resources; it is not the number of days of shifting. So, this is a common problem when we start working out problems we tend to think m is the number of days of shifting, it is not; it is the number of days of resources that are removed.

So, for example, here if I am move this by 4 days, my activity duration is only 3, I am only still shifting 3 days of resources. So, m is 3, even though the shift might be 4 days, we will come to this later, because this is the common problem in when we do these exercises. So, please keep this in mind.

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$M1 > M2$


$$\sum_{1}^m Xi^2 + \sum_{1}^m Wi^2 \gg \sum_{1}^m (Xi - r)^2 + \sum_{1}^m (Wi + r)^2$$

Expanding & Collecting Terms (& dividing by -2)

$$IF_{A,d} = r \left(\sum_{1}^m x_i - \sum_{1}^m w_i - mr \right)$$

$IF_{A,d}$ = improvement factor for shifting activity A d days out in time
 r = daily resource rate for the activity
 m = minimum number of days that the activity is shifted or the duration of the activity
 x_i = daily resource sum for the current time frame over which resources will be deducted
 w_i = daily resource sum for the time frame over which resources will be added

$IF > 0$ implies reduction in moment due to the shift
Magnitude of change is also important to select alternative



So, these are the variables and remember we wanted this, so we talked that we ideally want $M1$ to be greater than $M2$; that is, after my shift my resource load, my resource histogram has a lower moment. When we say this is; that means, there is improvement in the histogram, this is what I want to try to achieve with the shift. If the shift does not result in an improvement in the histogram, it means that I should look for another alternative shift which might result in an improvement, this current shift does not.

So, if I take, again put $M1$ on the left hand and $M2$ on the right hand, so these are the equations from the earlier thing, earlier slide. You then expand it, collect and you reformulate it and you get what is classically known as the improvement factor formula. There will be derivation for this, more details and the derivation I will give it in the hand out, but you know it would be good for you go through the derivation, but it is equally important for you to understand what this formula basically implies.

So, here we have improvement factor of an activity moved by d days is the resource level of the activity, the sum of the resource load for the days from which was moved, to the days it is going to be moved, multiplied by the number of resource days that have been moved again into the resource load of the activity. So, this is the improvement factor and as you can as you can imagine, improvement factor what we would like to say here, it is a kind of highlighted here, but improvement factor greater than zero implies reduction.

If I calculate it is improvement factor after shift and show that, it is greater than zero; I

mean and find that it is greater than zero; that means, there is a reduction in the moment of the histogram. The improvement factor = 0, then the moment of the histogram remains the same. If the improvement factor < 0 ; it means that my moment of the histogram has increased because of the shift. So, this gives me an indicator, whether I should make a shift or not.

So, the other point is that it is not just the reduction in moment due to the shift, it is also a magnitude of change. So, we are multiplying by r here which gives you the magnitude of change of a shift. So, this becomes important when we are comparing, you know when we want to compare between two three activities I shift one, I bring that one back, I shift the second one; might be both result in a positive value; I mean in terms of improvement factor, but which gives you a better a higher reduction in moment will now depend on the resource level which is required, so that is what is or the resource rate of the activity; that is what is r .

So, again just just re visiting these terms, high improvement factor for shifting is IF, daily resource rate for the activity, m is a minimum number of days activity shifted or the duration of the activity, the smaller minimum of the number of days or duration. Like we said earlier it is the number of resource days that was shifted from the from activities to the to activities and X_i is the daily sum for current time frame and W_i is the daily resource sum for the time frame over which resources will be added.

So, it might take some time and some reviewing to understand these variables clearly, but when you, also it will take some working out of exercises and reviewing it, so that is typical in for you know when we do minimum moment algorithm and its approach. Only through practice you will be able to understand what and get a good feel of these variables and be able to apply it effectively to minimize, to kind of get a level or to try to level the resource profile.