**Prestressed Concrete Structures**

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# Lecture - 39

# Compression Members

Welcome back to prestressed concrete structures. This is the fifth lecture of module nine on special topics. In today's lecture, we shall cover compression members.

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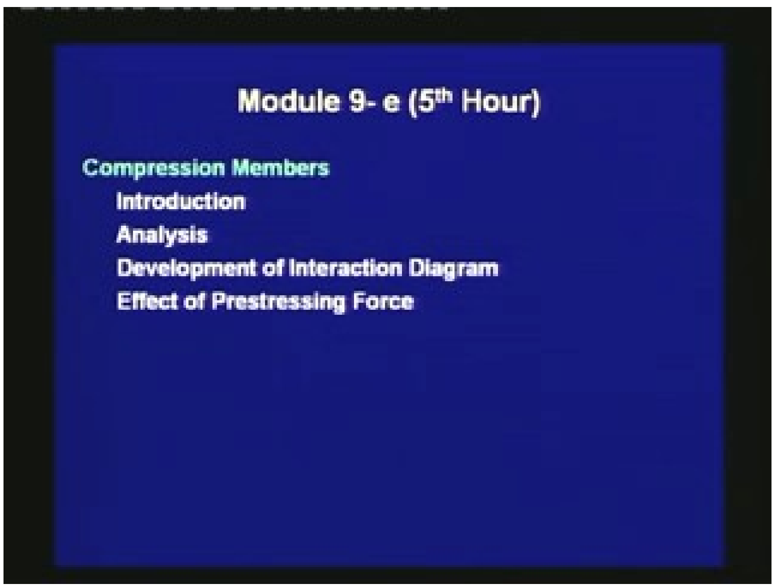


After the Introduction, we shall study about the analysis, development of interaction diagram and effect of prestressing force.

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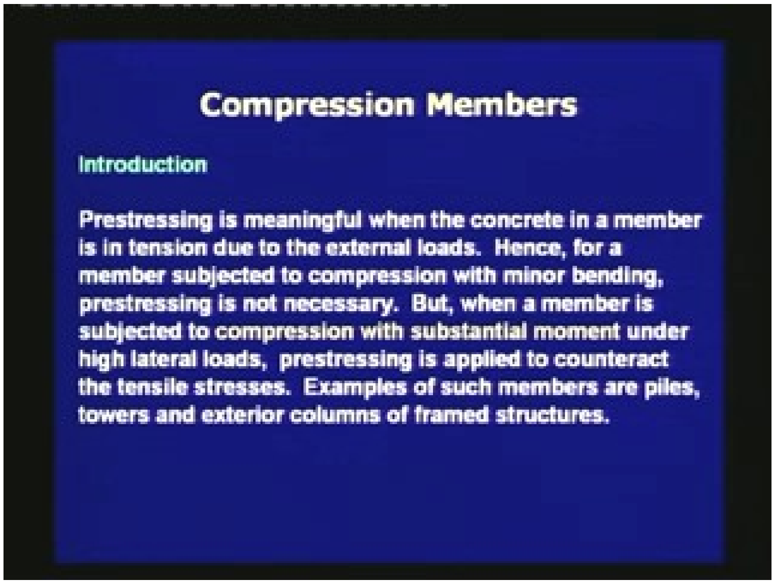


Prestressing is meaningful when the concrete in a member is in tension due to the external loads. Hence, for a member subjected to compression with minor bending, prestressing is not necessary. But when a member is subjected to compression with substantial moment under higher loads, prestressing is applied to counteract the tensile stresses. Examples of such members are piles, towers and exterior columns of framed structures. Earlier, when we studied the behaviour of axial members, we had known that the prestressing is effective only when the concrete is under tension. In fact, prestressing may be harmful if the concrete is under compression, because the compressive capacity of the member gets reduced. Thus, prestressing is applied to mostly tensile members; but if a compression member is subjected to substantial moments along with compression then prestressing may be meaningful. Thus, for compression members prestressing is applied to piles, towers or columns in a building only if they are subjected to high moments due to lateral forces. As the seismic forces are reversible in nature, the prestressing of piles or columns is concentric with the cross-section. Some typical cross-sections are shown below.

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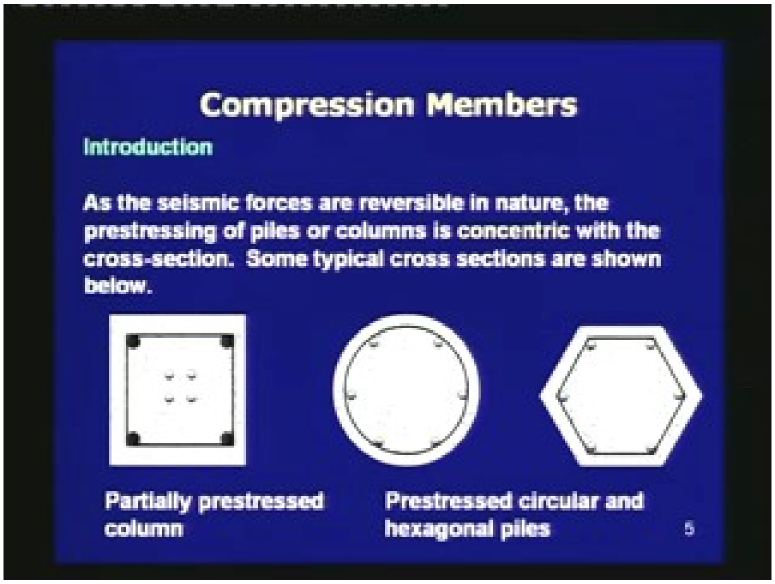


The one in the left is a partially prestressed column, where we are having non-prestressed reinforcement at the four corners and we also have prestressing tendons at the centre. The two figures on the right are piles. The first one is the circular section and the second one is the hexagonal section and here, the prestressing tendons are along the perimeter but the effect of prestressing force is concentric to the section.

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This is a figure which shows the lowering of the reinforcement cage for a pile.

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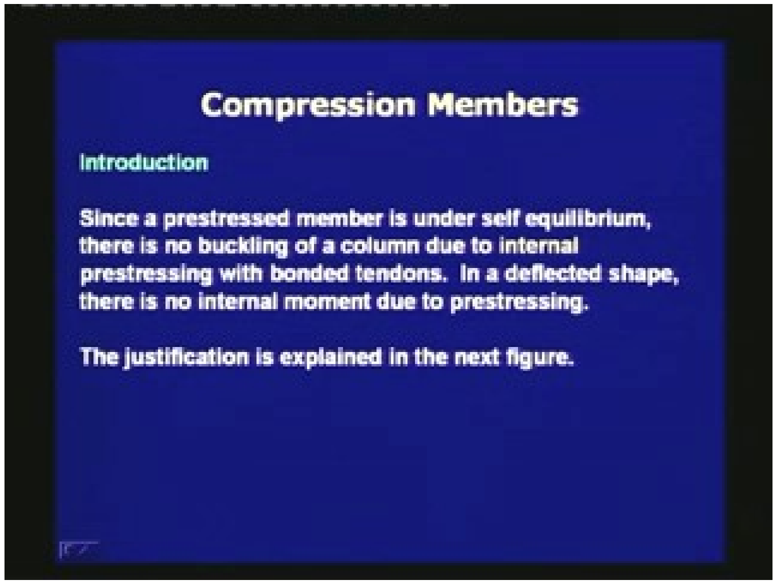


Since a prestressed member is under self-equilibrium, there is no buckling of a column due to internal prestressing with bonded tendons. In a deflected shape, there is no internal moment due to prestressing. The justification is explained in the next figure.

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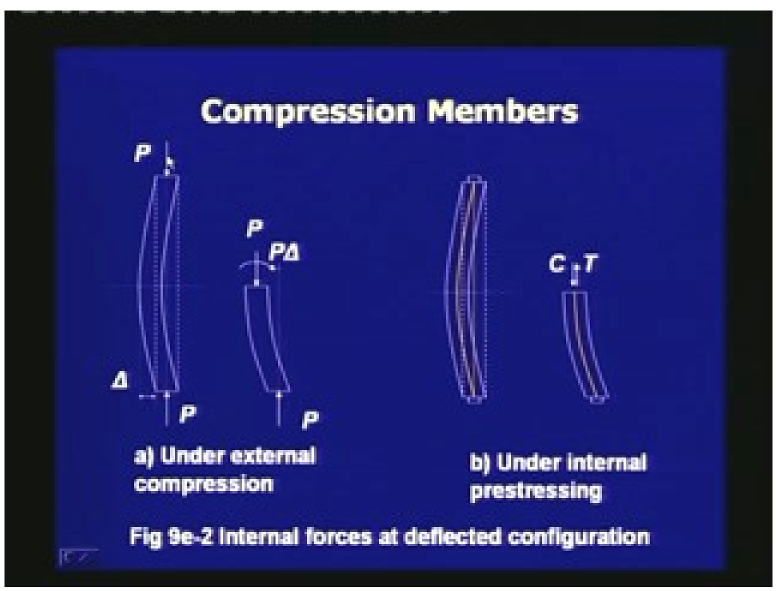

If a column or a pile is subjected to an axial force then, when it gets deflected there will be a moment due to the deflected shape. For a deflection of delta at the middle, the moment at mid-height is represented as P delta. This happens under the axial compression, whereas for an internal prestressing we observe that, even under the deflective shape, the internal prestressing is under equilibrium with the tension in the prestressing tendon and there is no internal moment generated under the deflected shape.

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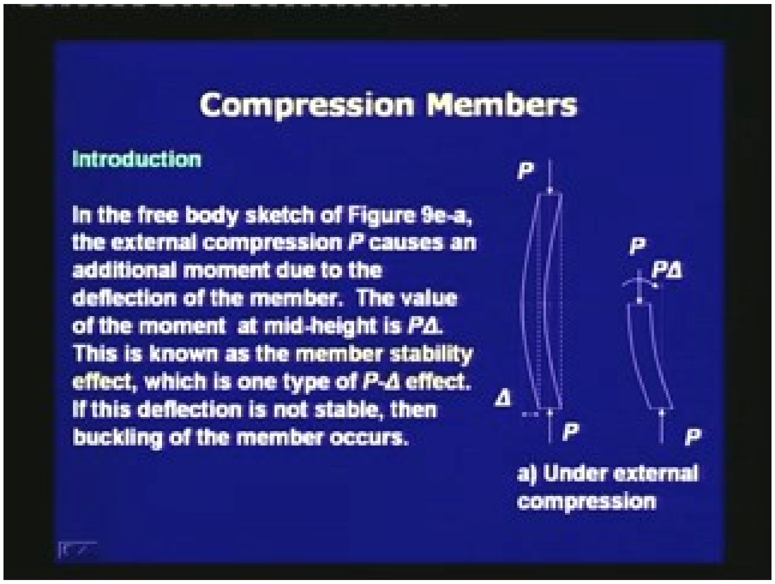


Thus, in the free body sketch of figure 9e-a, the external compression P causes an additional moment due to the deflection of the member. The value of the moment at mid-height is P delta. This is known as the member stability effect which is one type of P delta effect. If this deflection is not stable, then buckling of the member occurs. Thus, the external compression can cause buckling of a member depending upon the slenderness ratio.

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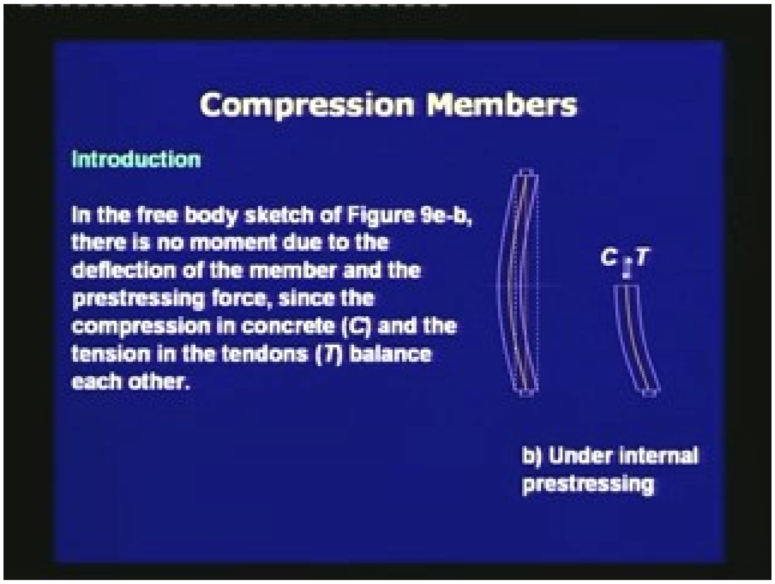


In the second case, for a prestressing in the free body sketch there is no moment due to the deflection of the member under the prestressing force, since the compression in concrete which is C and the tension in the tendons which is T, balance each other.

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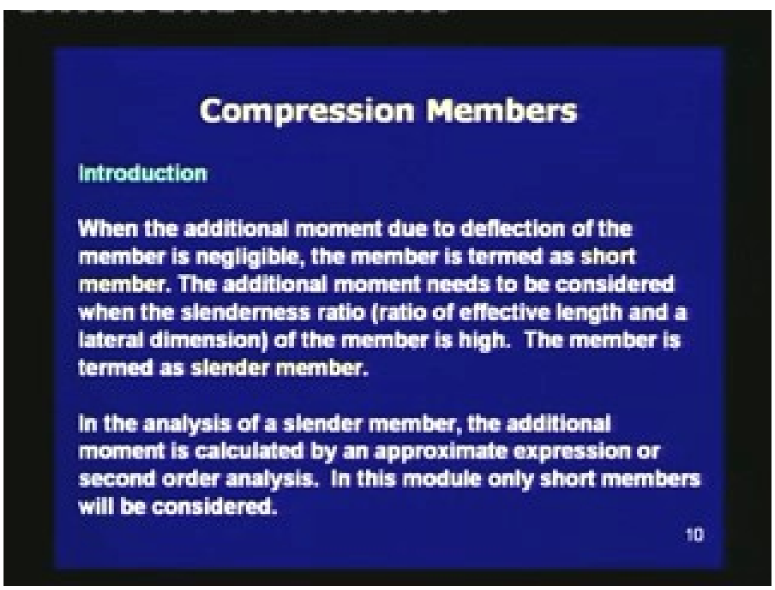


When the additional moment due to deflection of the member is negligible, the member is termed as short member. The additional moment needs to be considered when the slenderness ratio of the member is high. The slenderness ratio is the ratio of the effective length and lateral dimension. These types of members are termed as slender members, where the additional moments need to be considered. In the analysis of a slender member, the additional moment is calculated by an approximate expression or second order analysis. In this module, only short members will be considered. Thus, if we have a slender member depending upon the slenderness ratio greater than a certain value then we need to account for the additional moment generated due to the external compression. This can be estimated by an approximate expression or by second order analysis. In this lecture, we are considering only short members where the additional moment due to the deflection of the member is negligible.

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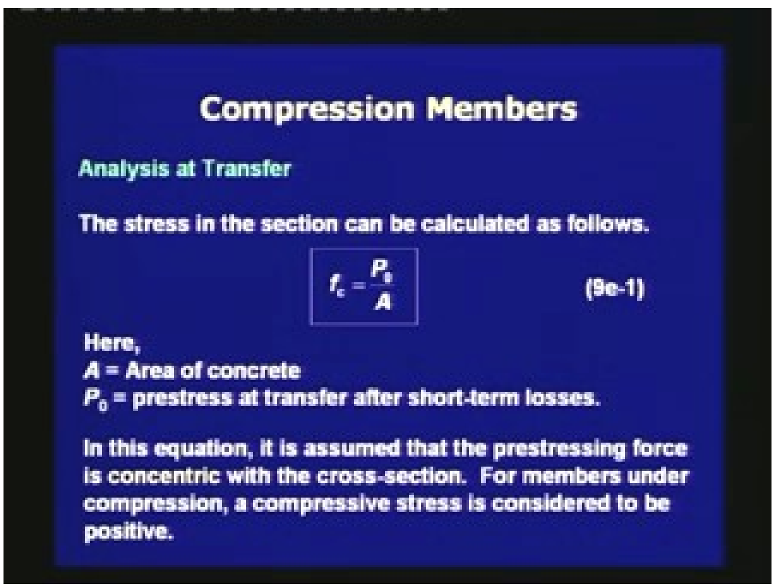


Next, we move on to the analysis of compression members. The stress in the section can be calculated as follows. $f_c$ is equal $P_0$ divided by A. Here, A is the area of concrete and $P_0$ is prestress at transfer after short-term losses. Thus, after the concrete is cast and then the prestressing is transferred to the member, we find that the stress in the member is compressive and it is uniform since the prestressing is concentric. This compression is given as the prestress at transfer, which is $P_0$ divided by the area of the concrete, which is A. In this equation, it is assumed that the prestressing force is concentric with the cross-section. For members under compression, a compressive stress is considered to be positive. In this lecture, we shall consider that the compressive stress is positive and the tensile stress is negative. This is the convention which is used for compression members.

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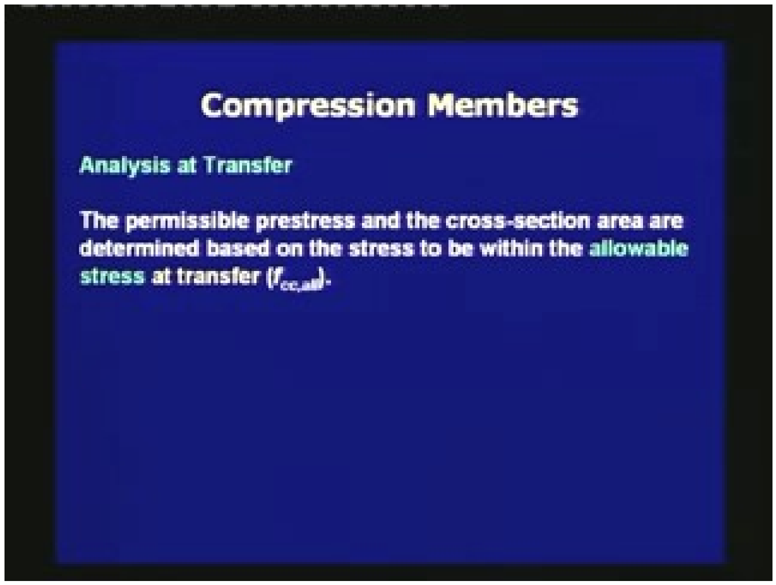


The permissible prestress and the cross-sectional area are determined based on the stress to be within the allowable stress at transfer $f_{cc,allowable}$. Thus, given the compressive stress to be within the allowable value, we can determine $P_0$ and A; a suitable combination of the two variables.

Next, we move on to the analysis at service loads. The analysis is analogous to members under flexure. The stresses in the extreme fibres can be calculated as follows.

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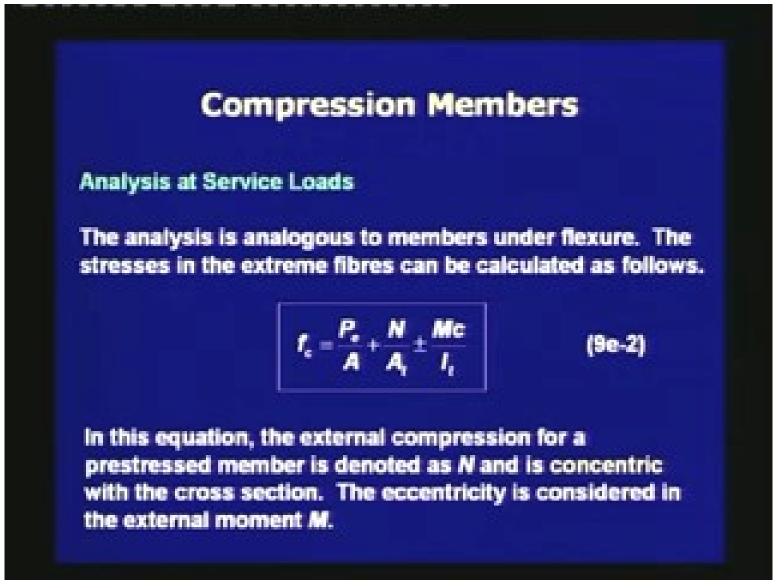


$f_c$ is equal to $P_e$ divided by A plus N divided by $A_t$ plus minus M times C divided by $I_t$. The first term is due to the prestressing force, which is now the effective prestress under service loads; the second term is due to the external compression which is represented as N and the third term is due to the external moment which is represented as M.

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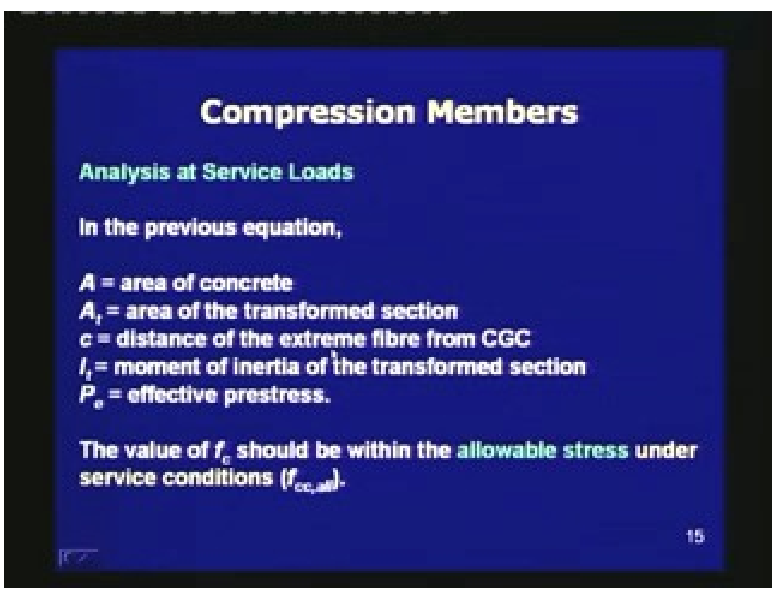

In this equation, the external compression for a prestressed member is denoted as N and is concentric with the cross-section. The eccentricity is considered in the external moment M. A is the area of concrete, $A_t$ is the area of transformed section; c is the distance of the extreme fibre from the centroid of the section which is CGC; $I_t$ is the moment of inertia of the transformed section; $P_e$ is the effective prestress. The value of the $f_c$ should be within the allowable stress under service conditions.

Next we move on to the analysis at ultimate.

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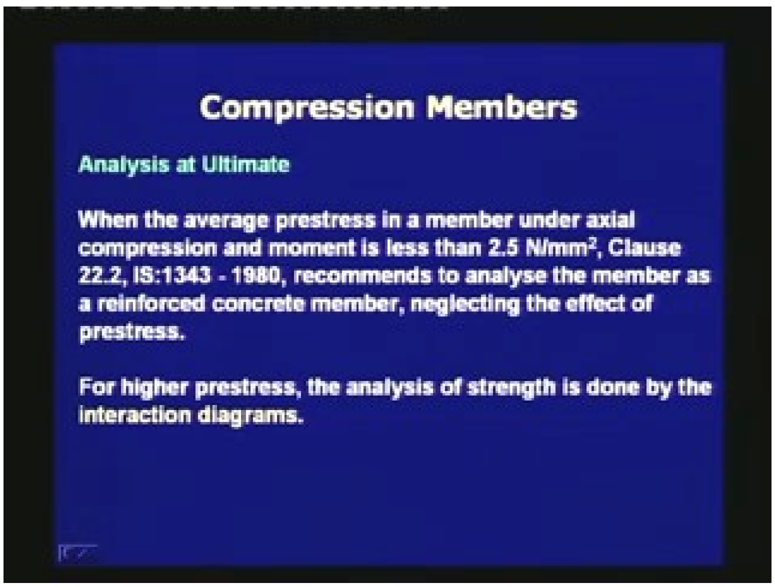


When the average prestress in a member under axial compression and moment is less than 2.5 Newton per millimeter square, clause 22.2 of IS: 1343 - 1980 recommends analyses of the member as a reinforced concrete member, neglecting the effect of prestress. Thus, if the prestress is small then the code allows us to analyze the prestress member as a reinforced concrete member, with just the amount of the external load acting on it. If the amount of prestress is greater than 2.5 Newton per millimeter square, then we analyze the prestress member including the prestress and with the help of interaction diagrams for the ultimate limit state. At the ultimate limit state, an interaction diagram relates the axial load capacity, which is represented as $N_{uR}$ and the moment capacity which is represented as $M_{uR,}$

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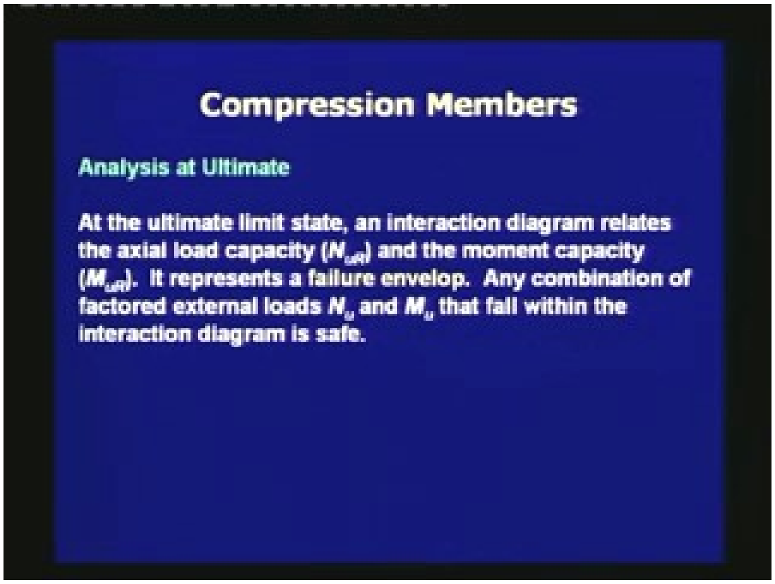


The R stands for resistance and the u stands for ultimate. It represents a failure envelop. Any combination of factored external loads $N_u$ and $M_u$ that fall within the interaction diagram is safe. Let us understand this by the help of a sketch. A typical interaction diagram is shown below.

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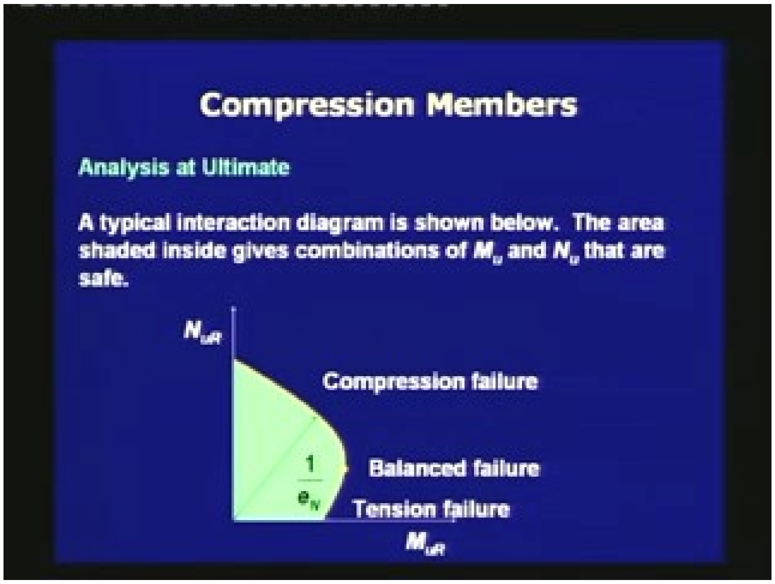

The shaded area inside gives combinations of external loads $M_u$ and $N_{u,}$ that is safe. Thus, this boundary is a failure envelope and any combination of the external loads which fall within this boundary is safe and the combination which falls outside this boundary is unsafe. In this boundary, we have two types of failure conditions. One is the compression failure and the other is the tension failure. The transition between them is called the balanced failure. The radial line in the previous sketch represents the load path. Usually, the external loads increase proportionally. At any load stage, M and N are related as follows: M is equal to N times $e_N$

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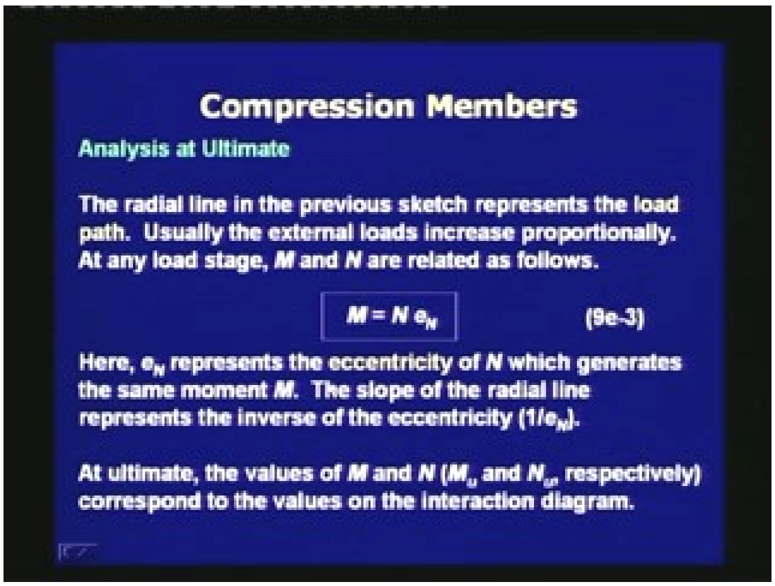


Here, $e_N$ represents the eccentricity of N which generates the same moment M. The slope of the radial line represents the inverse of the eccentricity, which is 1 by $e_{N.}$ Thus, in most of the cases, the axial load and the moment, when they increase they increase proportionally. Hence, the load path is a straight line which passes through the origin and move towards to the failure envelope. The slope of the line is given as 1 by $e_N$ where $e_N$ is the ratio of the moment M and the axial force N. At ultimate, the values of M and N, which are represented as $M_u$ and $N_u$ respectively, correspond to the values on the interaction diagram. As the load increases along the load path and finally, when at ultimate, the load state falls on the interaction diagram, then a failure occurs.

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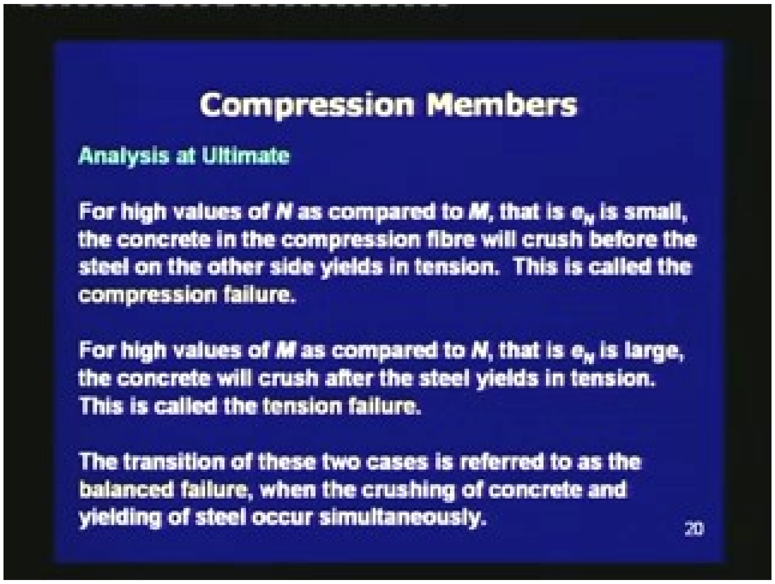


For high values of N as compared to M, that is $e_N$ is small, the concrete in the compression fibre will crush before the steel on the other side yields in tension. This is called the compression failure. For high values of M as compared to N, that is $e_N$ is large, the concrete will crush after the steel yields in tension. This is called the tension failure. The transition of these two cases is referred to as the balanced failure, when the crushing of concrete and yielding of steel occur simultaneously. Thus, the significance of compression failure is that, the concrete is crushing before the steel has the chance to yield and the tension failure means, the concrete is crushing after the steel has yielded. The transition between them is called the balanced failure but the crushing of concrete and yielding of steel occur simultaneously.

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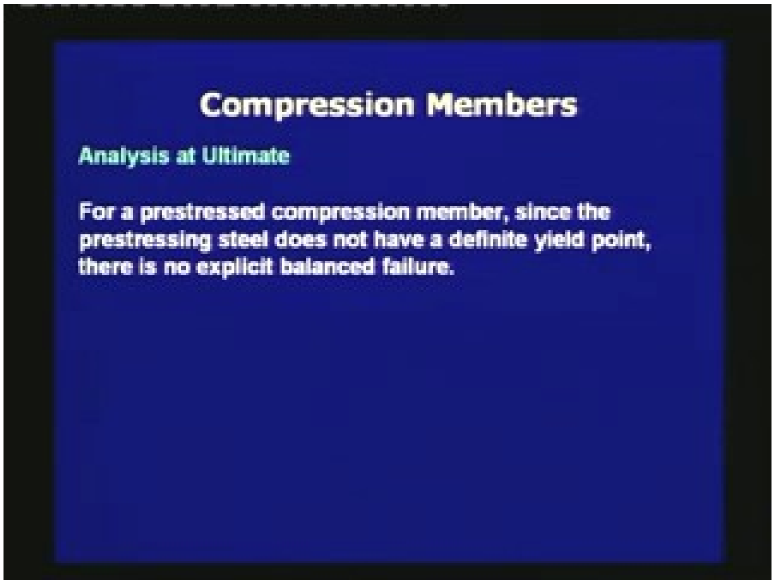


For a prestressed compression member, since the prestressing steel does not have a definite yield point, there is no explicit balanced failure. The steel for reinforced concrete member tend to have a sharp yield point, such as mild steel. Unlike that , the prestressing steel does not have a sharp yield point. Hence, the definition of a balanced failure is not explicit for a prestressed compression member, as compared to our reinforced concrete member under compression.

Next, we shall learn about the development of interaction diagram. An interaction diagram can be developed from the first principles using the non-linear stress-strain curves of concrete under compression and steel under tension.

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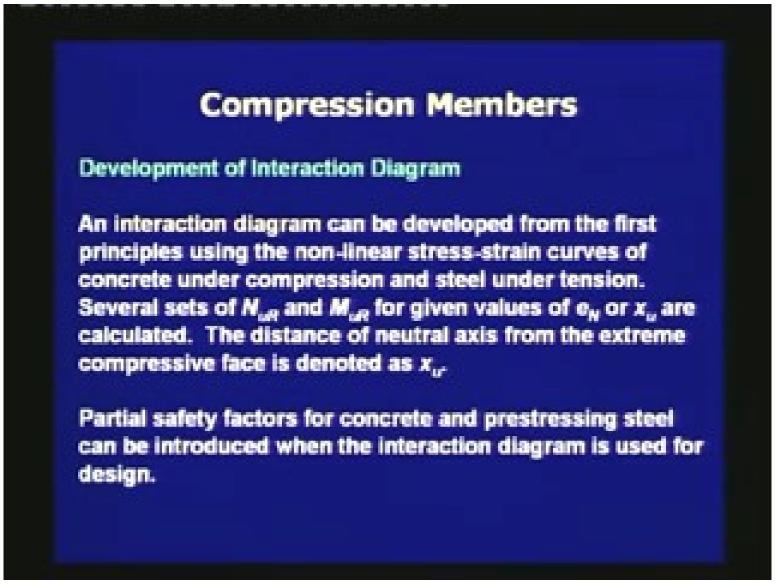


Several sets of $N_{uR}$ and $M_{uR}$ for given values of $e_N$ or $x_u$ are calculated. The distance of neutral axis from the extreme compressive face is denoted as $x_u$. Partial safety factors for concrete and prestressing steel can be introduced, when the interaction diagram is used for design. Thus, the interaction diagram is developed by calculating values of the capacities $N_{uR}$ and $M_{uR}$ for a given eccentricity $e_N$. Also, you can do the same calculation, for a given value of $x_u$, where $x_u$ is the depth of the neutral axis from the extreme compressive phase.

Once we calculate a set of $N_{uR}$ and $M_{uR}$ values, we can join those points to get the interaction diagram for that particular member. If we use partial safety factors, then this curve can be used for design. Here, the procedure is illustrated for a rectangular section with prestressed tendons placed at two opposite faces symmetrically and without non-prestressed reinforcement.

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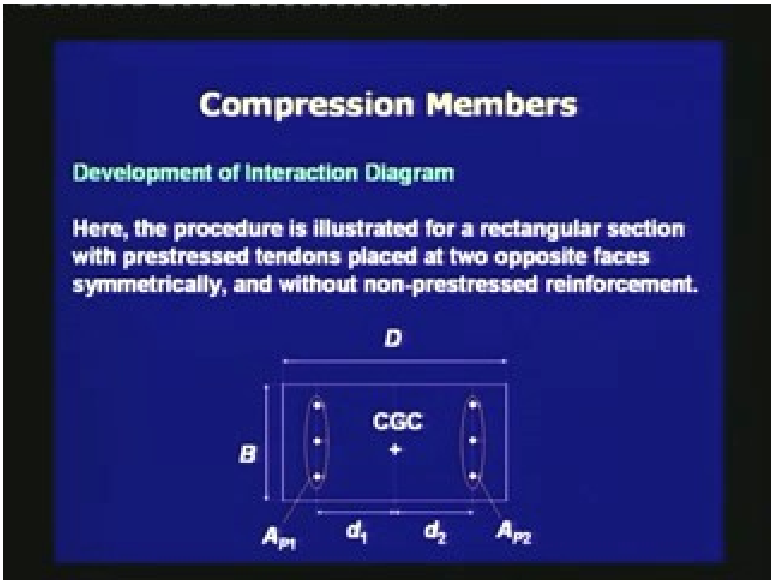


Thus, we shall study the development of interaction diagram for a rectangular section, whose lateral dimension about the direction of bending is the depth D and whose lateral dimension transfers to the direction of bending is B. The prestressing tendons are placed symmetrically about the centroid. The group on the left is denoted as $A_{p1}$ and the area on the right is denoted as $A_{p2}$; $d_1$ and $d_2$ are the distances of $A_{p1}$ and $A_{p2}$ respectively from the CGC.

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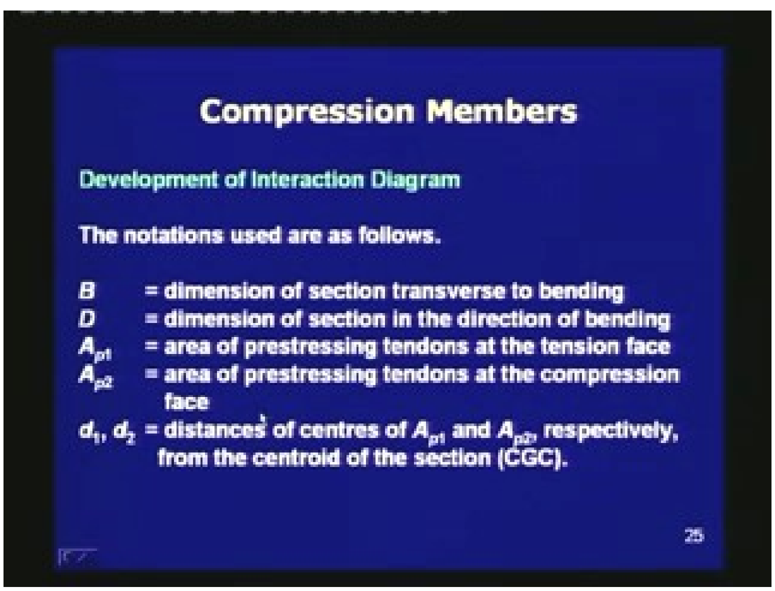

The notations thus used are as follows: B is the dimension of the section transverse to bending; D is the dimension of section in the direction of bending; $A_{p1}$ is the area of prestressing tendons at the tension face; $A_{p2}$ is the area of prestressing tendons at the compression face; $d_1$ and $d_2$ distances of centers of $A_{p1}$ and $A_{p2}$ respectively, from the centroid of the section which is CGC.

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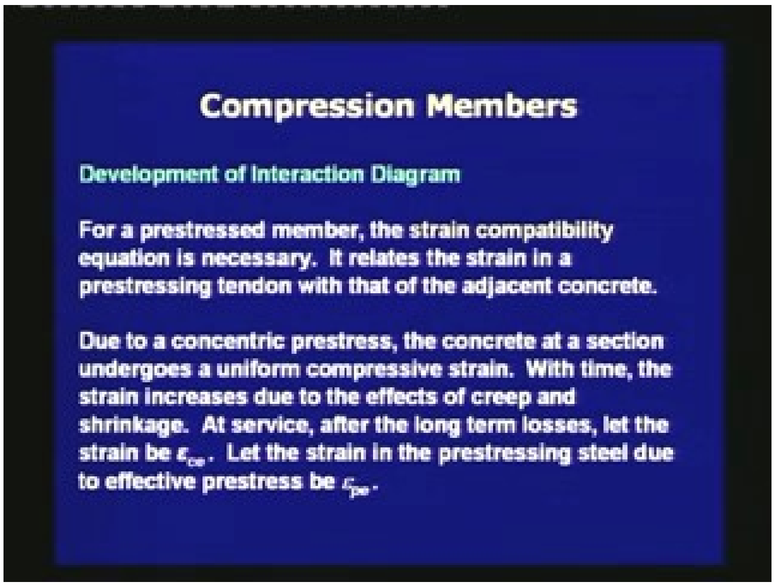


For a prestressed member, the strain compatibility equation is necessary. It relates the strain in a prestressing tendon with that of the adjacent concrete. Due to a concentric prestress, the concrete at a section undergoes a uniform compressive strain. With time, the strain increases due to the effects of creep and shrinkage. At service, after the long-term losses, let the strain in concrete be $\text{epsilon}_{ce}$. Let the strain in the prestressing steel due to effective prestress be $\text{epsilon}_{pe}$. Thus, just due to prestressing, the section undergoes a uniform compression which increases with time due to the losses and after the losses, let the compressive strain in the concrete be $\text{epsilon}_{ce}$ and the tensile strain in the prestressing tendons be $\text{epsilon}_{pe}$. This is the strain diagram that we are observing, due to the prestressing force alone under service conditions. The strain compatibility equation for the prestressed tendons is then given as follows.

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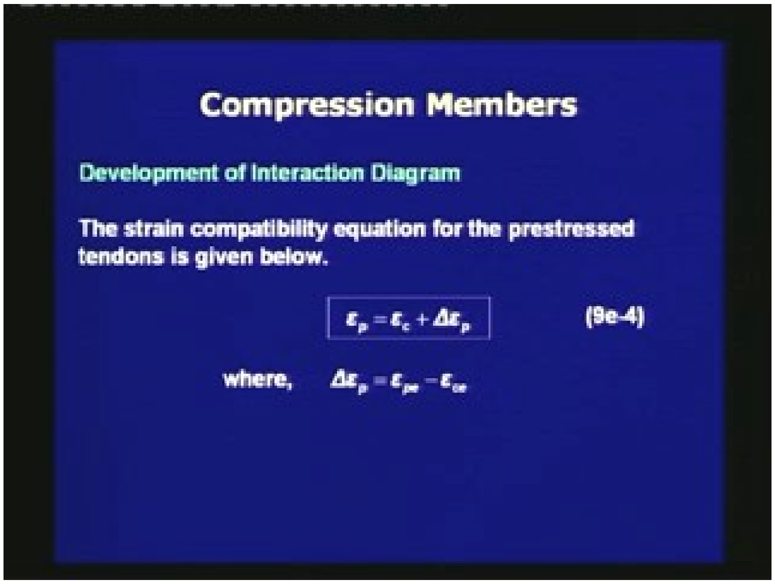


Epsilon$_{pe}$ is equal to epsilon$_c$ plus delta epsilon$_{p.}$ Thus, under any load the strain in the prestressing steel epsilon$_p$ is equal to the summation of strain in the concrete, which is epsilon$_c$ at that level of prestressing steel plus the strain differential which we denote as delta epsilon$_{p.}$ Delta epsilon$_p$ is calculated as epsilon$_{pe}$ minus epsilon$_{ce.}$ Thus we calculate, what is the uniform compressive strain in concrete under service conditions due to the prestressing force? What is the tensile strain in the tendons under service conditions? We take their difference, get this strain differential and we add this strain differential to the strain in concrete to get the strain in the prestressing steel at that level. This is called the strain compatibility equation, which relates the strain in the concrete with the strain in the prestressing steel which is lying at the same level. The following stress-strain curve for concrete under compression is used. It is parabolic and then, it is constant up to strain of 0.0035.

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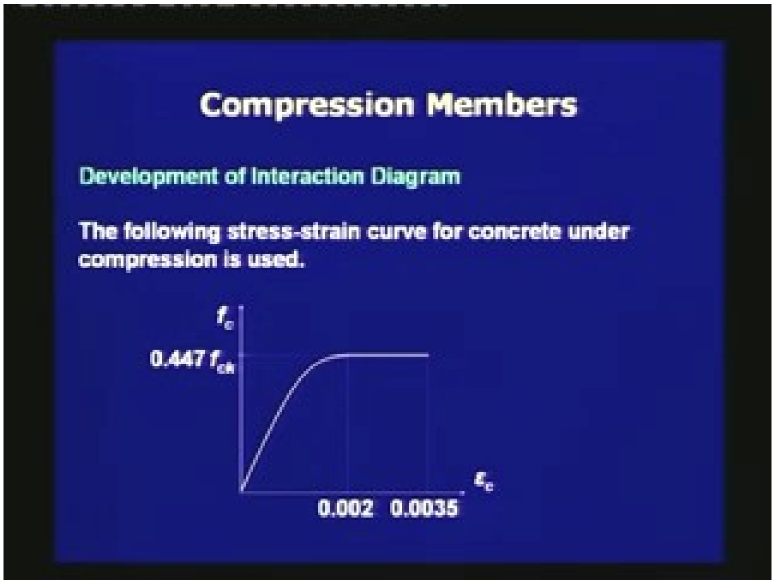


The stress-strain curve for the prestressed tendon under tension can be expressed in the following terms.

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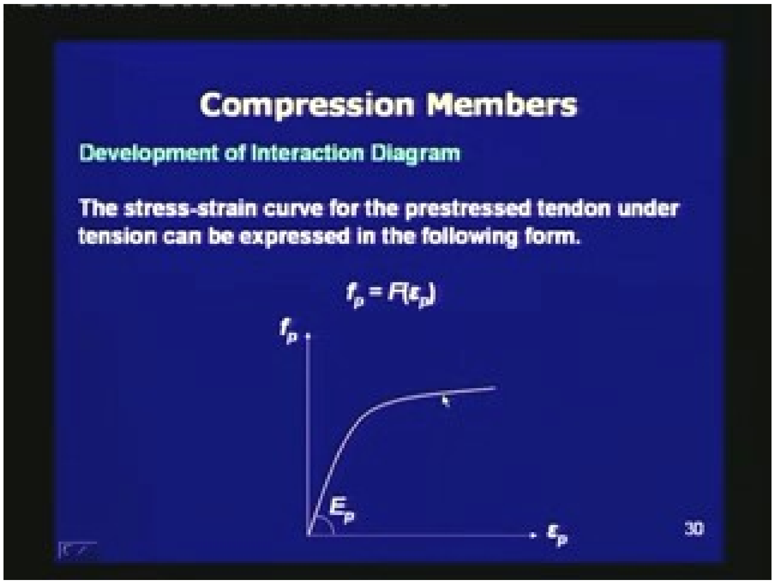


This curve is not elasto-plastic behaviour. It has a gradual transition from the elastic behaviour with increasing plastic strain. And this curve is symbolically represented as $f_p$

is equal to a function of epsilon$_p$. The calculation $N_{uR}$ and $M_{uR}$ for typical cases of $e_N$ or $x_u$ are illustrated.

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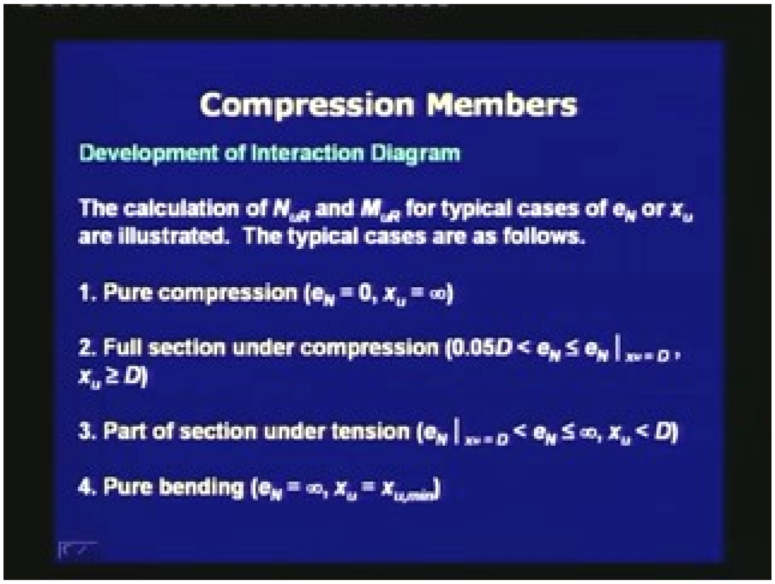


The typical cases are as follows. First, the case of pure compression where, $e_N$ is equal to 0 and $x_u$ is equal to infinity; second, full section under compression where, $e_N$ can vary between 0.05D and less than $e_N$ corresponding to $x_u$ is equal to D and $x_u$ is greater or equal to D; third, is part of section which is under tension, here, $e_N$ is greater than the value corresponding to $x_u$ equal to D, but it is less than infinite and $x_u$ is less than D. Finally, we have pure bending where, $e_N$ is equal to infinite and $x_u$ is equal to $x_{u,min}$, which is the minimum value of $x_u$. We shall study these cases as we move on.

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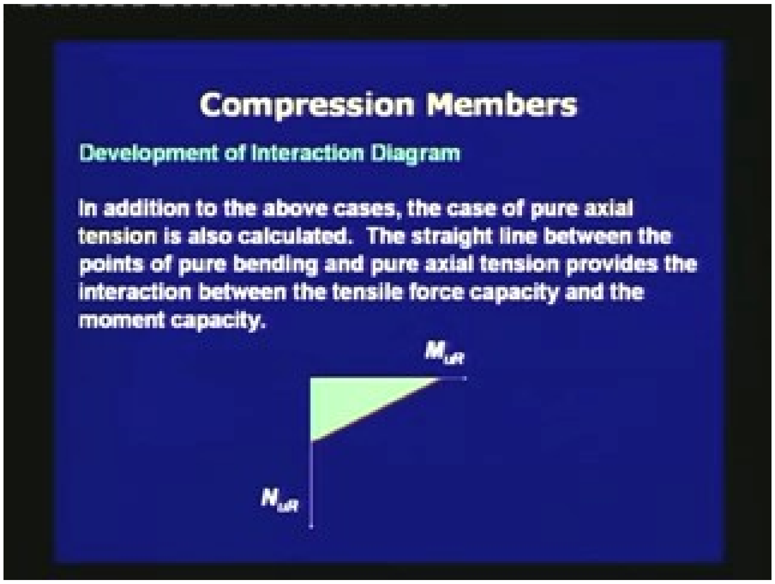


In addition to the above cases, the case of pure axial tension is also calculated. The straight line between the points of pure bending and pure axial tension provides the interaction between the tensile force capacity and the moment capacity. Thus, this part of the interaction diagram is in the lower quadrant, where the tensile axial force is given by a negative value. We calculate the moment capacity of the section under pure bending. We calculate the tensile force capacity of the member and then we plot these points, join them by a straight line; like that we get the interaction diagram for the quadrant which is for the tensile force applied on the section. Any combination of the external loads line within the green region is safe. Next, we move on to the development of interaction diagram for each of the typical cases separately.

First is the pure compression. Under pure compression, the strain profile is constant and at ultimate the strain is given as minus 0.002.

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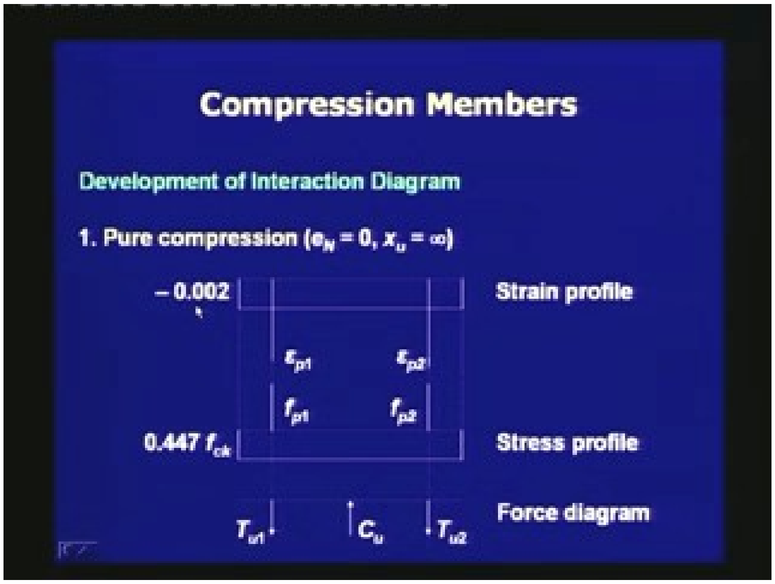


The eccentricity is 0 because there is no moment, whereas the depth of the neutral axes from the compression phase is infinite. Now the stress profile is, we have a stress of $0.447f_{ck}$ uniform along the concrete and the stress in the prestressing tendons are denoted as $f_{p1}$ and $f_{p2,}$ which we shall calculate from the strains $epsilon_{p1}$ and $epsilon_{p2.}$ From the strain profile, we go to the stress profile and then to the force diagram, where the compression in concrete is denoted as $C_u$ and the tension in the prestressing tendons is denoted as $T_{u1}$ and $T_{u2}$ for $A_{p1}$ and $A_{p2}$ respectively.

The forces are as follows. $C_u$ is equal to 0.447 $f_{ck}$ times $A_g$ minus $A_p$; $T_{u1}$ is equal to $T_{u2}$ is equal to $A_{p1}$ times $f_{p1}$ is equal to $A_{p1}$ times $E_p$ times minus 0.002 plus delta $epsilon_p$. Thus, the term in the bracket represents the strain compatibility condition that we have used to calculate the strain in the prestressing steel from the strain in the concrete, which is minus 0.002. The steel is in the elastic range and hence, we have used the hooks law which is the stress is equal to the modulus times the strain in the steel.

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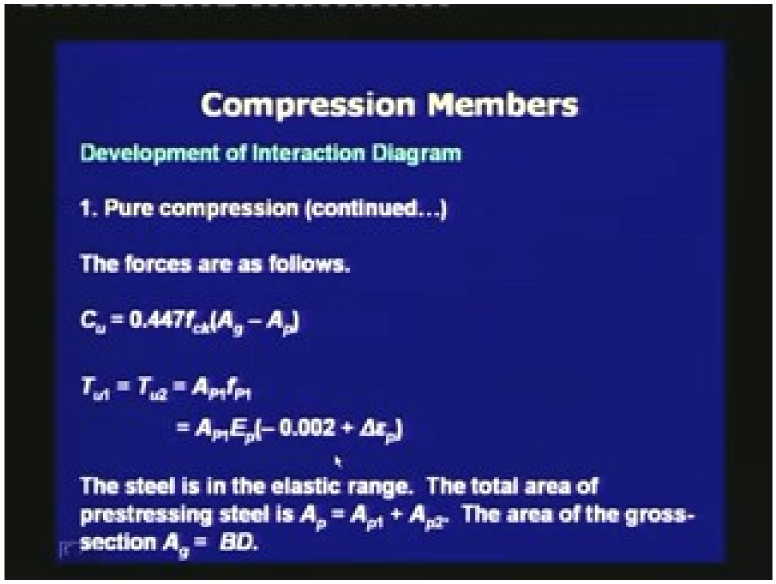


The total area of prestressing steel is denoted as $A_p$ which is equal to $A_{p1}$ plus $A_{p2}$. And the area of the gross-section is denoted as $A_g$ is equal to BD. Thus, we can calculate the values of $C_u$, $T_{u1}$ and $T_{u2}$ given the geometric variables of the section and the material properties. Next, we are calculating the axial load capacity. The moment capacity $M_{uR}$ for this case is 0; there is no moment, whereas axial load capacity is given as $C_u$ minus $T_{u1}$ minus $T_{u2}$. Again remember that, a compression is considered to be positive and a tension is considered to be negative for compression members. Once we substitute the values of $C_u$, $T_{u1}$ and $T_{u2}$, we get an expression of $N_{uR}$.

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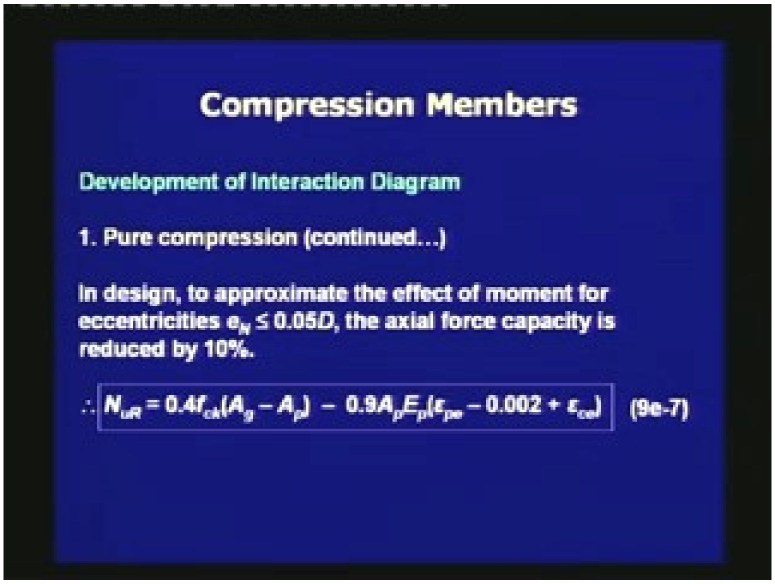


In design, to approximate the effect of moment for eccentricities $e_N$ less than or equal to 0.05D, the axial force capacity is reduced by 10%. In design if a member has small moment applied on the section, then we try to by-pass using the interaction diagram, by considering a reduced axial force capacity and neglecting the effect of moment. This is allowed when the eccentricity is less than 5% of the dimension D and then the axial force capacity is reduced by 10% to get an expression of $N_{uR}$. In that case, $N_{uR}$ is given as 0.4 $f_{ck}$ times $A_g$ minus $A_p$ minus 0.9 $A_p$ times $E_P$ times $E_p$ minus 0.002 plus epsilon$_{ce.}$

Next, we move onto the case of a full section under compression under simultaneous axial load and moment.

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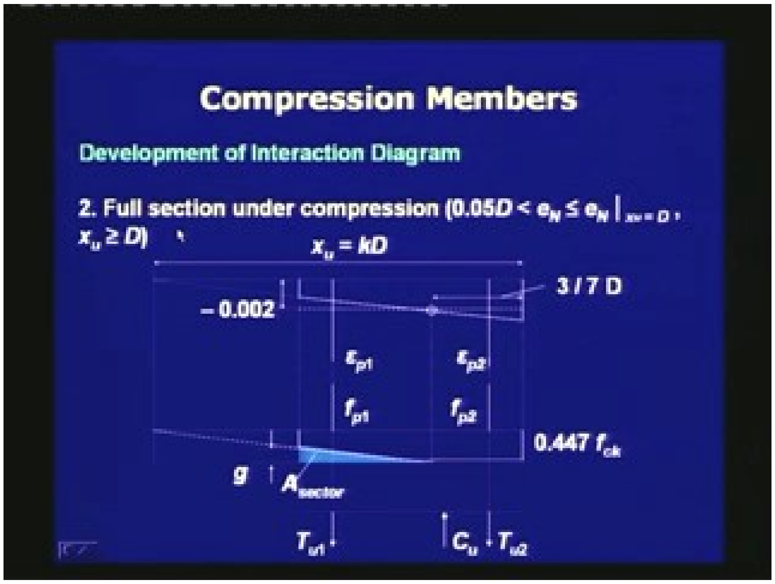


Note that here, this is applied when the eccentricity is larger than 0.05t but smaller than the case when $x_u$ is equal to D. Thus, whenever $x_u$ is equal to or greater than D, the full section is under compression. Let us try to understand the strain diagram. The depth of the neutral axis is outside the section that means the full section is under the compression. It is assumed that, the strain diagram pivots about a point which is at distance 3 by 7 D from the extreme compression face where the strain is minus 0.002. This is an important assumption to get the strain diagram for a given value of $x_u$ is equal to k times D. Note that, here, k is greater than 1. Now, given the strain diagram we calculate the stress diagram. Here, the parabolic part of the stress diagram in concrete goes outside the section. Hence, the computations are a bit more involved. We need to calculate the area under this curve and for that we first calculate the area under the rectangular region from which we subtract the area of this shaded part which we denote as A sector. We also calculate the tension in the prestressing tendons from the strains.

The limiting case for full section under compression corresponds to $x_u$ is equal to D, when the neutral axis lies at the left edge of the section.

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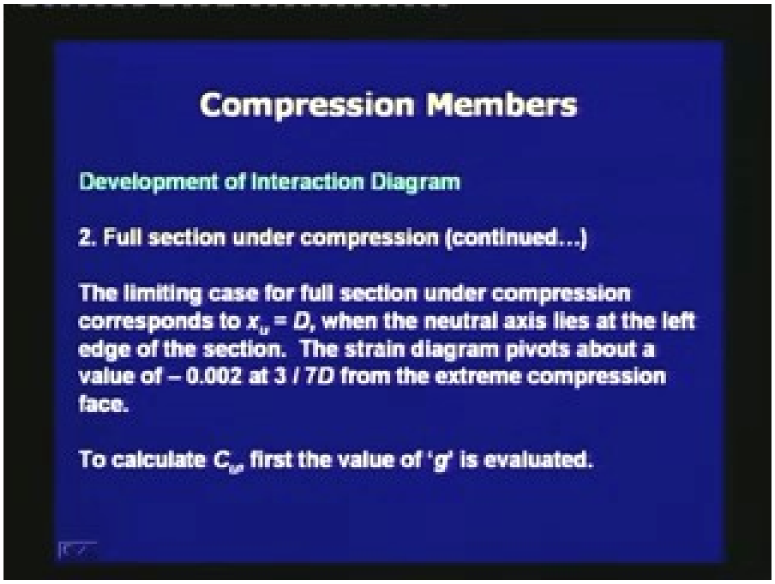


The strain diagram pivots about a value of minus 0.002 at 3 by 7D from the extreme compression face. To calculate $C_{u,}$ first, the value of g is evaluated. This g is the intercept of the parabolic curve from the value of $0.447f_{ck.}$ Based on the second order parabolic curve for concrete under compression, the expression of the g is as follows.

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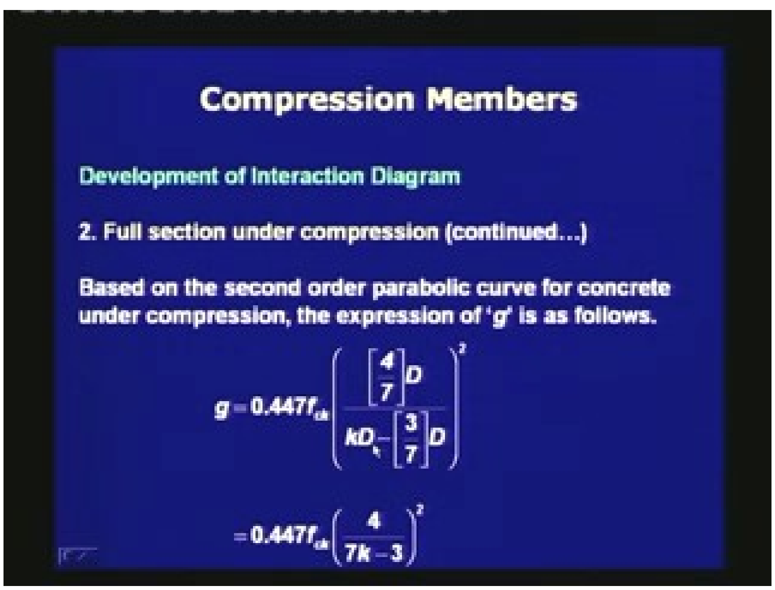

This expression can be derived by assuming a second order parabola; g is equal to $0.447f_{ck}$ times 4 by 7D divided by $x_u$, which is equal to kD minus 3 by 7D. This whole term is within bracket and raised to the power 2. This can be simplified to g is equal to $0.447f_{ck}$ times 4 divided by 7k minus 3 whole square. The area of the sector is given as follows. This is the area of a parabolic section and it is given as one-third times g times the distance to the apex from the distance to the base which is 4 by 7D.

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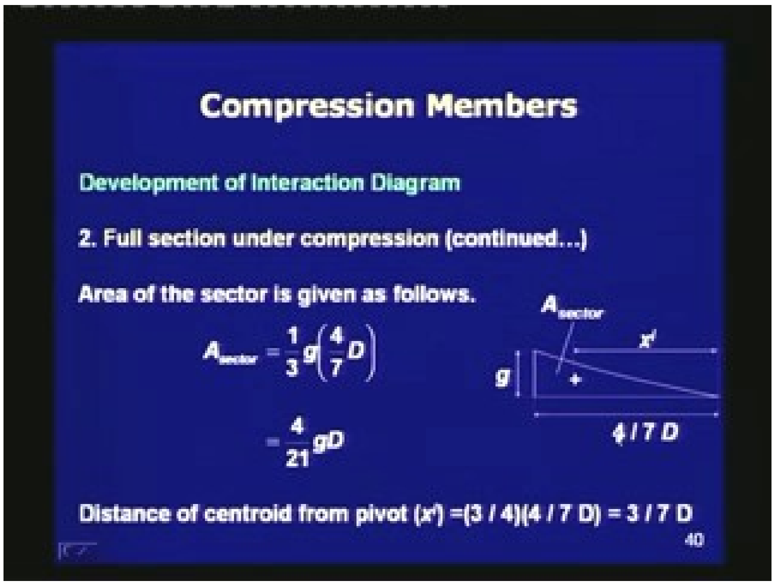


Thus, the area of the sector is equal to 4 by 21 g times D. Once we know g, we can calculate the area of the sector. The distance of the centroid of this area from this apex is denoted as x prime and this is equal to three-fourth of 4 by 7D, which is equal to 3 by 7D.

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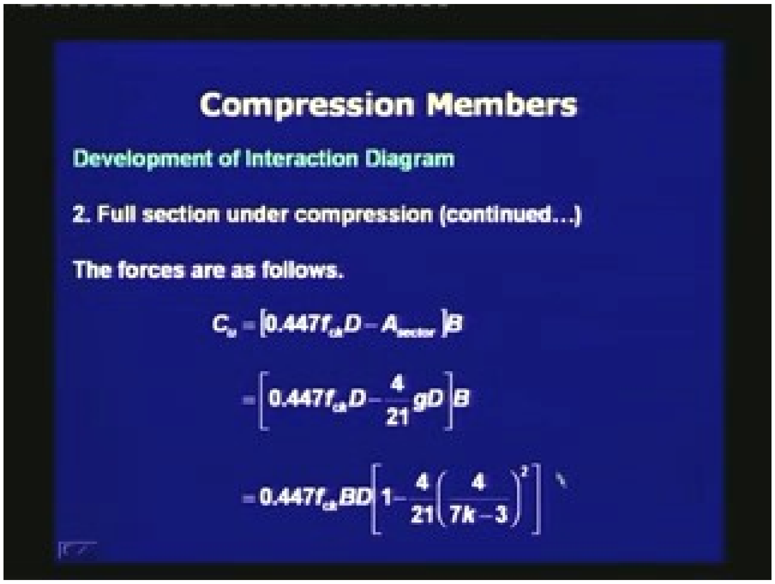


Then $C_u$ is given as the area of the rectangular section $0.447f_{ck}$ times D minus area of the sector A sector whole times B. Once we substitute the value of the area of the sector, we get an expression of $C_u$ for the given value of k.

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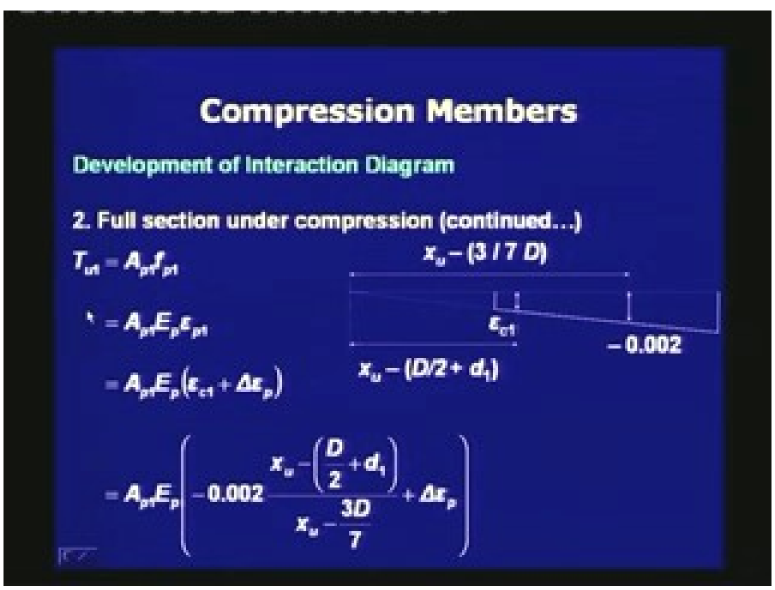


Next, we are finding out $T_{u1,}$ which is $A_{p1}$ times $f_{p1}$ and again here $T_{u1}$ is within the elastic limit and we can substitute the value of the strain in the concrete by this expression, which we can derive from the strain diagram. That means, once we know the strain in the pivot point and we know that the strain at the neutral axis is 0 from the similarity of the triangles, we can find out what is the value of the epsilon$_{C1}$ at the distance of $x_u$ minus D by 2 plus $d_1$.Thus, this expression has come from the similarity of the triangles. To the strain of the concrete, we add the strain differential to get the strain in the prestressing steel. From that, we calculate the stress and then we calculate the force in the prestressing steel $A_{p1}$.

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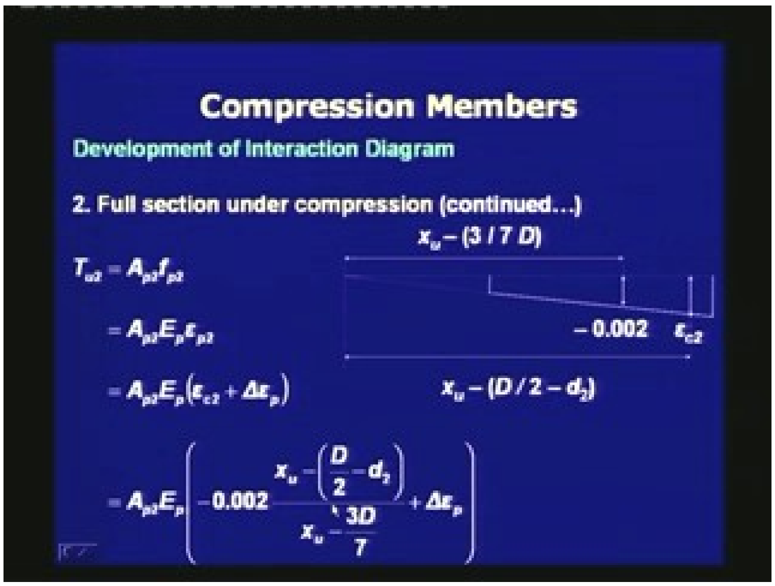


Similarly, we can calculate $T_{u2,}$ which is from the strain in the concrete at the level of $A_{p2}$ and here also, we find that the strain at epsilon$_{C2}$ can be determined from the similarity of the triangles. That means, based on this value of minus 0.002 at the pivot point, we can calculate what the strain at this level is. Again this expression is derived from this similarity of the triangles. Thus, after these calculations the moment and the axial force capacities are as follows.

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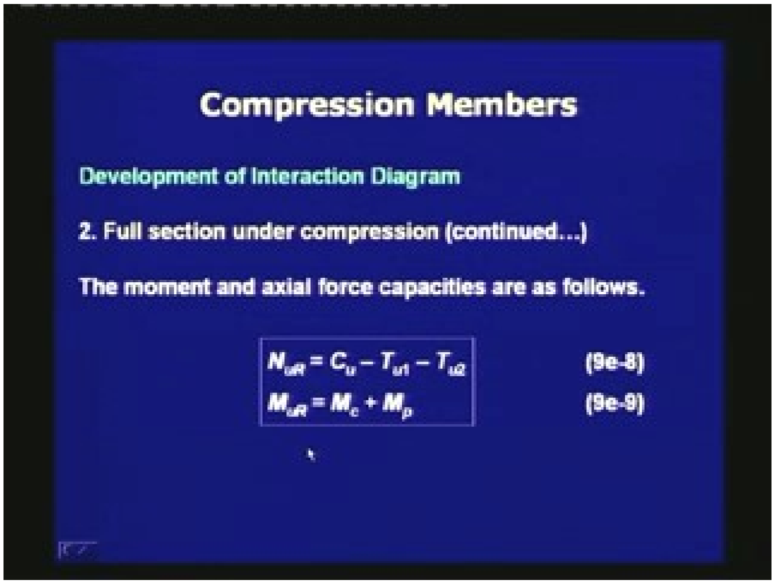


$N_{uR}$ is equal to $C_u$ minus $T_{u1}$ minus $T_{u2}$, and $M_{uR}$ is equal to plus $M_c$ plus $M_p$; the $M_c$ is the moment due to the force in the concrete and $M_p$ is the moment due to the forces in the prestressing tendons. The expression of $M_c$ and $M_p$ about the centroid of the section are given below.

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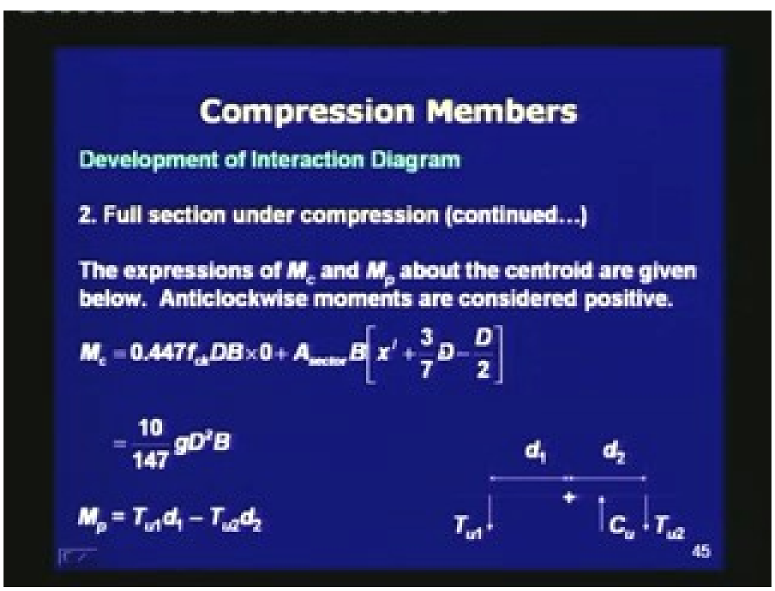

Anti-clock wise moments are considered to be positive in this derivation. $M_c$ is equal to 0.447 $f_{ck}$ times DB times 0. This is the area of the rectangle, whose centroid lies at the centroid of the section and hence this rectangular stress block does not create any moment about the centroid. The $M_c$ is only due to the area of the sector, A sector, which is given as $A_{sector}$ times B within bracket x prime plus 3 by 7 D minus D by 2. That is the distance of the centroid of the sector from the centroid of the cross-section. Thus, this term on the right hand side gives the value of $M_{c,}$ which, when simplified is equal to 10 divided by 147 gD square times B. $M_p$ is given by, taking the moments of $T_{u1}$ and $T_{u2}$ based on the centroid and since, these are the distances $d_1$ and $d_{2,}$ $M_p$ is equal to $T_{u1}$ times $d_1$ minus $T_{u2}$ times $d_{2.}$

Next, we move to the case where, part of the section is under tension. Note that, in the strain diagram, from the left hand side, we have some tension. $e_N$ is less than the value corresponding to $x_u$ is equal to D whereas, $e_N$ is less than infinite, which is for the case of pure bending.

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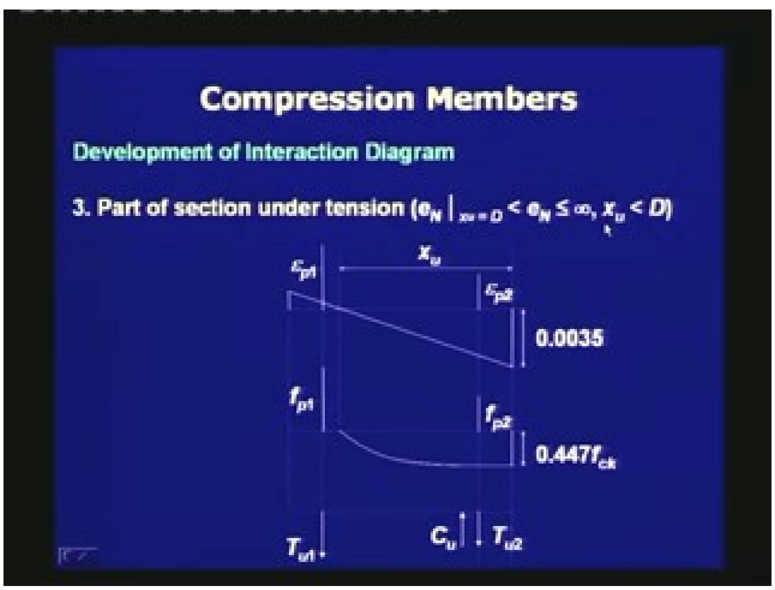


$x_u$ which is depth the of the neutral axis, is lower than D. Once $x_u$ is lower than D, we have some tension on the left side. From the strain diagram, we get the stress diagram. Note that, the extreme compression is now 0.0035 and the stress diagram is similar to a reinforced concrete section. That means once the section cracks, the analysis is similar to a section under flexure. The forces are as follows. $C_u$ is equal to 0.36 $f_{ck}$ times $x_u$ times B, which is the resultant of the stress block in concrete; $T_{u1}$ as before, is given as $A_{p1}$ times $f_{p1.}$ Now here, $f_{p1}$ should be calculated from the stress in curve. Now, the concrete around $A_{p1}$ is under tension and the prestressing tendon may go to the yield region. Hence, we should not use the elastic values without checking whether the steel is yielding or not. For $T_{u2}$ the steel need not yield and we can use the elastic relationship.

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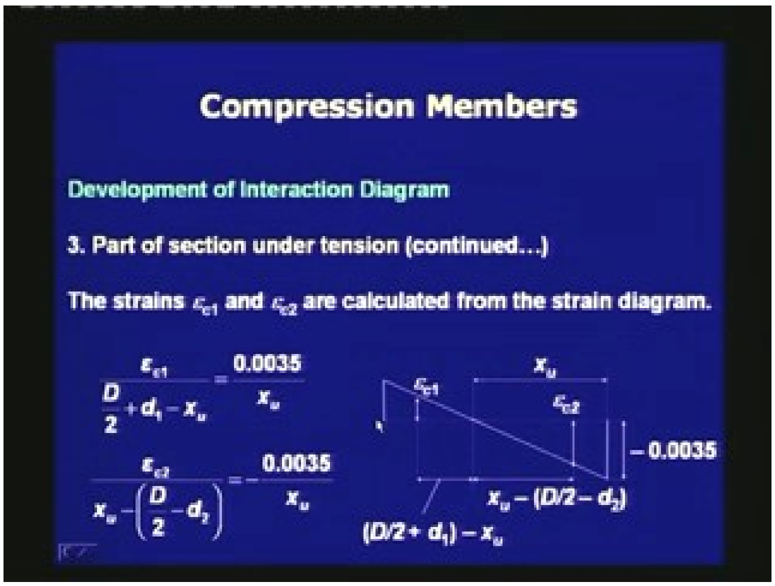


The strains $epsilon_{c1}$ and $epsilon_{c2}$ are calculated from the strain diagram as follows. These expressions are based on the linearity of the strain diagram. From the similarity of triangles given this extreme strain minus 0.0035 and given the location of the 0 strain, which is $x_{u,}$ we can find out what epsilon $c_1$ is. Similarly, we can find out what epsilon $c_2$ is, knowing these distances. Thus given the distances of $A_{p1}$ and $A_{p2}$ from the neutral axis, we can find out what the value of epsilon $c_1$ and epsilon $c_2$ is. For $epsilon_{c1}$ and $epsilon_{c2}$ we calculate $epsilon_{p1}$ and $epsilon_{p2}$ from which we calculate $f_{p1}$ and $f_{p2}$ and then we get the forces. Finally, the moment and the axial force are given by these equations, which are same as we had seen before that, $N_{uR}$ is $C_u$ minus $T_{u1}$ minus $T_{u2}$ and $M_{uR}$ is equal to $M_c$ plus $M_{p.}$ The expression of $M_c$ about the centroid is given by the stress block times the lever arm, which is D by 2 minus 0.42 $x_u$ and the expression of $M_p$ is same as before.

Finally, we are coming to the case of pure bending, where the eccentricity is now infinite, because there is no axial force coming and the depth of the neutral axis has the minimum value for all these cases.

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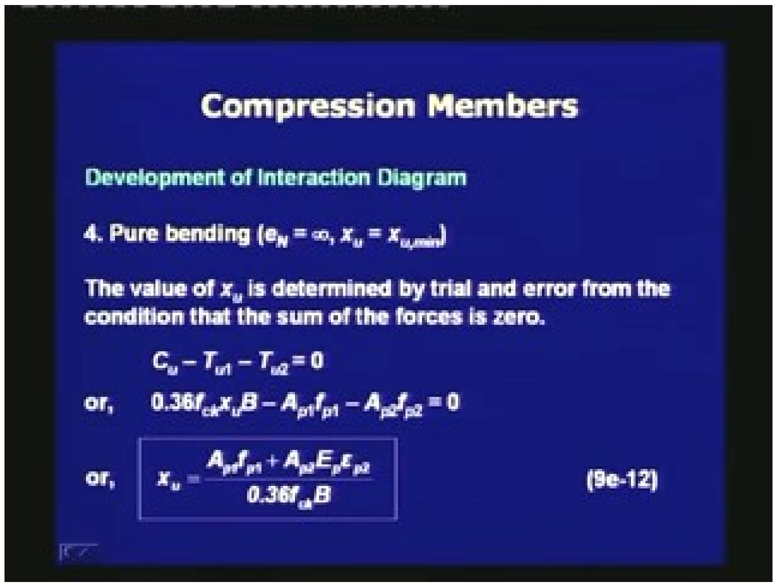


The value of $x_u$ is determined by trial and error from the condition that the sum of the forces is 0. Thus, we write the generic expression $C_u$ minus $T_{u1}$ minus $T_{u2}$ is equal to 0. You substitute the values of $C_u$ and then from this, we get the expression of $x_{u.}$ The stress epsilon$_{p1}$ and epsilon$_{p2}$ are calculated from the strain compatibility equations.

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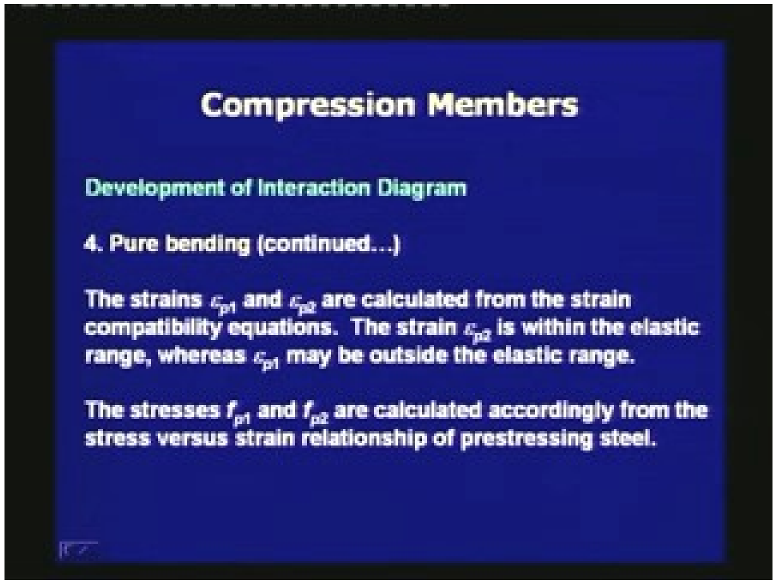


The strain eplison$_{p2}$ is within the elastic range, where as epsilon$_{p1}$ may be outside the elastic range. The stresses $f_{p1}$ $f_{p2}$ are calculated accordingly from the stress versus strain relationship of prestressing steel.

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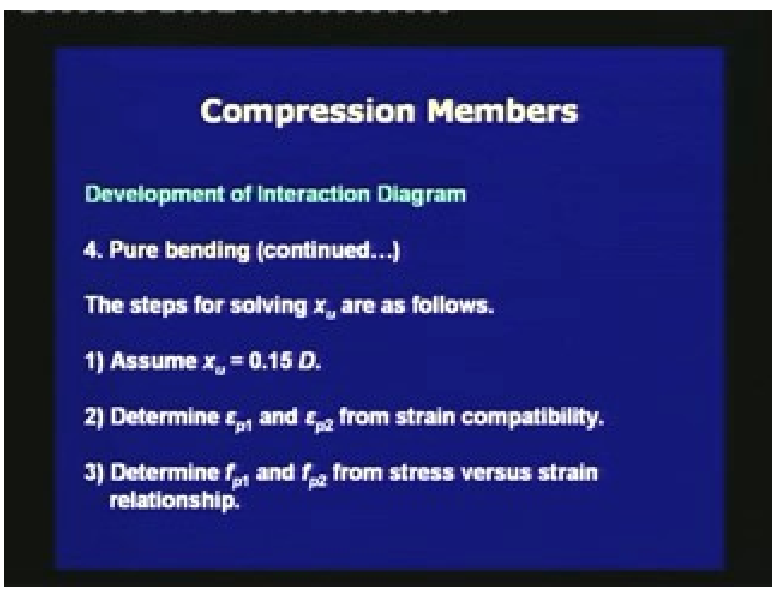


The steps for solving $x_u$ are as follows. Assume some value for $x_u$ say, 15% of the total depth; then determine epsilon$_{p1}$ and epsilon$_{p2}$ from strain compatibility; determine $f_{p1}$ and

$f_{p2}$ from the stress versus strain relationship for the prestressing tendon. Calculate $x_u$ from the expression which satisfies that, the axial force in the section is 0 and then, compare this $x_u$ with the assumed value. If it does not satisfy, iterate till we converge. That means, we are satisfying the condition that the axial force in the section is equal to 0. The moment and the axial force capacities are as follows. The axial force capacity is 0 for pure bending and the moment is equal to $M_c$ plus $M_p$, but the expression of $M_c$ and $M_p$ are same as the previous case.

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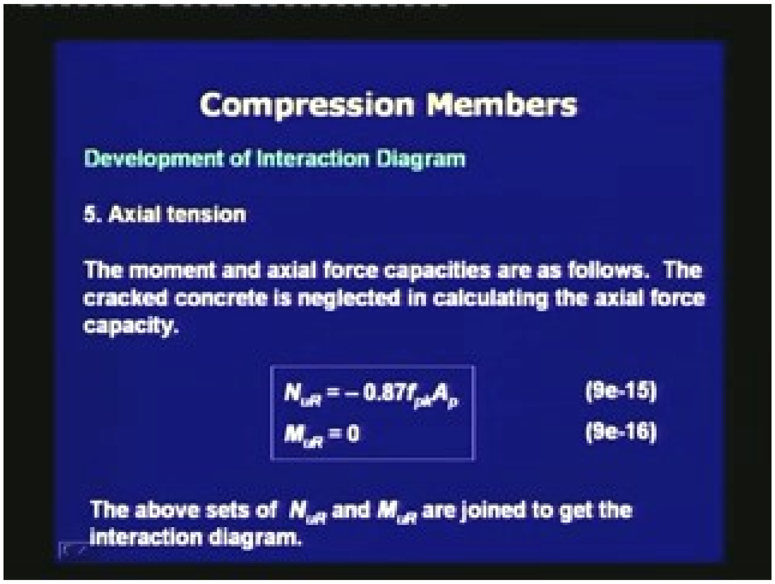


Last, we come to the axial tension, where the moment and the axial force capacities are given directly. The cracked concrete is neglected in calculating the axial force capacity. Thus, the axial force capacity is equal to minus 0.87 times $f_{pk}$ times the total prestressing steel, which is $A_p$ and $M_{uR}$ is equal to 0. The above sets of $N_{uR}$ and $M_{uR}$ are joined to get the interaction diagram.

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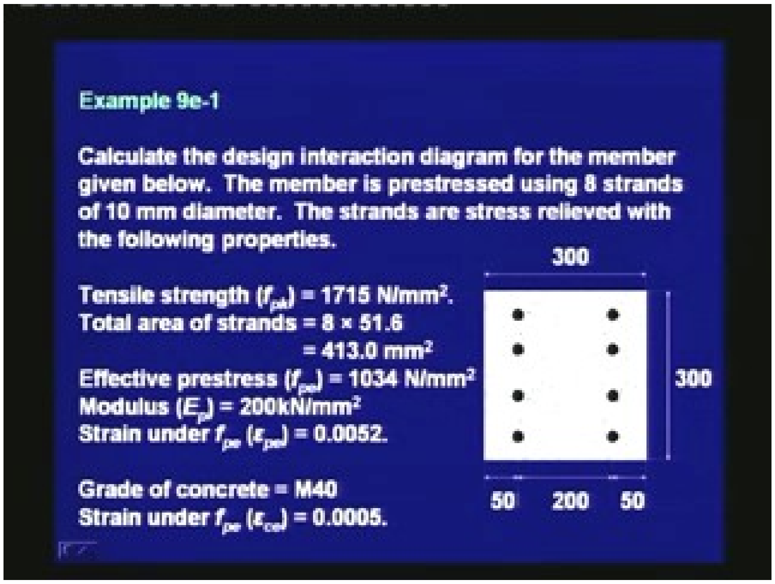


Let us calculate the design interaction diagram for the member given below. The member is prestressed using eight strands of 10 millimeter diameter. The strands are stress relieved with the following properties: the tensile strength is equal to 1715 Newton per millimeter square; the total area of tendon is 8 times 51.6 is equal to 413 millimeter square; effective prestress is 1034 Newton per milimeter square; modulus of elasticity, $E_P$ is 200 Kilonewton per millimeter square; the strain under $f_{pe}$ which is epsilon$_{pe}$ is given as 0.0052; the grade of concrete is M40 and the strain under $f_{pe}$ which is epsilon$_{ce}$ form the concrete is 0.0005.

The dimensions of the sections are as follows: D is equal to 300; B is equal to 300 and the distances from the edges to the centre of the prestressing steel is 50.

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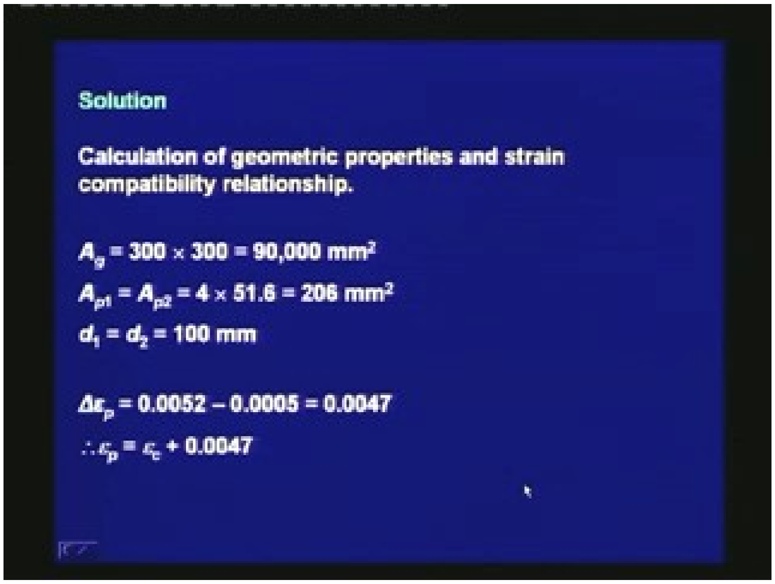


First is the calculation of geometric properties and strain compatibility relationship. $A_g$ is 90,000 millimeter square; $A_{p1}$ and $A_{p2}$ is 206 millimeter square; $d_1$ and $d_2$ is equal to 100 millimeter; Delta epsilon$_p$ is equal to 0.0052 minus 0.005 which is equal to 0.0047. Thus, our strain compatibility equation is epsilon$_p$ is equal to epsilon$_c$ plus 0.0047.

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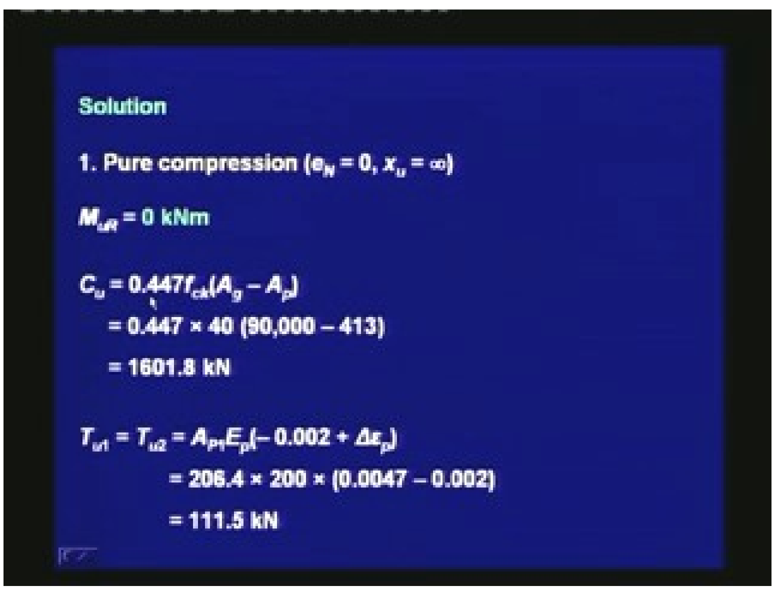

We are calculating for the case for pure compression. For this case $M_{uR}$ is equal to 0. We are calculating the expression of $C_u$ from the previous expression and it is 1601.8 Kilonewtons; $T_{u1}$ and $T_{u2}$ come out to be 111.5 Kilonewtons.

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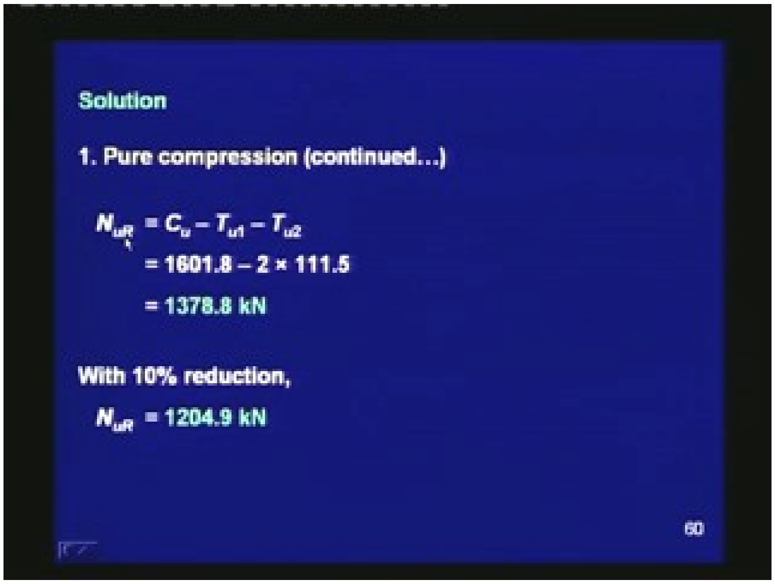


We calculate the value of $N_{uR}$ and we get $N_{uR}$ is equal to 1378.8 Kilonewtons. If we reduce by 10% to consider eccentricities less than 0.05t, then $N_{uR}$ is equal to 1204.9 Kilonewtons.

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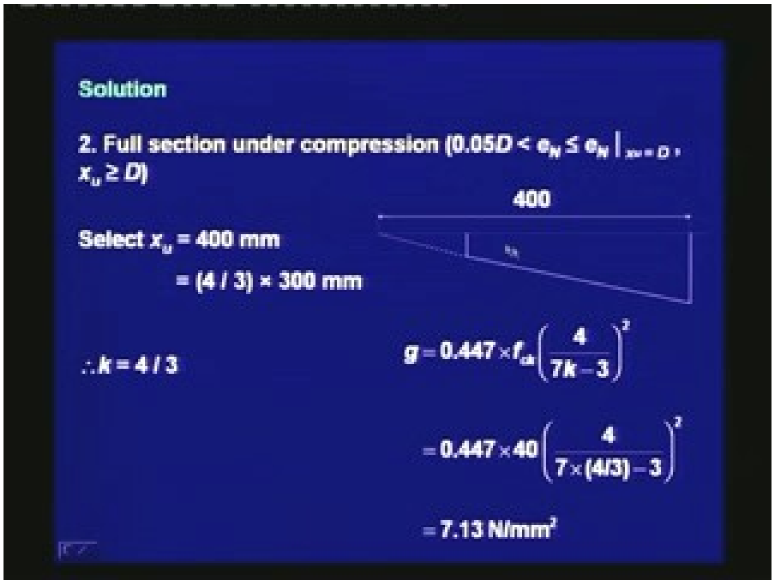


Next, we are calculating for the full section under compression. We are selecting $x_u$ is equal to 400 millimeter. That means, the depth of the section is 300; whereas, $x_u$ lies 100 outside section, where k is given as 4 by 3. We can calculate the value of g by the previous expression.

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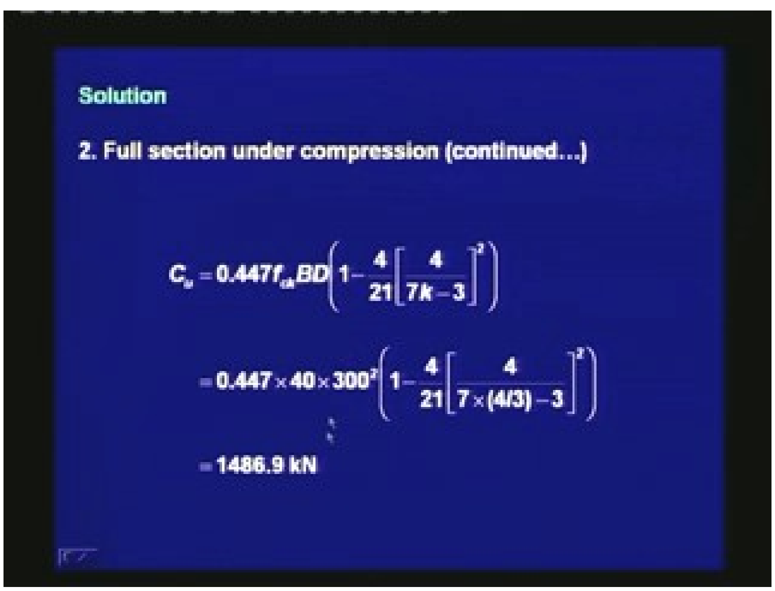

Then, we get the value of $C_u$, again substituting in the previous expression, we get $c_u$ is equal to 1486.9 Kilonewtons.

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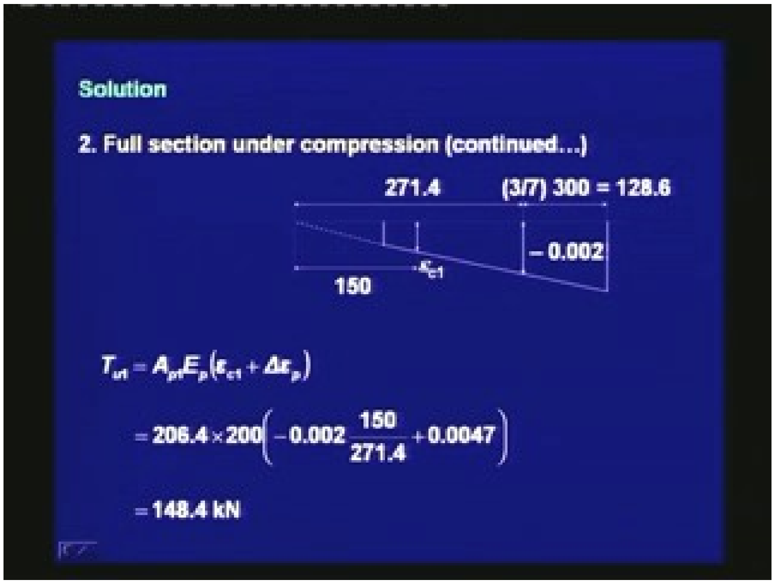


We are calculating the strain in $A_{p1}$ from which we get $T_{u1}$ is equal to 148.4 Kilonewtons.

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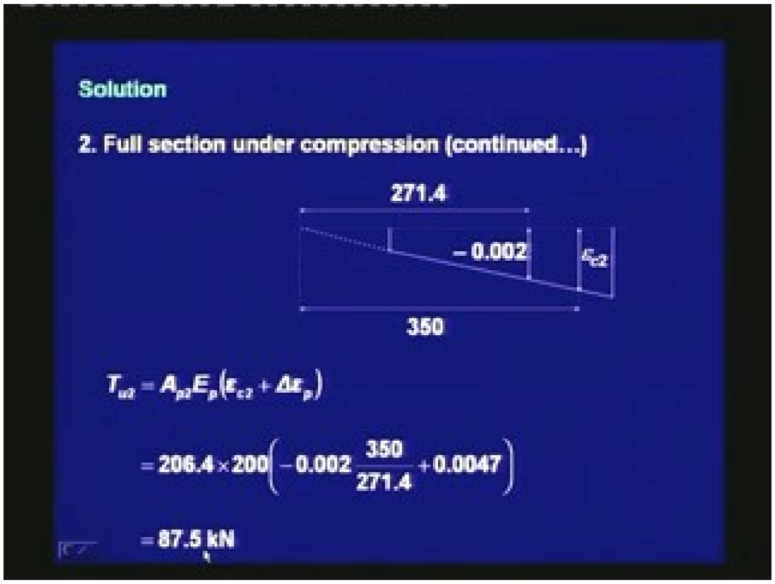


Similarly, we are calculating the strain in $A_{p2}$ from which $T_{u2}$ is equal to 87.5 Kilonewtons.

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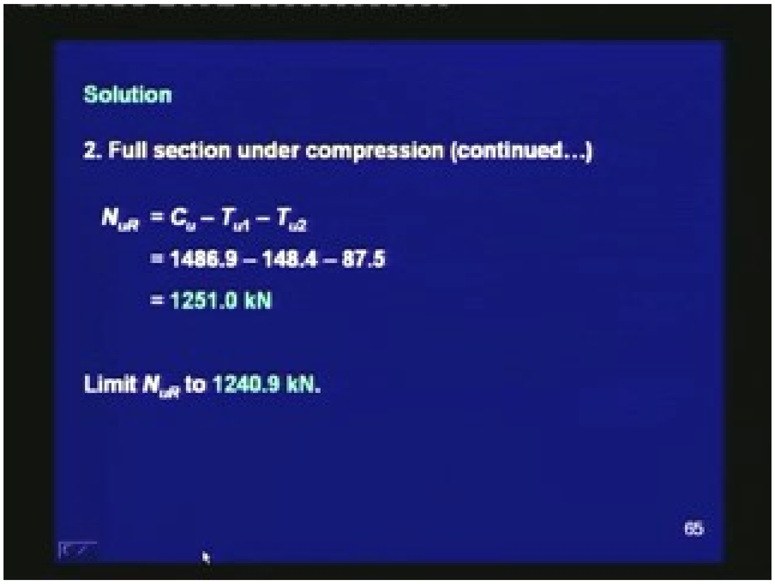


Thus, $N_{uR}$ is equal to $C_u$ minus $T_{u1}$ minus $T_{u2}$. Substituting that we get $N_{uR}$ equal to 1251 Kilonewtons. We can limit that $N_{uR}$ to the value corresponding to e is equal to 0.05 D and that value is 1240.9 Kilonewtons.

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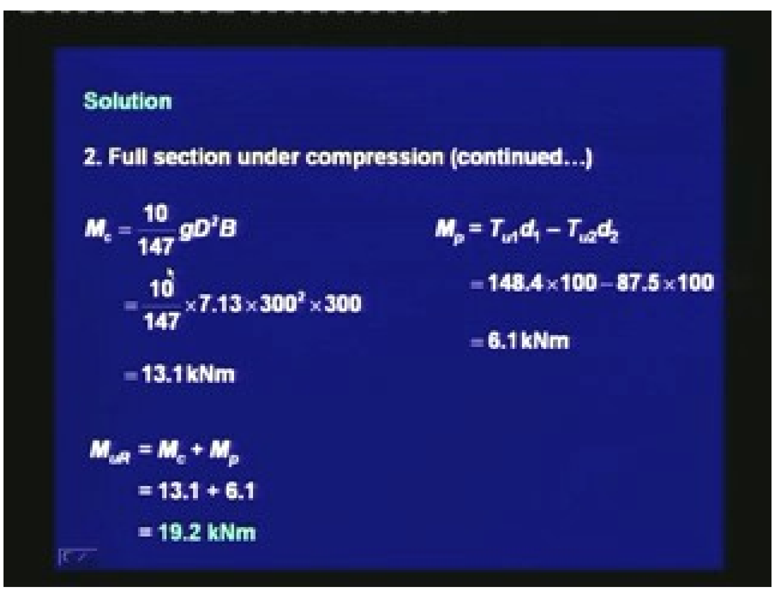


We are calculating the value of $M_c$ from the previous expression. We get $M_c$ is equal to 13.1 Kilonewton meters and $M_p$, we get 6.1 Kilonewton meters. When we add them up, we get $M_{uR}$ is equal to 19.2 Kilonewton meters. Thus, for this case we got the values of $N_{uR}$ and $M_{uR}$.

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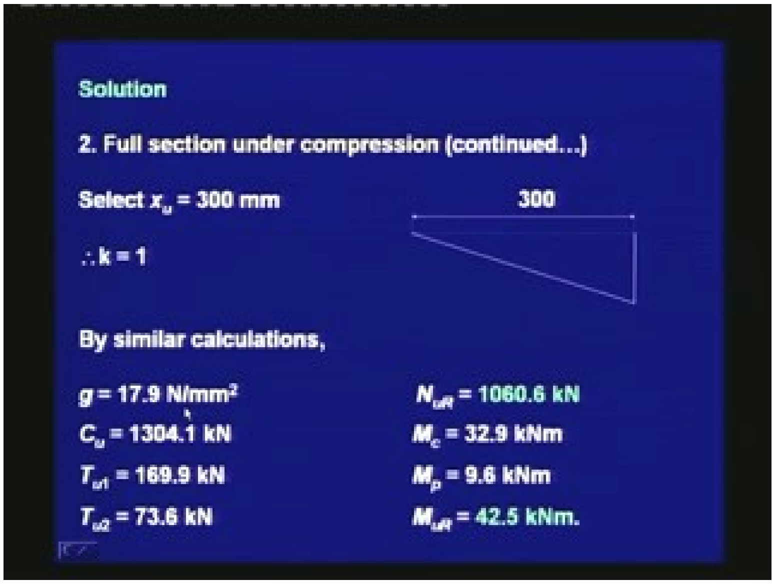


We are selecting another case, where $x_u$ is equal to 300. That means the neutral axis is lying right at the edge of the section. Here, k is equal to 1. By similar calculations we can find the values of g, $C_u$, $T_{u1}$, $T_{u2}$. $N_{uR}$ comes out 1060.6 Kilonewtons and $M_{uR}$ is equal to 42.5 Kilonewton meters.

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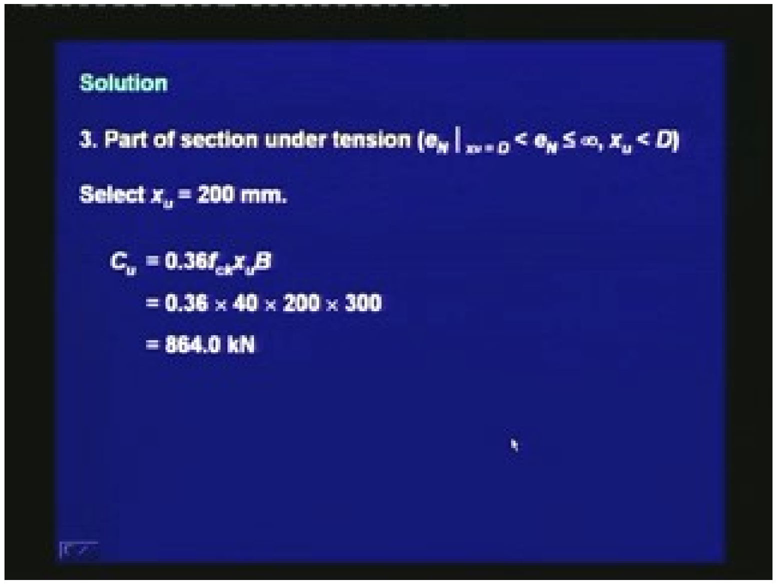


Next, we move on to the case of a part of the section under tension. We are selecting $x_u$ is equal to 200 millimeters. Note that, now the neutral axis lies within the section. $C_u$ is given by the expression, which is 864 Kilonewtons.

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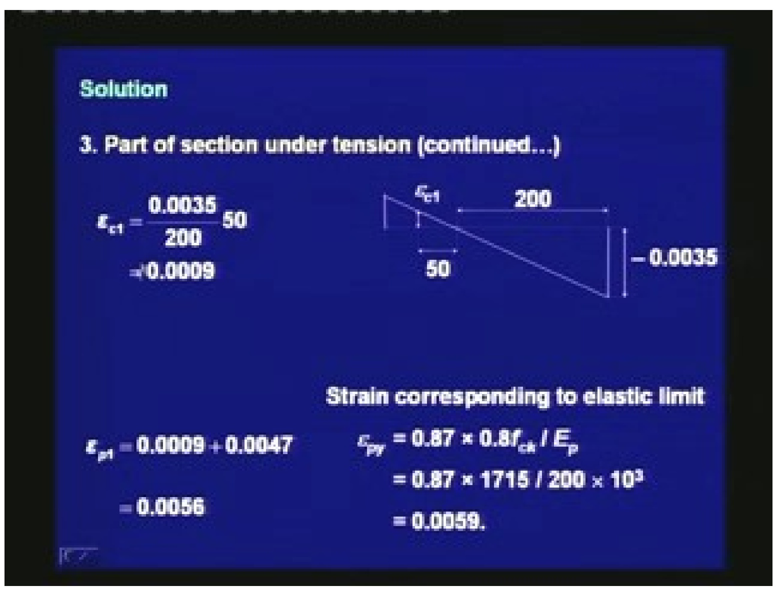


We are calculating epsilon$_{c1}$ from the strain diagram. Given the two distances 200 and 50, we can calculate what the value of epsilon$_{c1}$ is. Epsilon$_{p1}$ is equal to epsilon$_{c1}$ plus the strain differentials 0.0047 which is equal to 0.0056. Since, the strain is within the elastic limit, now the elastic limit is given as 0.87 of $0.8f_{ck}$ divided by $\text{epsilon}_p$, which is equal to 0.0059. Note that, $\text{epsilon}_{p1}$ is less than $\text{epsilon}_{py}$ and hence, we can use the elastic relationship.

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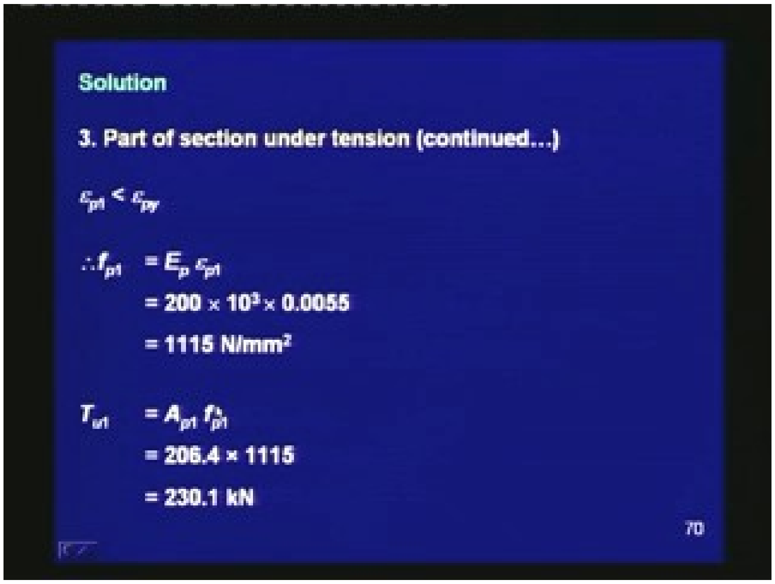


$f_{p1}$ is equal to $\text{epsilon}_{p1,}$ we get 1115 Newton per millimeter square, from which we get $T_{u1}$ is equal to 230.1 Kilonewton.

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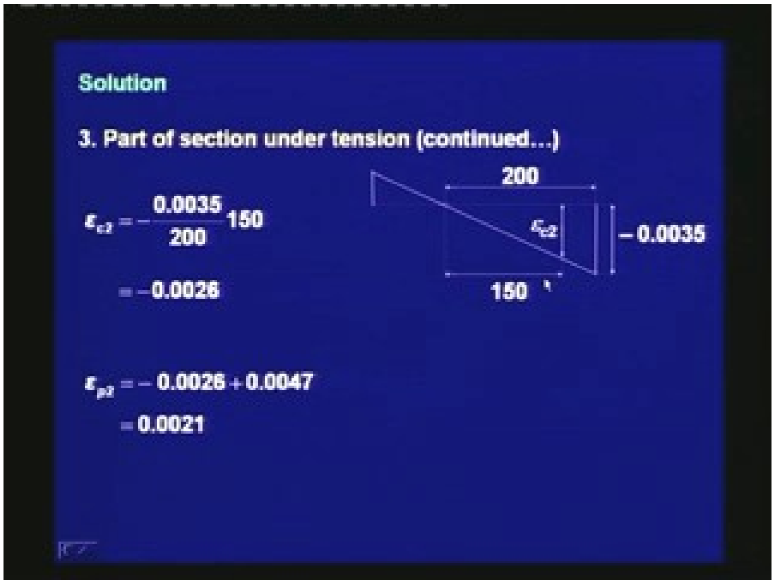


We can calculate the strain in $A_{p2}$ similarly, from another strain diagram from the similarity of triangles and from epsilon$_{c2}$ we calculate epsilon$_{p2}$ and that is equal to 0.0021.

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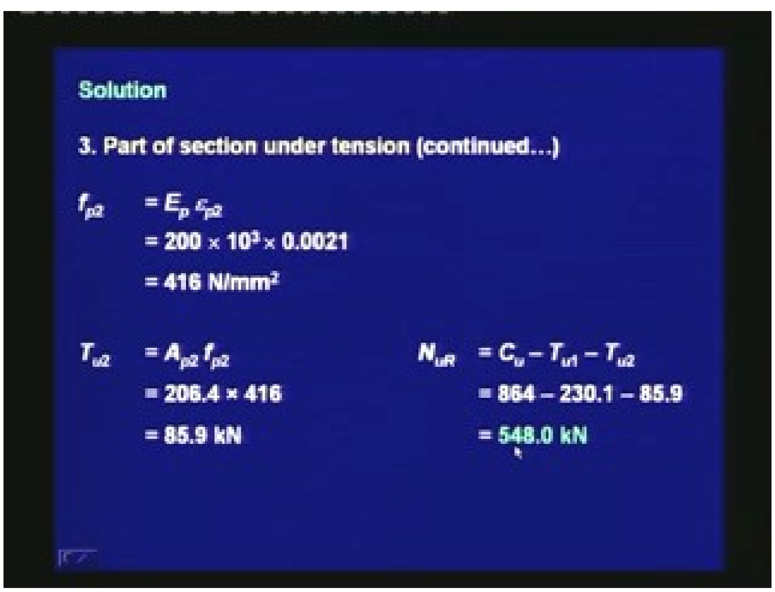


$f_{p2}$ is calculated from the elastic relationship and $T_{u2}$ is equal to 85.9 Kilonewtons. Thus, $N_{uR}$ is equal to 548 Kilonewton.

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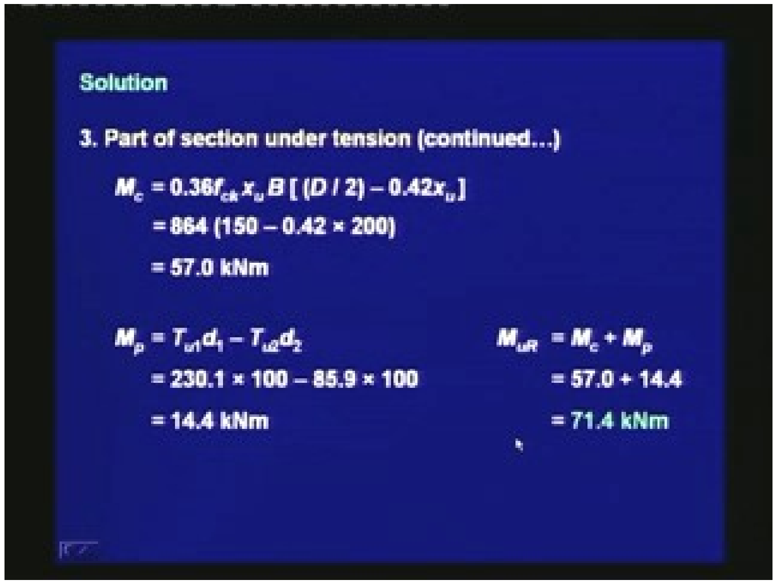


$M_{c,}$ we can substitute the values and that is equal to 57.0 Kilonewton meter. $M_p$ is equal to 40.4 Kilonewton meters. Thus, $M_{uR}$ is the summation of $M_c$ and $M_{p,}$ which is equal to 71.4 Kilonewton meters.

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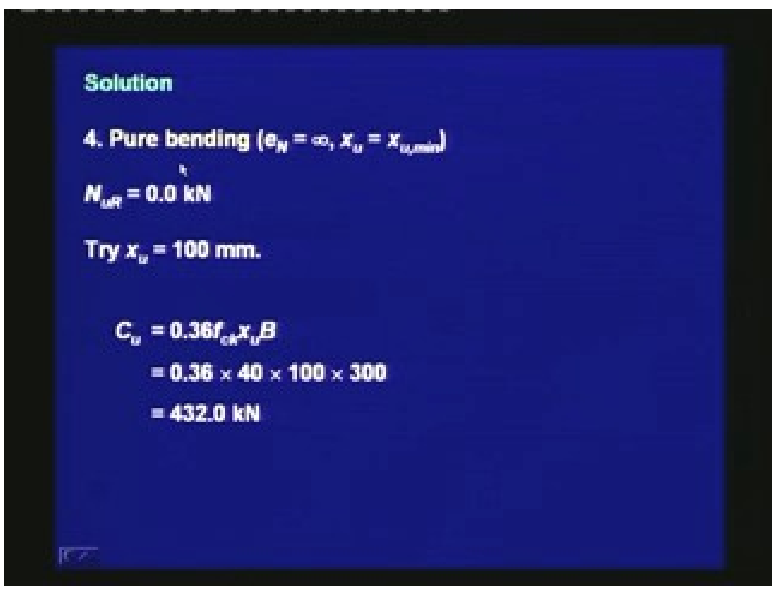


For the pure bending case, $N_{uR}$ is equal to 0 and now, we are trying to solve the equations by trial and error. We select $x_u$ is equal to 100. We calculate the value $C_u$, which is equal to 432 Kilonewtons.

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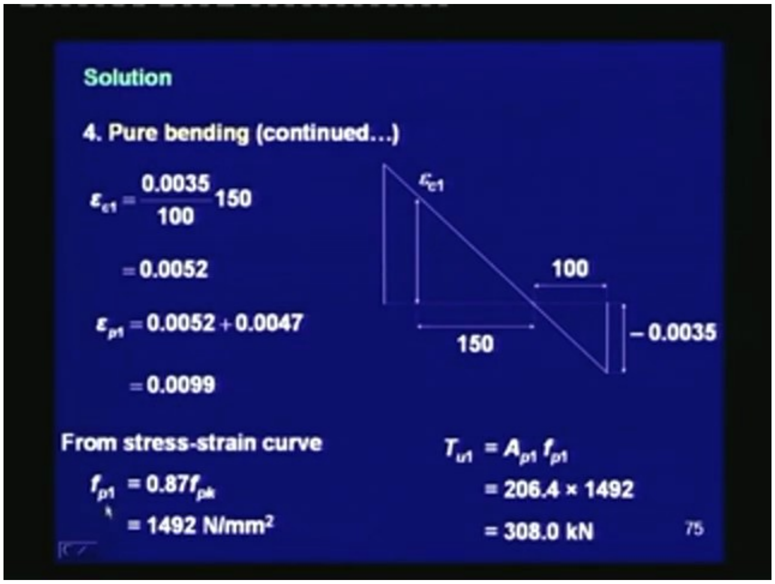


We are calculating the values of epsilon$_{c1}$ from the strain compatibility relationship. Then we calculate epsilon$_{p1}$. From the stress-strain curve we find the f$_{p1}$. This stress strain curve has to be used for the stress relieved type of steel and f$_{p1}$ is equal to 1492 Newton per millimeter square. T$_{u1}$ is equal to 308 Kilonewtons.

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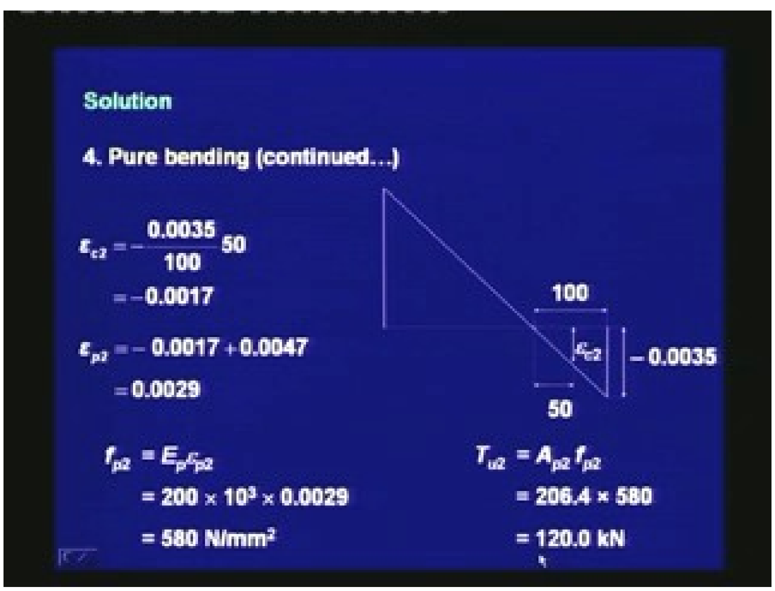

Similarly, we can calculate $epsilon_{c2}$ from the strain diagram and we calculate $epsilon_{p2}$, from which we can calculate $f_{p2}$ and finally, $T_2$ is equal to 120 Kilonewtons.

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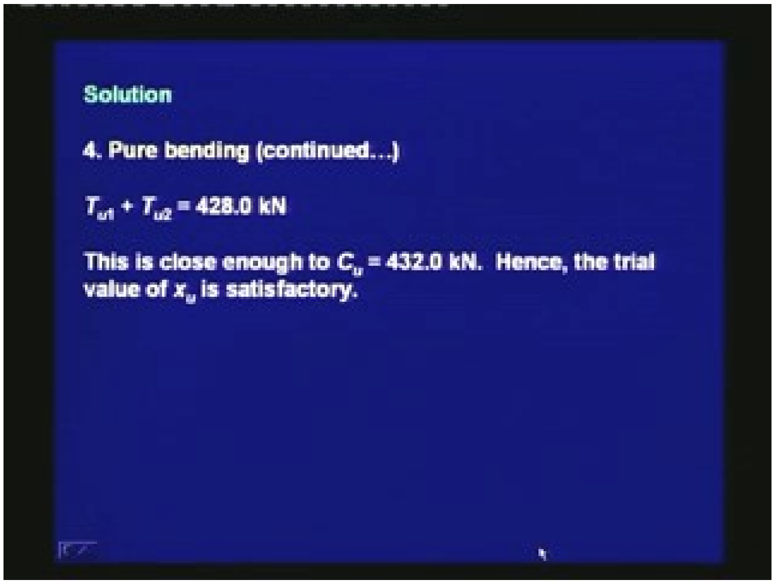


$T_{u1}$ plus $T_{u2}$ is equal to 428.0 Kilonewton, which is close enough to $C_u$. Hence, the trial section of $x_u$ is satisfactory.

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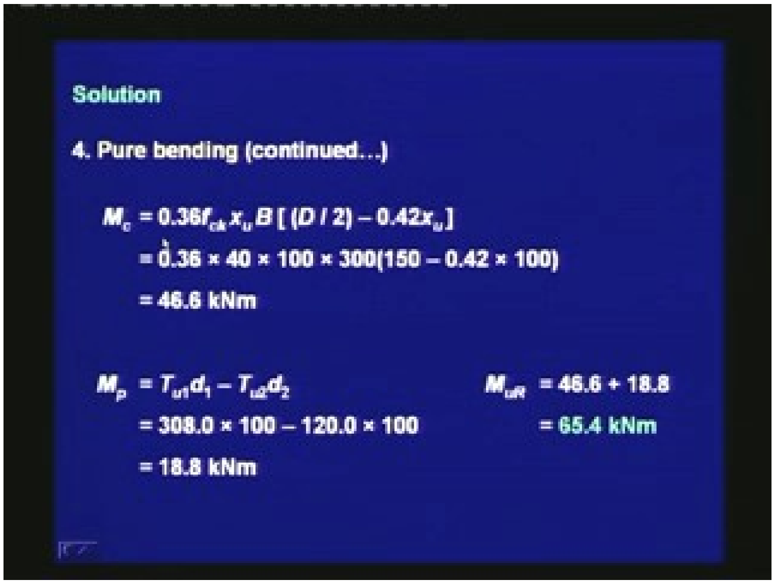

$M_c$, by the substitution of the values is 46 Kilonewton meter and finally, we get $M_{uR}$ is equal to 65.4 Kilonewton meters.

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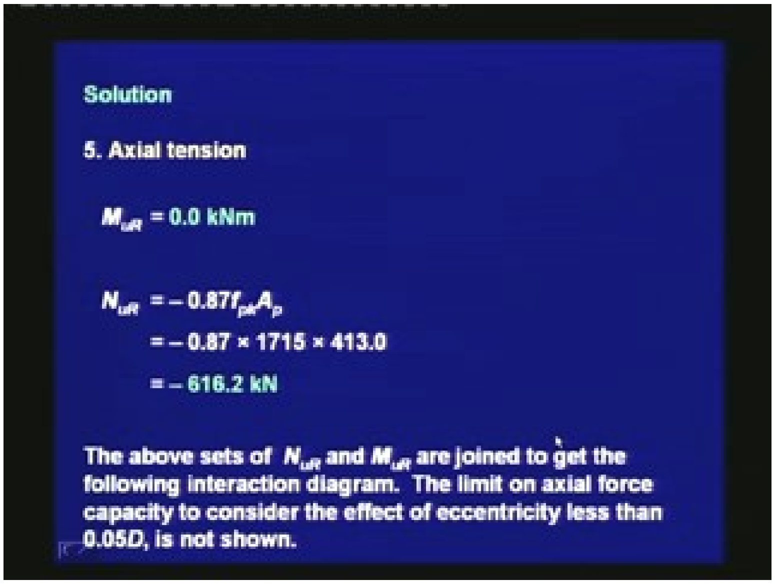


For axial tension, $M_{uR}$ is equal to 0 and $N_{uR}$ is equal to minus 616.2 Kilonewtons. The above sets of $N_{uR}$ and $M_{uR}$ are joined to get the following interaction diagram. The limit on axial force capacity to consider the effect of eccentricity less than 0.05 D is not shown in this diagram. This is the interaction curve for the section and once we have plotted the points, we get this failure envelope.

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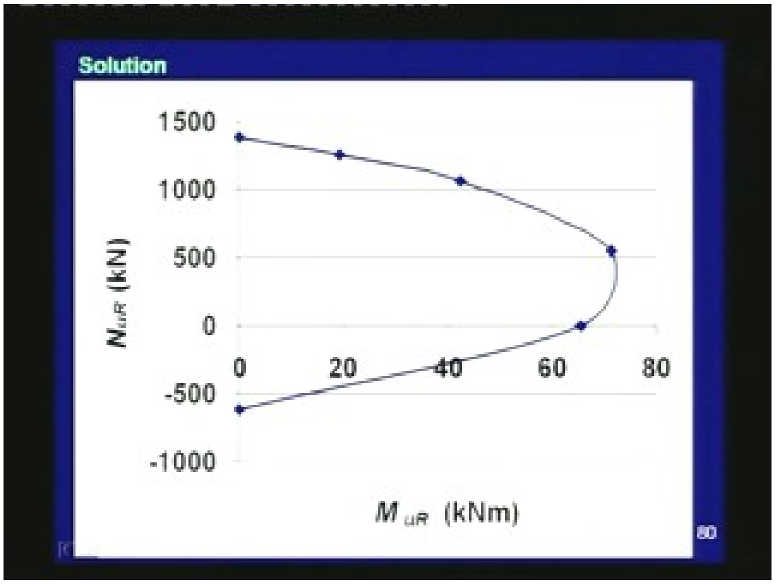


To see the effect of prestressing force, let us compare the results with two equivalent reinforced concrete sections. For the first section, the blue line is represented as prestressed concrete section.

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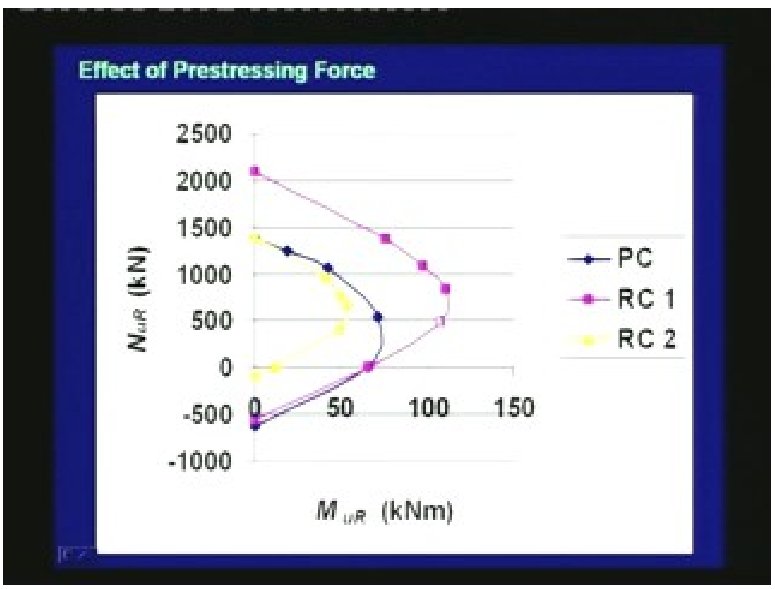


The pink line above is the reinforced concrete section with the same flexural capacity and the yellow line is the reinforced concrete section with the same axial load capacity.

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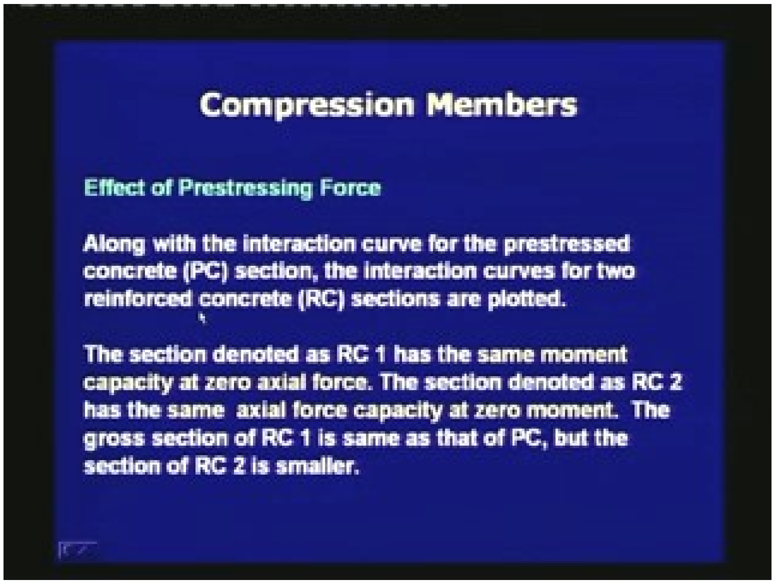


Along with the interaction curve for the prestressed concrete section, the interaction curves for two reinforced concrete sections are plotted. The section denoted as RC 1 has the same moment capacity at zero axial force. The section denoted as RC 2 has the same axial force capacity at zero moment. The gross section of RC 1 is same as that of PC, but the section of RC 2 is smaller.

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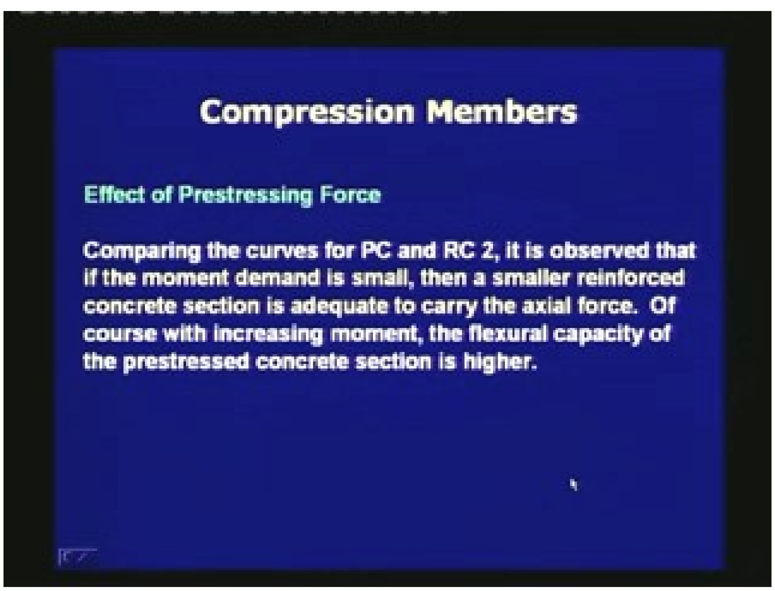

Comparing the curves for PC and RC 2, it is observed that if the moment demand is small then a smaller reinforced concrete section is adequate to carry the axial force. With increasing moment, the flexural capacity of the prestressed concrete section is higher.

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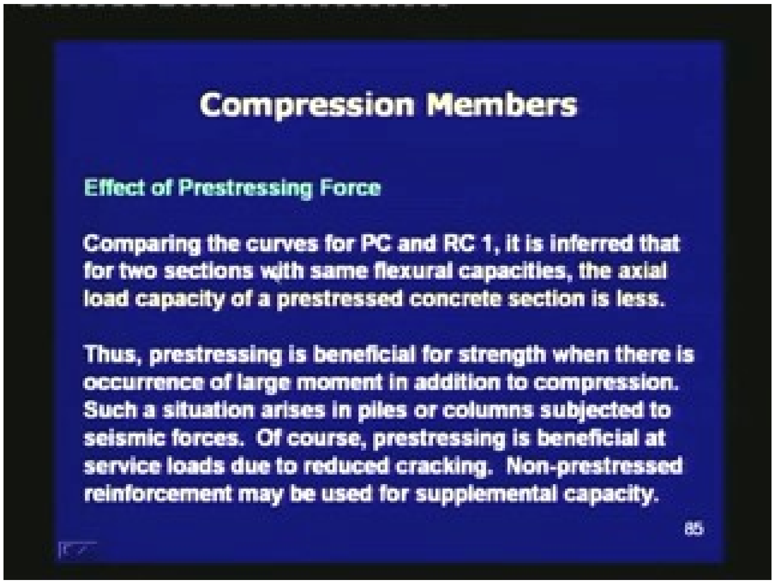


Comparing the curves for PC and RC1, it is inferred that, for the two sections with same flexural capacities, the axial load capacity of a prestressed concrete section is less. Thus, prestressing is beneficial for strength, when there is occurrence of large moment in addition to compression. Such a situation arises in piles or columns subjected to seismic forces. Prestressing is beneficial at service loads due to reduced cracking. Non-prestressed reinforcement may be used for supplemental capacity.

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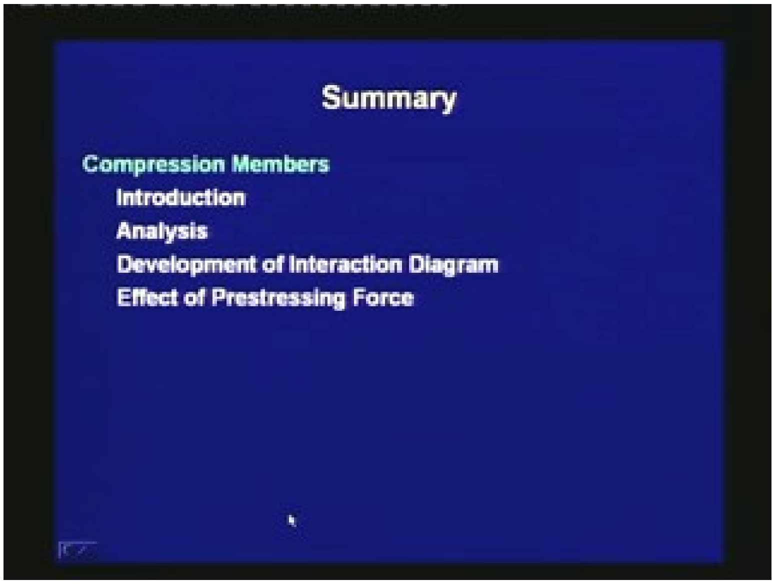


Today, we covered the compression members. After the introduction of the different types of application of prestressing in compression members, we went on to the analysis of compression members. We first saw the analysis at transfer, then at service, then finally, at the ultimate state. For the ultimate state, we need the interaction diagrams and we learnt of the development of the interaction diagrams. We came to know that the effect of prestressing is beneficial only when there is high moment along with axial compression; otherwise it may not be economical. With this we are ending the module on compression members. Thank you.