

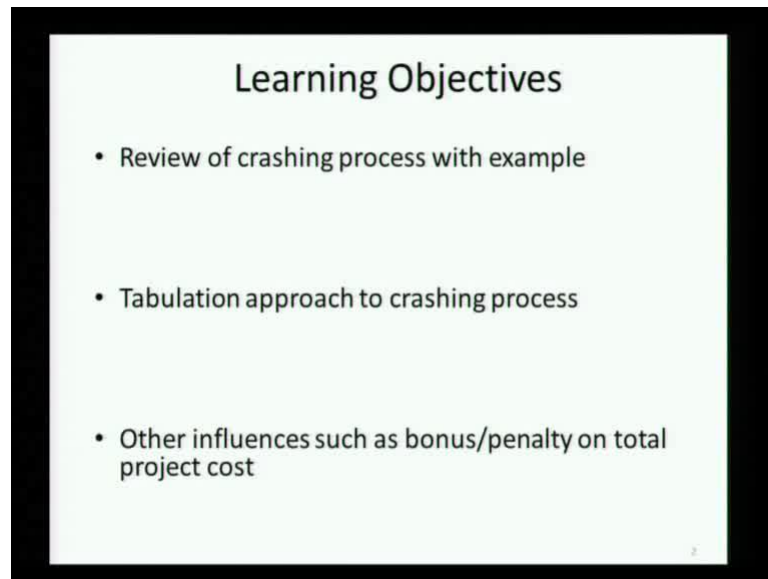
**Project Planning & Control**  
**Prof. Koshy Varghese**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Madras**

**Lecture – 30**

**Lesson - 04**

**Time-Cost trade-off: Problem- 3, Tabulation Approach**

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**Learning Objectives**

- Review of crashing process with example
- Tabulation approach to crashing process
- Other influences such as bonus/penalty on total project cost

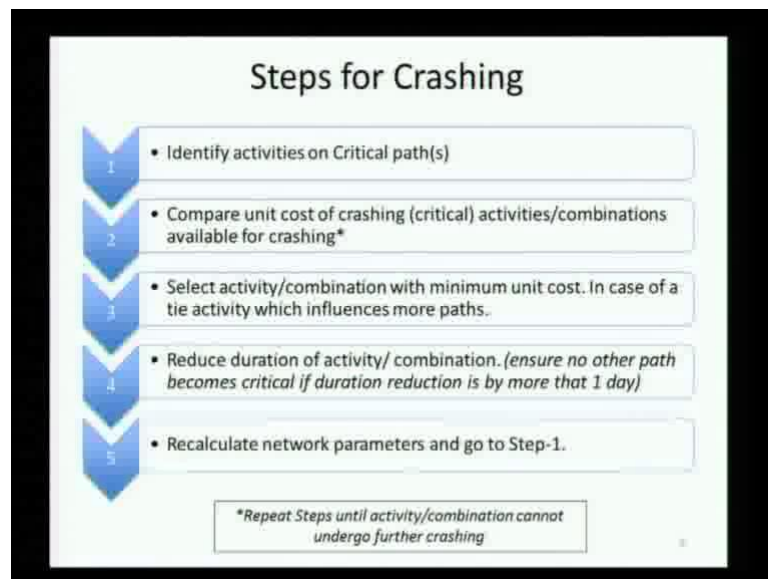
In this lecture we will cover Time Cost trade off again, we will look at some different aspects and we will first review, what we had done last time, we will use an example, so that the process of how we go about time cost trade off is a kind of reinforced. We will look at a tabulation approach; this is a systematic way of doing this. You know we will look at the, how we can use the tabulation to systematize it and then we will look at other influences such as bonus and penalty and how it can affect the time cost trade off and specially overall project cost.

(Refer Slide Time: 00:50)

Activity	Preceded by	Crash cost (CC) (u)	Normal cost (NC) (u)	Crash duration (CD) (days)	Normal duration (ND) (days)	Slope	Duration Reduction
A	-	3,900	3,600	6	7	300	1
B	A	6,500	5,500	3	5	500	2
C	B	7,200	6,350	7	9	425	3
D	B	4,900	4,700	18	19	200	1
E	B	2,200	2,050	9	10	150	1
F	C	1,700	1,200	6	8	250	2
G	F	7,200	7,200	5	5	NA	0
H	E	10,000	9,450	10	11	600	1
I	D,G,H	4,700	4,500	6	7	200	1
<b>Total Cost</b>			<b>44,500</b>				

So, if we you recall this was a problem that was given last time. We will just go through this problem; we will first solve it by hand, because we will be able to review the process through that.

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Now, if we go into the next level which is, if you recall the process of time cost trade off was to be able to the various steps if you remember. First we had to identify activities and critical path and then, compare the unit cost of crashing for each, for the different combinations of activities and critical path and then, select the combination with minimum cost and in case of ties, which you know in case of a tie activity select the one which influences the path.

And then, they reduce the duration, then you have to redo the network analysis and then go back to the step 1 and as we had, as it is specified here. So, if we take this problem ((Refer Time: 01:46)) the first step is to actually go ahead and calculate the cost slopes, so you may recall that we calculated the cost slopes here. Do you remember what the cost slope was?

**Student:** ((Refer Time: 02:00))

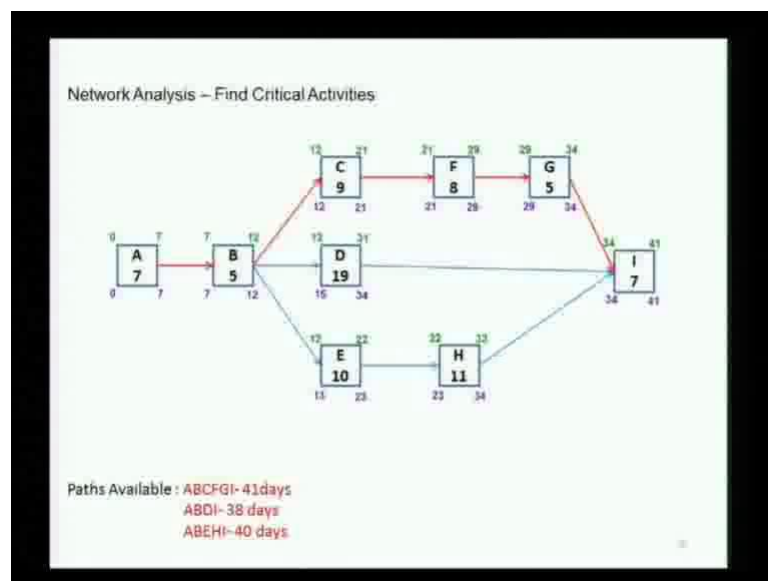
No, no, what is a cost slope?

**Student:** 
$$\frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal duration} - \text{Crash duration}}$$

So, we are looking at the cost per day of crash. So, that was the slope. So, here in this table you will be, we remember we go back to the problem that was given; we had given you crash cost, normal cost, crash duration, normal duration. We have added two more columns to that, one is the cost slope and two is the maximum amount by which the duration can be reduced. So, these are the two columns that have been added.

So, here we will see a number like 300 is obtained by subtracting 3900 from 3600 and the number of days is only 1, you can reduce it only by 1 day, so it is 300. So, similarly the remaining values have been calculated. Now, the value here 44500 is the total value, total cost of the project if it everything is done in normal duration. So, this forms a basis for a crashing process.

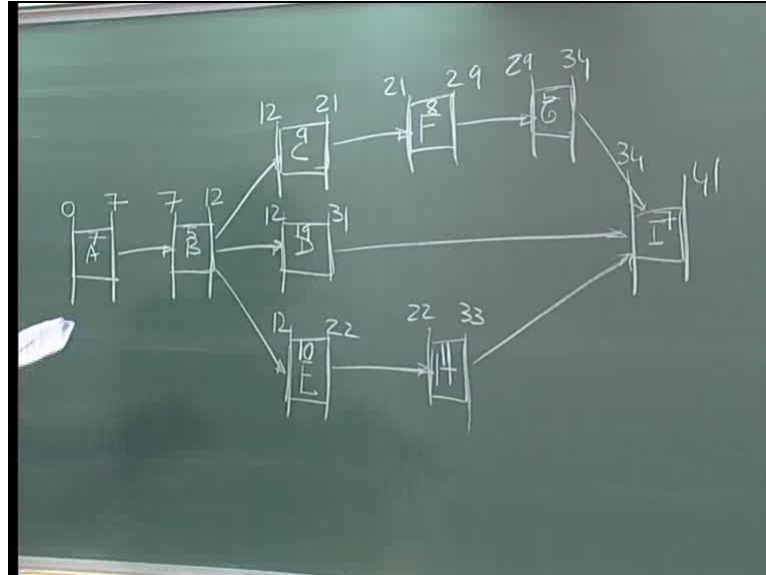
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And we have this as the basic network diagram. So, if you follow the precedence

relationship that was given there, this is the basic network diagram. Now, let us use this as a basis and solve the time cost trade off.

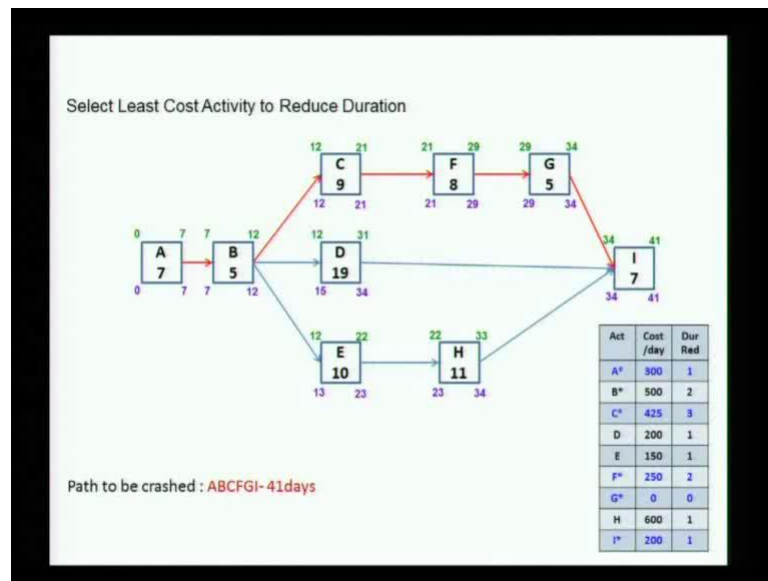
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So, we have A, is that ok? And we have the durations at, we have actually written it in the box system we have duration of 7, B has the duration of 5, C has the duration of 9, D 19, E 10, 8, 11, 5 and 7. We have the values as you can see the forward pass and backward pass have been done and if you can tell me the values as we go, so that you are doing the participating in it, so 12, 12 and 21, 12 and 31, 21 and 29, 29, 34, 34, 41. Okay. The backward pass value is also given in the network on the power point, but I am not going to calculate that here, because we can do this with this simple forward pass itself we will do this and then we will do go to the tabular method.

Now, we can see the critical path A B C F G I, so you can see in the slide here it shown, there are three possible paths, you have A B C F G I which is 41 days, A B D I which is 38 days or A B E H I which is 40 days and we see that; obviously, the 41 days Is critical, which means my next step will be to take the activities on this path okay and...

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So, if you see I have put the cost of the activities of the crash cost of the activities per day on the side. So, we can see if I take the activities A B C F G I you can see the various costs 300, 500, 425, G cannot actually be crashed and I is 200. So, we know that we have to select the minimum cost and yes, minimum and on critical path and this case it could be I. So, if you go back to I and we will see why, how many days can we actually reduce I.

**Student:** 1 day.

1 day, so we will go to I and we will reduce it by the 1 day will make I 6 ((Refer Time: 07:45)); which means this goes to 40, so that was the process we use. Now, we know that also if you go back to the basic where we calculated all the parameters, you can see I can only go up to 6, it cannot go any further, so I has been crashed to the maximum. So, we now, once we crash the I ((Refer Time: 08:18)), has the critical path changed?

**Student:** No, to critical path change.

Has the critical path changed? See I is now with the, if I go back to the diagram you will be able to see that, yes now I change. I change 34, this to 6 and it became...

**Student:** 40

40, will it change anything else?

**Student:** No.

No, critical path is still the same, every activity in before I has remained the same. So,

the critical path leading to I will remain the same, the critical path will only change if I change any of the activities which are in the parallel paths. For example, if I change B, nothing will change, because it is predecessor to all the other activities, it will all be pulled; similarly I is successor to all the activities, so there will be no change in the critical path.

So, right now I we have taken out and the path is still the same and we still have this table value. So, we took I last time, now with 40 days what is the activity you would choose. Your choices are A B C F and remember G, G was cannot be crashed any way, so G also cannot be crashed. So, we have F, F is at...

**Student:** 250

250 days, F is at 250 is the amount and we can crash it by how much.

**Student:** 2

So, I can take F straight away to 2 days, no.

**Student:** By 1 day.

By 1 day, because if I take F to 2 days what happens.

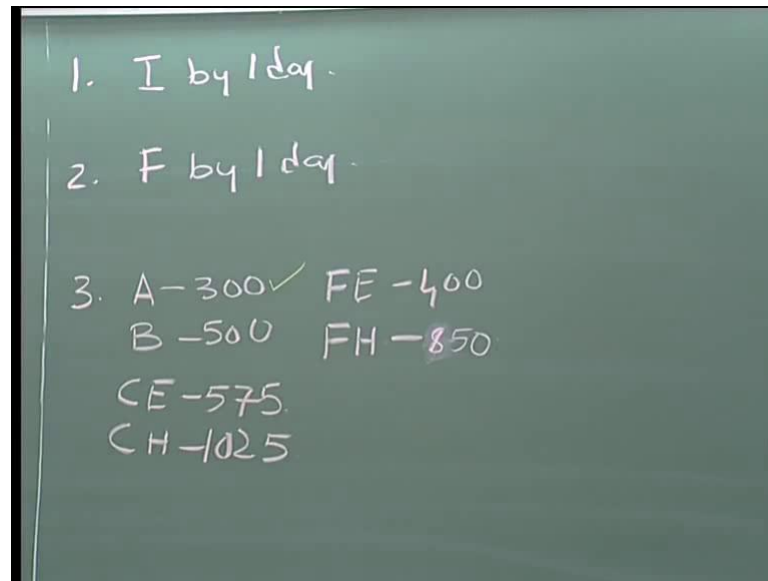
**Student:** ((Refer Time: 10:08))

Now, before F goes down to 6 ((Refer Time: 10:11)), by the time F goes down to 7, E and H also becomes critical. So, I have to take it by one step first, so I take F to 7, now this goes to 28. So, what has happened now? Now, we find this path also, so this is at 33 this is at 33, this E and H have also become critical. So, any further crashing requires what.

**Student:** Simultaneously.

Simultaneously, so my combinations now are A alone, B alone or C, E C H, F E or F H. So, let me write that down, so now, that it is getting a little more involved.

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I will write it down when we take it to the, so if we... Let us just recollect, step 1 was what, I by 1 day; step 2, step 3 we have a choice, either A B, we have then we said C E, C H, F E or F H. So, what are the costs? You can read it from there, A would be 300, C E is 575, F E.

**Student:** 400

So, we have all these combinations and; obviously, A at 300 is the least expensive. How much can we move A by?

**Student:** By 1 day.

By 1 day, so we will move A to 6 and this will impact the whole network. Any change in critical path?

**Student:** No.

No, because it an activity A that moved, it is predecessor to all of it. So, now, we have and we have actually crashed A also to the max. What are all options now?

**Student:** F and E.

F and E, so in the next option it is F and E, so I will have to move F to 6 and E to 9. So, if it is F and E there, so F to, yes. So, now, I have reduced, ((Refer Time: 14:22)) say F has gone down by 2 and E has gone down by 1, so both are crashed to the maximum, let me calculate the result 20. So, here we go to 26, 37, so, now we have crashed out E and F. What is the only, so if you look at the combination now what do we have left.

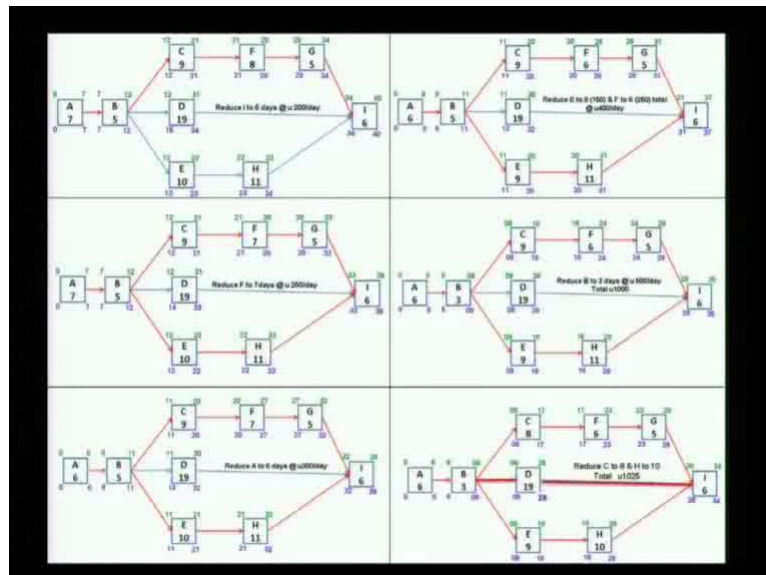
**Student:** We have B.

We have B, so I am, this is basically for us to recollect the process which we went through, so you get the idea. Now, we have B; once I take B in, I can take it by how many days, I can take it by 2 days, again predecessor to everything, it will come down to 35 and if my step after B, I am only left with.

**Student:** C E.

See, E is crashed to the maximum, so only C H. F H is also gone, because F is crashed to the maximum, we will come to C H. So, we can do this process and you know the principle on which is based. Now, as we do a process like this you will see that we could put it into a series of diagrams of what we have shown.

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So, here is we have reached, up to here we have crashed A by 8, we have brought A down to 6 days and then we just brought E and F down. We can go to the next step which is to reduce B and the final step is to reduce C and H, so you can see in this we can go through the series of processes and finally, the all three parts become critical. Now, D is also become critical and there is no further reduction possible, because everything has been, the other two paths have been crashed to the maximum.

So, if we were doing this the way we did it in the last class we would go through this network diagrams and it is very good to understand the network, because that is really the basis on which crashing is done. But, sometimes it becomes really messy to be able to do it in this form.



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Cost table : Direct, Indirect, Total cost

Duration (days)	Direct Cost Change	Direct cost (u)	Indirect cost/day	Total indirect cost (u)	Total cost (u)
41	0	44,500	200	8,200	52,700
40	+200	44,700	200	8,000	52,700
39	+250	44,950	200	7,800	52,750
38	+300	45,250	200	7,600	52,850
37	+400	45,650	200	7,400	53,050
35	+1000	46,650	200	7,000	53,650
34	+1025	47,675	200	6,800	54,475

Minimum total cost = u52,700  
Duration for minimum cost = 41 or 40 days.

So, what we have introduced is a tabular approach. but let me show you this tabulation.

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Activity	Paths requiring time reduction			Cost Slope (u/day)	Available Crash time (ND-CD)	Iteration					
	ABCF GH	ABDI	ABE HI			1	2	3	4	5	6
<del>X</del> A	1	1	1	300	1	1	1	0	0	0	0
<del>X</del> B	1	1	1	500	2	2	2	2	2	0	0
C	1	0	0	425	2	2	2	2	2	2	1
D	0	1	0	200	1	1	1	1	1	1	1
<del>X</del> E	0	0	1	150	1	1	1	1	0	0	0
<del>X</del> F	1	0	0	250	2	2	1	1	0	0	0
<del>X</del> G	1	0	0	0	0	0	0	0	0	0	0
H	0	0	1	600	1	1	1	1	1	1	0
<del>X</del> I	1	1	1	200	1	0	0	0	0	0	0
Initial Path Length:	41	39	40	Iteration	Action	Iteration cost	Cum-cost				
Path Length:	41	38	40	0			44500				
	40	37	39	1	Crash I by 1 day	200	44700				
	39	37	38	2	Crash C by 1 day	250	44950				
	38	36	38	3	Crash A by 1 day	300	45250				
	37	36	37	4	Crash F by 1 day	400	45650				
	35	34	35	5	Crash B by 2 day	1000	46650				
34	34	34	6	Crash C,H by 1 day	1025	47675					

So, here what we have is the various activities of the network, the possible paths. So, you remember the three paths A B C F G I, A B D I, A B E H path and we will write a cost slopes for each of these and the available crash time and for each step of reduction, we will go through an iteration and we will update it as we go. So, let us see first we enter the basic values, you can see that these are the, this is the amount by which I can reduce the activity and this is the current path length.

So, for example, D does not participate in the A B C F G I activity and F does not

participate in this activity, H does not participate in this path. So, take a look at the table and tell me, ask me if you have questions. So, 1 indicates that, that activity takes place is participating in the path, the cost slope you are familiar with it; that is the 300, the 500 all that you are familiar with it, the available time is the maximum related crash, the path length is what is given at the bottom is the length of the each point.

So, our base is that this is the length of each path and we know that G cannot be crashed any further, so we have putting an X and this is the total cost of the normal duration at it. So, we go to the first step which was, now you have to still use the network, you have to understand from the network what is happening, because you will have to keep path, but basically you have to, it gives you a way to express what you are doing. You have a crash I by 1 day; which means you update what happens here.

So, you are going A B C F I now, you have crashed I by 1 day, there is no more space left with crash I and your path is reduced to 40 and this path is reduced to 40, all the paths are reduced by 1 and so your cost is increased by 200 come to 44700. Similarly, we did the second step, so this was the, this is the stage next and then we remember we had I crash to the maximum, then we crashed F if you recall by 1 day and this is the same process it is iteration 2.

It went up by 250 and this is the resulting direct cost, then A by 1 day and this is the result. Now, we can keep track of cost, the cumulative cost, direct cost as we go to the process. We then crashed F and E by 1 day and resulted in this and you can see the changes in, as we go through each iteration the available time in each activity has also been updated, then we went B by 2 days and the duration went down to 35 and the final was C and H by 1 day and we will end up with this final path.

So, it is basically a systematic way of entering the values as you crash when you are doing it in a manual approach, if the network gets complex. I would still recommend using this approach if the network is small and you are able to understand and you are able to keep track of the numbers as you go through. Any questions ask.

**Student:** The network analysis without network we go for it.

You can, but as your network, what to say, as your network critical paths change and things like that, it is also nice to have a visual as to seeing where the critical path is going and how it is going. We will take a class later, where we will do all this in a spread sheet. So, you can set up things in such a way that you have been very algorithmic in your

fashion and not using a pictorial representation at all, but then you have to be very systematic in how you go about it in the algorithm fashion.

So, what I would say is different people have different preferences, if you want a very structured approach where we are looking at just the tables and the numbers and computing this would be preferred by some of you. Others who prefer a more visual approach and seeing what happens might be, this would be people prefer. But, without a doubt this can get extremely messy and prone to mistake if the network is large and there is a lot of crashing options. So, which case then the tabulation is preferred, but setting up the table can take quite a bit of times, especially if you are doing, in an exam situation.

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Cost table : Direct, Indirect, Total cost  
Indirect @ u200/day

Duration (days)	Direct Cost Change	Direct cost (u)	Indirect cost/day	Total indirect cost (u)	Total cost (u)
41	0	44,500	200	8,200	52,700
40	+200	44,700	200	8,000	52,700
39	+250	44,950	200	7,800	52,750
38	+300	45,250	200	7,600	52,850
37	+400	45,650	200	7,400	53,050
35	+1000	46,650	200	7,000	53,650
34	+1025	47,675	200	6,800	54,475

Minimum total cost = u52,700  
Duration for minimum total cost = 41 or 40 days

Now, this is what we land up with as the, our total. So, you remember the indirect cost was given at 200 per day. So, you can see the, we have done this calculation where we are showing the direct cost as it increases through the indirect cost with 200 and the total indirect cost and the total direct plus indirect cost and you can say from here that there are two durations at which the cost is minimum amount, why is that.

**Student:** optimal activity increase in cost.

Yes, I have two durations.

**Student:** I, the crashing of I, the cost was 200. So, if you take and if you crash it I by 1 day and pay 200, you gain another 200.

So, because it was 200 and say same as the slope value of this, you added up with two

solutions. So, it starts a unique minimum solution, we have two minimum, minima in this particular case. Are there any questions? So, this is pretty straight forward and you can, if you go to the 41 or 40 both give you the same time and same cost.

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And you will land up with the total cost graph which looks like this. So, both 40 and 41 the value of total project cost is the same. Now, let us take say the indirect cost is now 300 per day. So, take a look and say what would you, what I mean what is the change that would happen.

**Student:** It gone up 39 and 38.

So, it has gone up to 39 and 38 and again you have two values, because one of the slope values have direct cost change happens to coincide with your indirect cost. So, because the value is exactly the same, like you said earlier what you lose by charging for the direct cost you gain exactly the same amount, so both days it remains the same. Now, what if the indirect cost was say something like 350?

**Student:** It has a unique cost between 300.

It will have a unique between 300, because so long as I spent 300 and gained 350 am gaining, I spent 400, gained 350 am losing, so it would be on that line. So, here we have the same values, now this is we have seen this is shifted to as because it is 300. So, you need to get a feel of how the indirect cost influences the overall project cost and why the, how the value of the indirect cost and the relationship of the slope of the direct cost values.