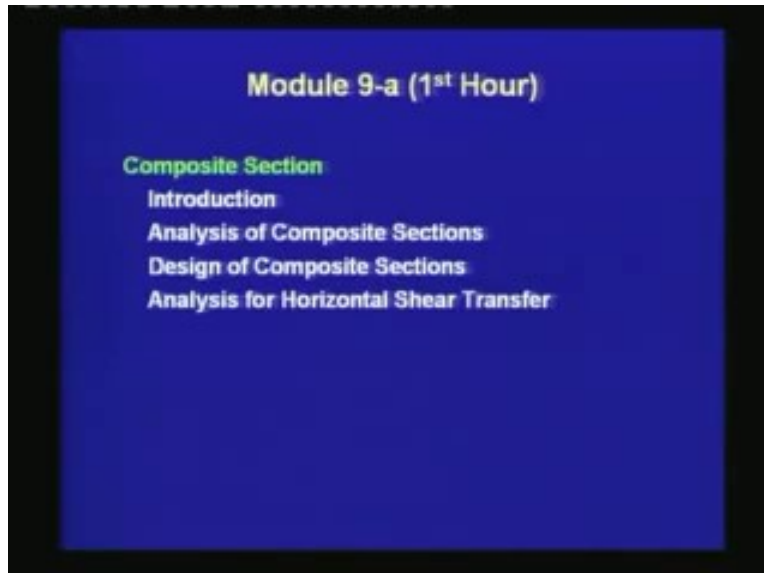


**Prestressed Concrete Structures**  
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**Indian Institute of Technology, Madras**

**Module - 9 Lecture - 35**  
**Composite Sections**

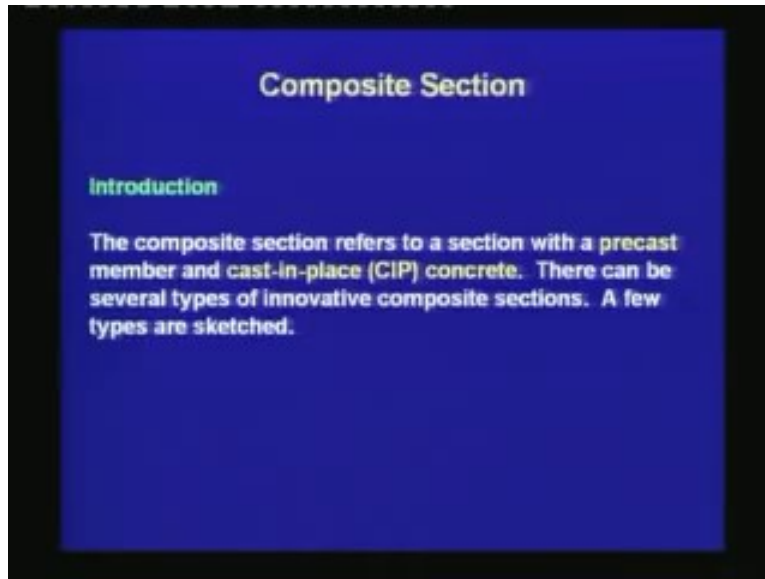
Welcome back to prestressed concrete structures. This is the first lecture of the module nine on special topics.

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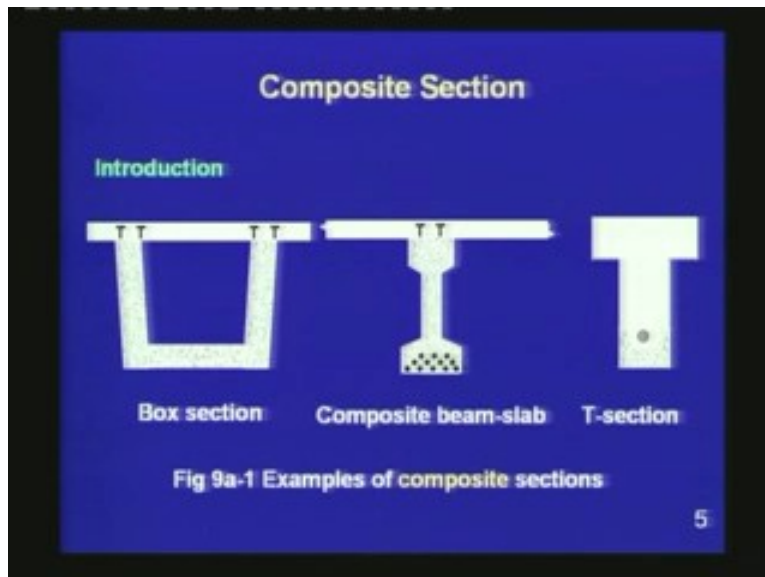
In this lecture, we shall study about composite section. This is the first special topic that we are covering in this program. First, we shall introduce the composite section and then we shall know about the analysis of composite sections; we shall move on to the design of composite sections and finally, we shall see the analysis for horizontal shear transfer.

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The composite section refers to a section with a precast member and cast-in-place concrete; the cast-in-place will be denoted as CIP. There can be several types of innovative composite sections. A few types are sketched.

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In the left most figure, we have a precast web with a bottom flange. Once it has gained the strength, it is taken to the site and then the top flange has been cast inside. Similarly,

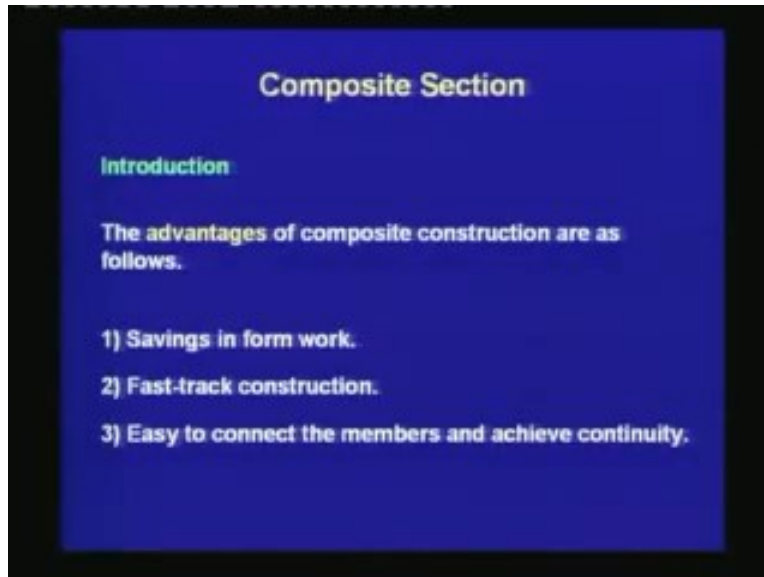
for the figure in the middle, I girder is a precast girder which has been cast in the yard and then the top slab has been cast in the site. In the figure, on the right side the precast part is a small part, which acts like a form work and the cast-in-place part is quite substantial. Here we can see that, the precast member can act as a form work for pouring the concrete for the top part. There can be different types of composite sections and it depends on the innovation, the type of application and the construction equipments and personnel available for producing the section.

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In this photograph we can see that a precast bridge girder has been placed in the site, the reinforcement has been fabricated, the ducts for post tensioning has been placed and you can observe that the transverse reinforcements have been projected upwards, which will act as the shear transverse reinforcement for the composite section. Once the concrete is cast and hardened this precast girder will be transported to the site, it will be placed on the pillars and then the deck will be cast. Thus, in bridges the composite section is extensively used which helps us both in the construction as well as in the functioning of the member.

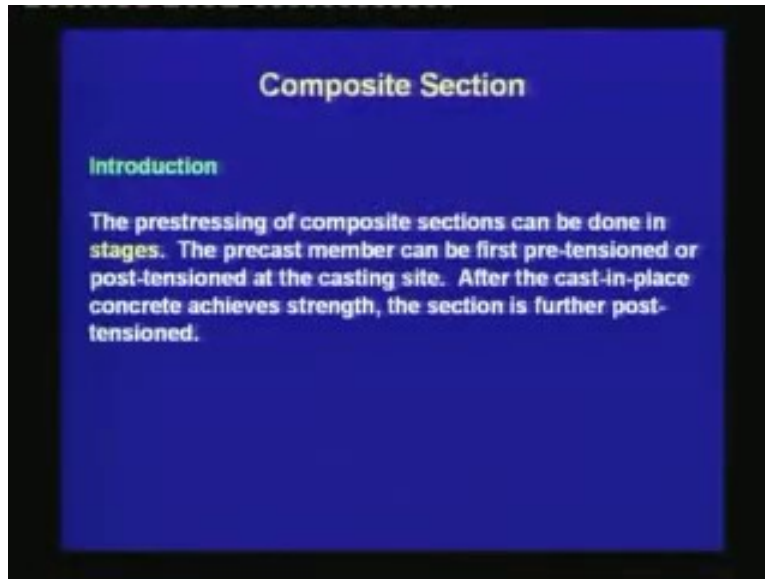
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The advantages of composite construction are as follows. First, it can lead to savings in form work, as I had said before sometimes, the precast member itself acts as a part of the form work and hence the form work can be saved. Second, it is a fast-track construction. If the same section have to be cast in site, then the quality control and the time of construction would not have been satisfactory. The composite construction helps us to expedite the construction, where the precast part is constructed in a control environment without creating any disturbance at the site and once the precast member has been strengthened, then it will be taken to the site for the casting of the top flange.

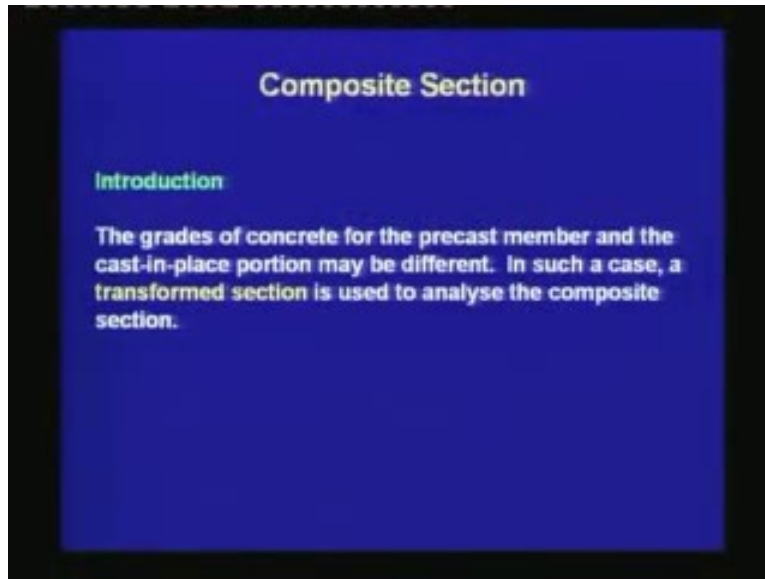
The third advantage is it is easy to connect the members and achieve continuity. In my lecture on continuous beams, we had discussed that introducing continuity is a difficult post-tensioning operation. It can lead to substantial friction loses, but if precast members are used then partial continuity can be introduced by connecting the post tension cables or providing some non-prestressed reinforcement in the support region. This construction is easier than trying to introduce complete continuity among the members.

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The prestressing of composite sections can be done in stages. The precast member can be first pre-tensioned or post-tensioned at the casting site. After the cast-in-place concrete achieves strength, the section is further post-tensioned. That means, first the precast member is prestressed in the casting yard itself. Then it is transported to the site and after the cast-in-place concrete has been placed and after that concrete has hardened it can further be prestressed. Hence, the prestressing operation is done in stages which are based on the analysis of the different sections at the different times and this also leads to a more efficient section.

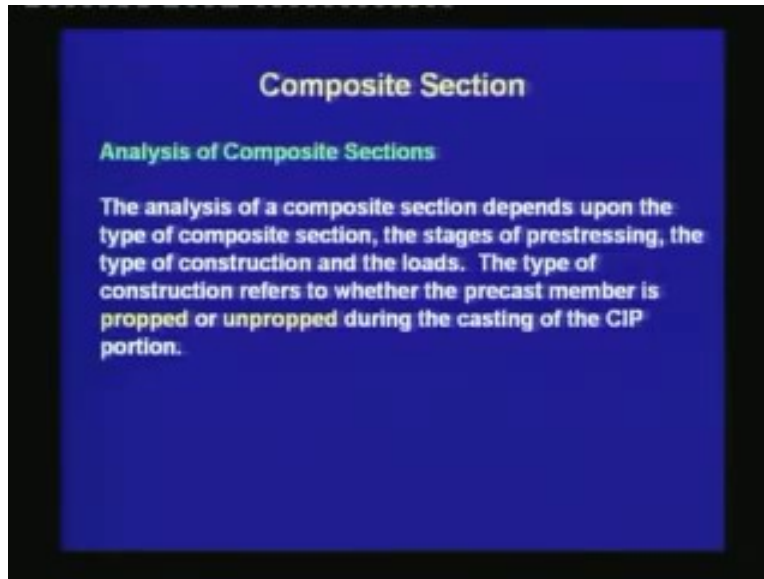
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The grades of concrete for the precast member and the cast-in-place portion may be different. In such a case, a transformed section is used to analyse the composite section. If you are having two different grades of concrete say M20 for the precast member and say M25 for the cast-in-place member, then there will be some discontinuity in the stress between the precast parts and the cast-in-place part, but we can assume that the full section is of the lower grade concrete and simplify our calculations.

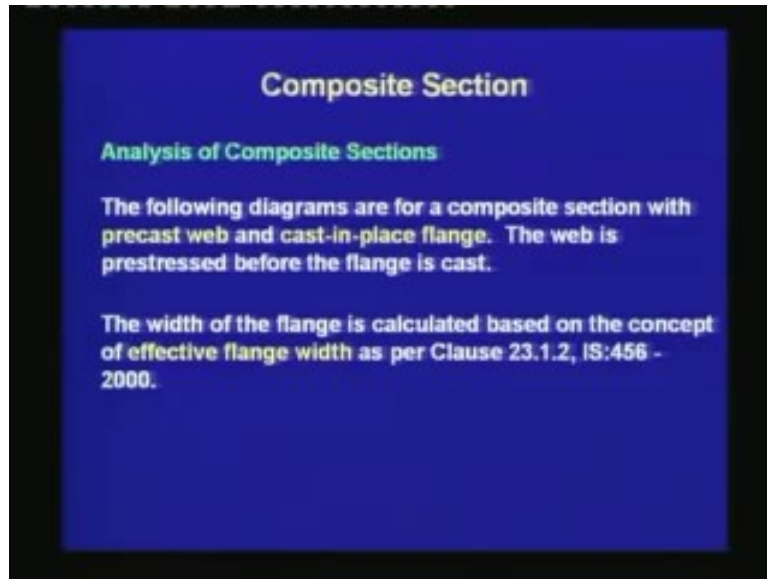
Next, we are moving on to analysis of composite sections.

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The analysis of a composite section depends upon the type of composite section, the stages of prestressing, the type of construction and the loads. The type of construction refers to whether the precast member is propped or unpropped during the casting of the cast-in-place portion. Thus, the analysis is very much dependant on the type of composite section. Are we having a web as precast and as a flange as the cast-in-place or is it something else? It depends on that. Next is the type of construction. That means, when the concrete in the cast-in-place flange is poured, is the web being supported by some props? If it is supported, then the stress block would be something. If it is not supported or unpropped, then the stress block will be something else. Thus, we have to know the type of construction that is being used for making the composite section and the analysis will depend on that. The third important thing is the stage of prestressing. Are we applying the prestress at one go or are we applying the prestress at stages? That will also lead a difference in the analysis.

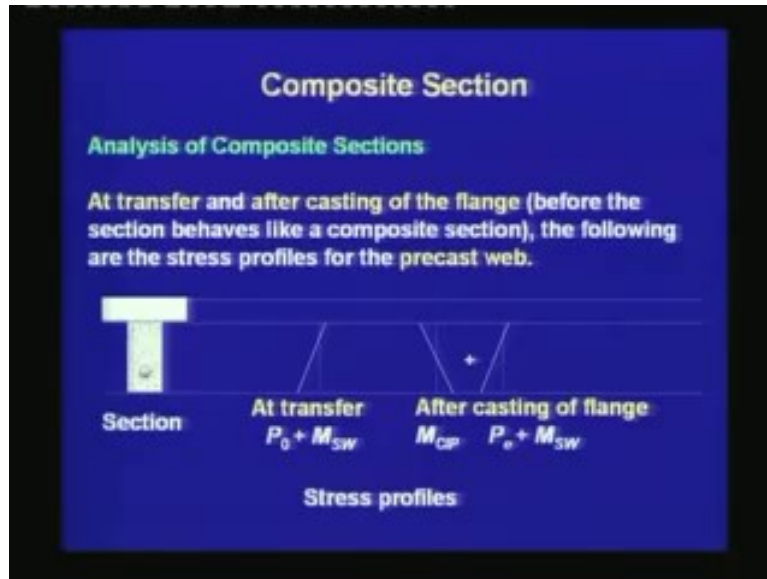
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The following diagrams are for a composite section with precast web and cast-in-place flange. The web is prestressed before the flange is cast. This is a common type of composite section where the web is precast and the flange is cast-in-place. The web is prestressed before the cast-in-place flange is poured. The width of the flange is calculated based on the concept of effective flange width as per clause 23.1.2, IS: 456 – 2000. That means IS: 456 – 2000 gives us, like if we have flange which is quite wide, we should take only a part of the flange in which the stress can be assumed to be uniform and that part of the flange is called the effective width of the section. Based on that clause, we determine, what is the width of flange in our analysis?



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At transfer and after casting of the flange during which the section behaves like a composite section, the following are the stress profiles for the precast web. That means, first of all, when we are transferring the prestress, the section is just the web; there is no flange at the time and the stress block is somewhat like this, it is compressive at the bottom. There can be some tensile stress at the top and at transfer the nodes to be considered are the prestress at transfer, which is  $P_0$  and the self weight member which is  $M_{sw}$ . When the flange is cast at the time the section resisting the gravity load is only the web. The flange has not gained the strength yet and hence the moment created due to the weight of the cast-in-place concrete is carried only by the web and not by the composite section and effective prestress and the moment due to self weight are also carried only by the web portion.

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**Composite Section**

**Analysis of Composite Sections**

Here,

- $P_0$  = Prestress at transfer after short term losses
- $P_e$  = Effective prestress after long term losses
- $M_{sw}$  = Moment due to self weight of the precast web
- $M_{CIP}$  = Moment due to weight of the CIP flange.

In this **previous** figure,  $P_0$  is the prestress at transfer after short term losses,  $P_e$  is the effective prestress after long term losses,  $M_{sw}$  is the moment due to self weight of the precast web,  $M_{CIP}$  is the moment due to weight of the cast-in-place flange.

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**Composite Section**

**Analysis of Composite Sections**

At service (after the section behaves like a composite section) the following are the stress profiles for the full depth of the composite section.

If propped

Section

At service

$M_{LL}$        $P_e + M_{sw} + M_{CIP}$

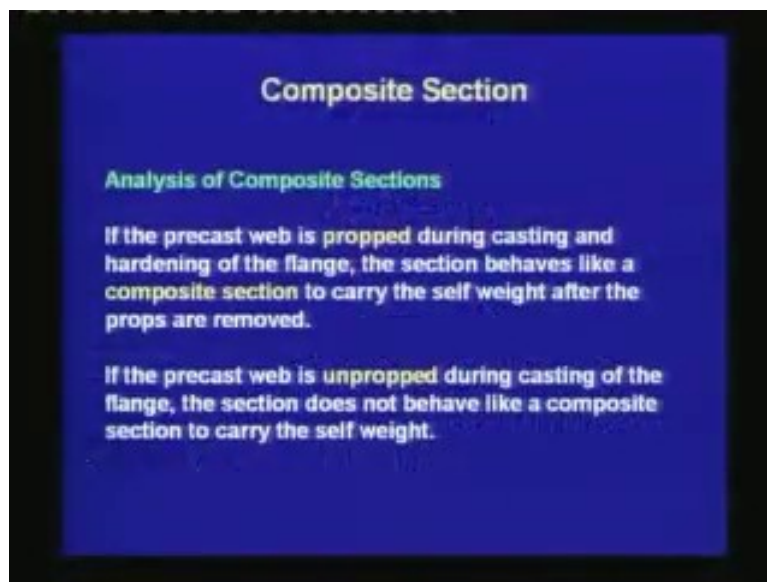
Stress profiles

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At service, after the section behaves like a composite section, the following are the stress profiles for the full depth of the composite section. Now, we are moving on to the service

load stage; by the time the flange has hardened and the full section can now act as a composite section. Hence, the additional load that comes during the service period that is shared by both the web and the flange because the full section is acting like a composite section. Thus the stress block is that due to the live load, the moment is carried along the full depth of the section; whereas, the prestress and the self weight of the full composite section is carried by the web itself, because this has stayed from the stage during the casting of the flange. That means the stress due to the prestressing and the stress due to the self weight of the section have been locked in the web portion; whereas, additional load that comes during the service life, that is resisted by the full composite section. There can be a variation that if the web is propped during construction then, after the flange hardens and if the props are removed after that, then the self weight of the member will be carried by the full section. This is shown as the figure on the right; if the web is propped then the self weight is carried by the full composite section. Thus, whether the construction is propped or unpropped, this is necessary to analyse the composite section after the flange has hardened.

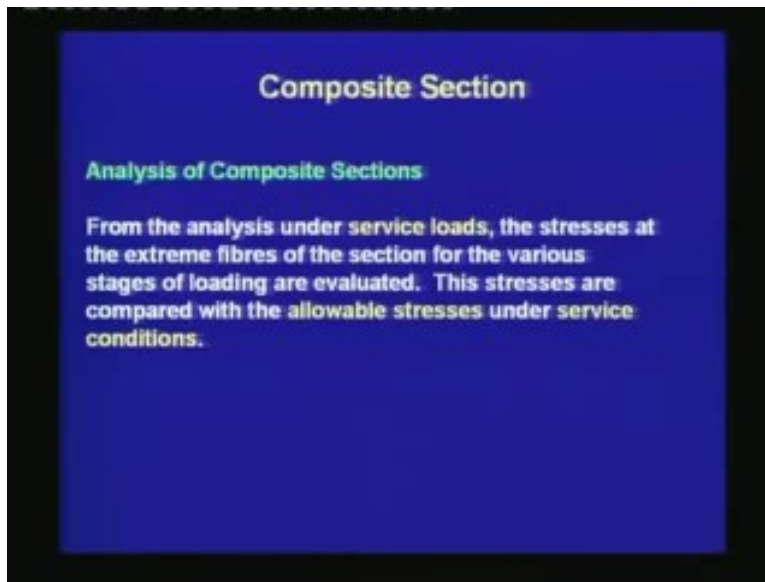
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If the precast web is propped during casting and hardening of the flange, the section behaves like a composite section to carry the self weight after the props are removed. If the precast web is unpropped during casting of the flange, the section does not behave

like a composite section to carry the self weight. Thus, it is essential to know whether the section is propped or unpropped during the casting of the flange.

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From the analysis under service loads, the stresses at the extreme fibers of the section for the various stages of loading are evaluated. This stresses are compared with the allowable stresses under service conditions. That means, once we find out the stresses at the extreme edges in the concrete, then we compare them with the allowable stresses given in the code and make sure that these stresses are within the allowable values.

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**Composite Section**

**Analysis of Composite Sections**

**Stress in precast web at transfer**

$$f = -\frac{P_0}{A} + \frac{P_0ec}{I} + \frac{M_{sw}c}{I} \quad (9a-1)$$

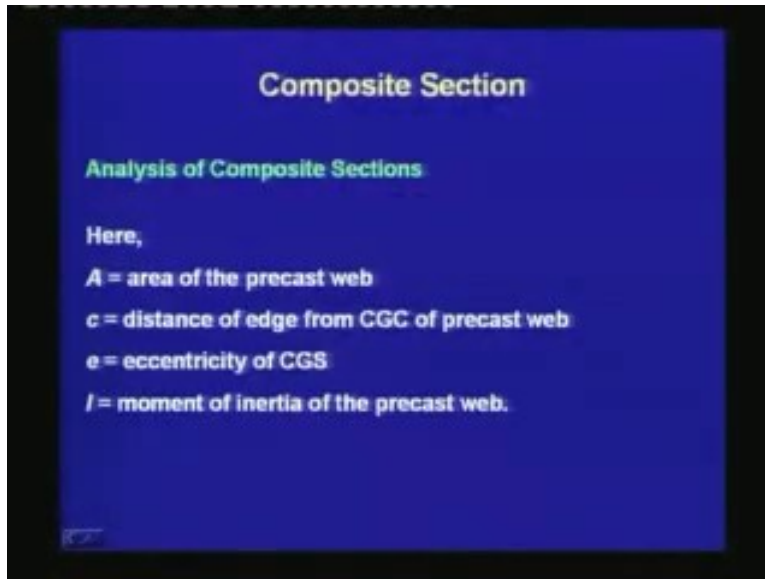
**Stress in precast web after casting of flange**

$$f = -\frac{P_e}{A} + \frac{P_eec}{I} + \frac{(M_{sw} + M_{CIP})c}{I} \quad (9a-2)$$

The stress in precast web at transfer is given by the expression that we learned during the analysis based on the stress concept. The stress first consists of a uniform component which is due to the prestressing  $P_0$ , and this is compressive. The second component is due to the eccentricity of the prestressing. The eccentricity is  $e$ , the moment due to the prestressing force is  $P_0$  times  $e$ ,  $C$  is the distance of the extreme edge from the CGC of the precast web,  $I$  is the moment of inertia of the precast web. Then the third term is the term due to self weight of the section, where  $M_{sw}$  is the moment due to self weight and other terms have been explained just before.

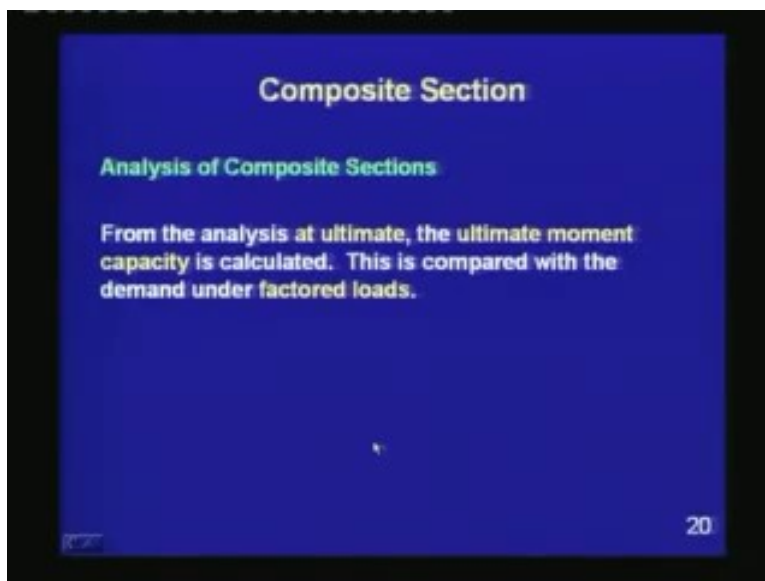
The stress in precast web, after casting a flange has certain differences. First,  $P_0$  has dropped to  $P_e$ , due to the long term losses because usually, there is a time difference between the prestress transfer and the cast-in-place flange construction. If the time difference is more than a month, then we can assume that the prestress will have substantial loss and it will drop down to the effective prestress. The other difference is that the momentum term, now includes not only the self weight of the web, but it also includes the weight of the cast-in-place flange. Thus, the stress is given as minus  $P_e$  by  $A$ , which is the uniform part, then plus minus  $P_e$  times  $e$  times  $c$  divided by  $I$ , which is the part due to the eccentricity of the prestressing force and the third term is plus minus  $M_{sw}$  plus  $M_{CIP}$  times  $c$  divided by  $I$ . This third term is due to the self weight of the section.

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In this expressions - A is the area of the precast web, c is the distance of edge, whether it is the top or the bottom from CGC of precast web, e is the eccentricity of the CGS with respect to the CGC of the precast web; moment of inertia of the precast web is denoted as I.

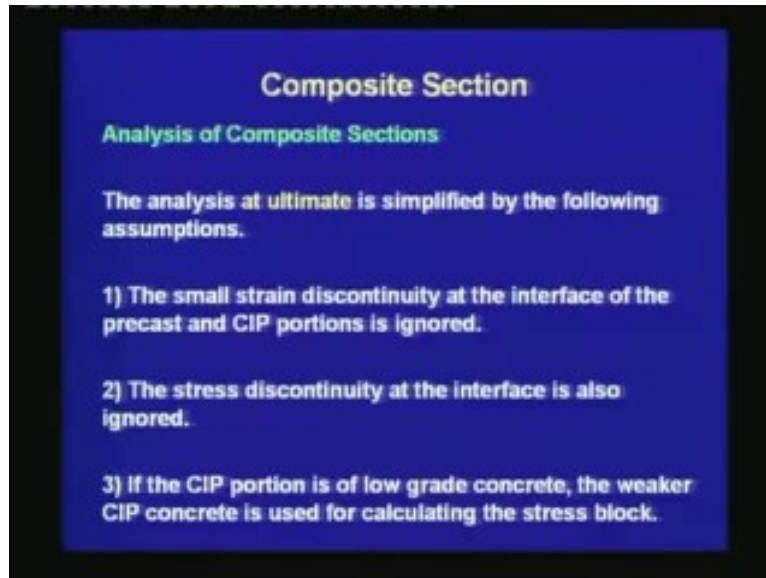
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Next, we do also the analysis at ultimate. From the analysis at ultimate, the ultimate

moment capacity is calculated. This is compared with the demand under factored loads. The procedure is similar to the conventional sections that we have studied earlier, that the analysis is for two stages. First, at transfer and at service which are based on elastic analysis. Then, we do another analysis for the ultimate state, which is based on the non-linear behavior of both concrete and the prestressing steel.

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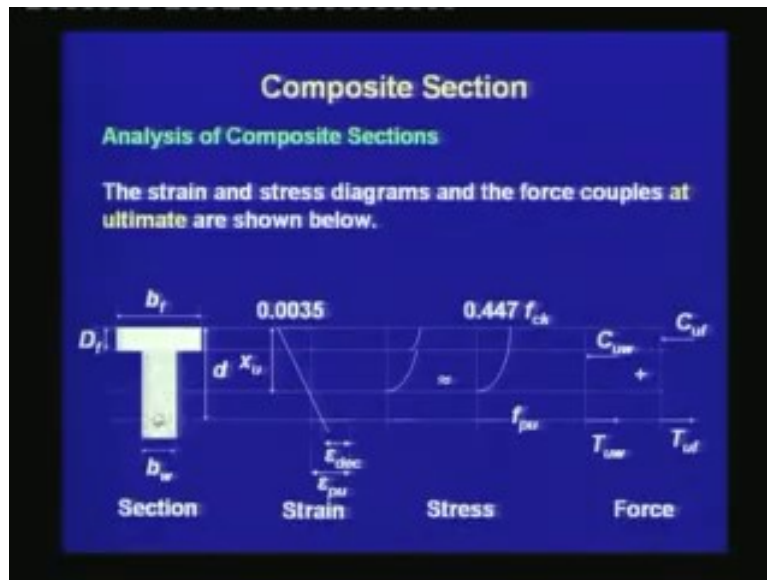


The analysis at ultimate is simplified by the following assumptions. The small strain discontinuity at the interface of the precast and cast-in-place portions is ignored. As I said that, when the flange is cast there is some stress which is interlocked in the web and due to that there is some strain in the top portion of the web. That means there is a strain discontinuity at the interface of the precast portion and the cast-in-place portion. But usually that is ignored in the analysis, for convenience. The second assumption is that the stress discontinuity at the interface is also ignored. Because the web has a stress before the casting of the flange, there will be a stress discontinuity at the interface of the web and the flange. That can also be ignored at the ultimate state, because the material is going into its non-linear plastic behavior. Hence, the discontinuity can be ignored at the ultimate state.

The third is, if the CIP portion is of low grade concrete, the weaker CIP concrete is used

for calculating the stress block. That means if the grades of the web and the flange are different, then the lower grade is selected for analysis.

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The strain and stress diagrams and the force couples at ultimate are shown below. For this composite section,  $b_f$  is the width of the flange and  $D_f$  is the depth of the flange; the flange is cast-in-place.  $b_w$  is the width of the web and  $D$  is the effective depth of the prestressing steel. The strain diagram at ultimate is considered to be linear based on Bernouli's hypothesis and the ultimate strain at the top fiber is considered to be 0.0035,  $x_u$  is the depth of the neutral axis,  $\epsilon_{psu}$  is the strain in the prestressing steel at the compression of concrete and  $\epsilon_{psu}$  is the strain in prestressing steel at ultimate. The evaluation of  $\epsilon_{psu}$  has been covered in the analysis of members. Hence, we are not repeating that calculation in this module.

From the strain diagram, we move on to the stress diagram. In the stress diagram, as I said that there is a discontinuity of the stress in the concrete, but we are neglecting that. We are considering a one stress block with a value of  $0.447 f_{ck}$  at the top most fiber and  $f_{ck}$  is the characteristic strength of the lower grade of concrete,  $f_{psu}$  is the stress in the prestressing steel at ultimate and again the code gives us some recommendation to find out  $x_u$  and  $f_{psu}$ , which were discussed under the analysis of flange sections. So, once we



know the stress diagrams, we now move on to the force diagrams. We split the tension into two components: one part balances the compression in the web and the other part balances the compression in the outstanding flanges.

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**Composite Section**

**Analysis of Composite Sections**

The expressions of the forces are as follows.

$$C_{uw} = 0.36 f_{ck} x_u b_w \quad (9a-3)$$
$$C_{ufl} = 0.447 f_{ck} (b_f - b_w) D_f \quad (9a-4)$$
$$T_{uw} = A_{pw} f_{pu} \quad (9a-5)$$
$$T_{ufl} = A_{pf} f_{pu} \quad (9a-6)$$

These forces are given as follows.  $C_{uw}$  is  $0.36 f_{ck}$  times  $x_u$  times  $b_w$ , this is available from the stress block that we have assumed for throughout the depth of the web.  $C_{ufl}$  is  $0.447 f_{ck}$  times  $b_f$  minus  $b_w$  times  $D_f$ , which is the compression in the outstanding flanges.  $T_{uw}$  is equal to  $A_{pw}$  times  $f_{pu}$ , which is the part of the tension which is balancing the compression in the web and  $T_{ufl}$  is  $A_{pf}$  times  $f_{pu}$ , which is balancing the compression in the outstanding flanges.

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**Composite Section**

**Analysis of Composite Sections**

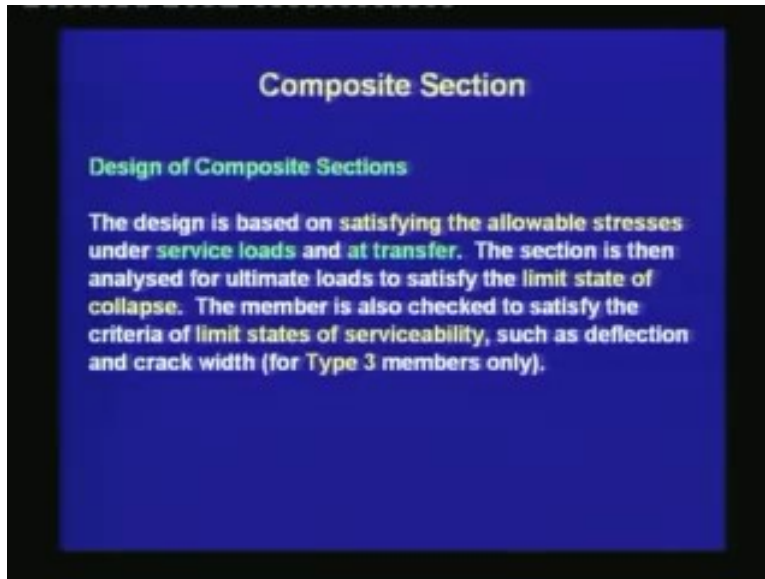
The equilibrium equations are given below. The ultimate moment capacity ( $M_{ur}$ ) is calculated from the second equation.

$$\sum F = 0$$
$$\Rightarrow (A_{pw} + A_{pf})f_{pu} = 0.36f_{ck}x_u b_w + 0.447f_{ck}(b_f - b_w)D_f \quad (9a-7)$$
$$\sum M = 0$$
$$\Rightarrow M_{ur} = A_{pw}f_{pu}(d - 0.42x_u) + A_{pf}f_{pu}(d - 0.5D_f) \quad (9a-8)$$

The equilibrium equations are given below. The ultimate moment capacity  $M_{ur}$  is calculated from the second equation. The first equation is used to calculate  $X_u$  accurately, if you are using the strain compatibility method and the first equilibrium is the equilibrium of the axial forces. This equation is  $A_{pw}$  plus  $A_{pf}$  times  $f_{pu}$ , which is the total tension, is equal to the two compressions, which is  $0.36f_{ck}$  times  $x_u$  times  $b_w$  plus  $0.447f_{ck}$  times  $b_f$  minus  $b_w$  times  $D_f$ ; this is the first equilibrium equation of the axial forces. The second equilibrium equation is regarding the moment.  $M_{ur}$  is equal to the moment due to the first couple, which is  $A_{pw}$  times  $f_{pu}$  times  $d$  minus  $0.42x_u$  plus the moment due to the second couple which is  $A_{pf}$  times  $f_{pu}$  times  $d$  minus  $0.5D_f$  and this equation we have used in the analysis of flange sections. We are not going into the details of this equation in this module.

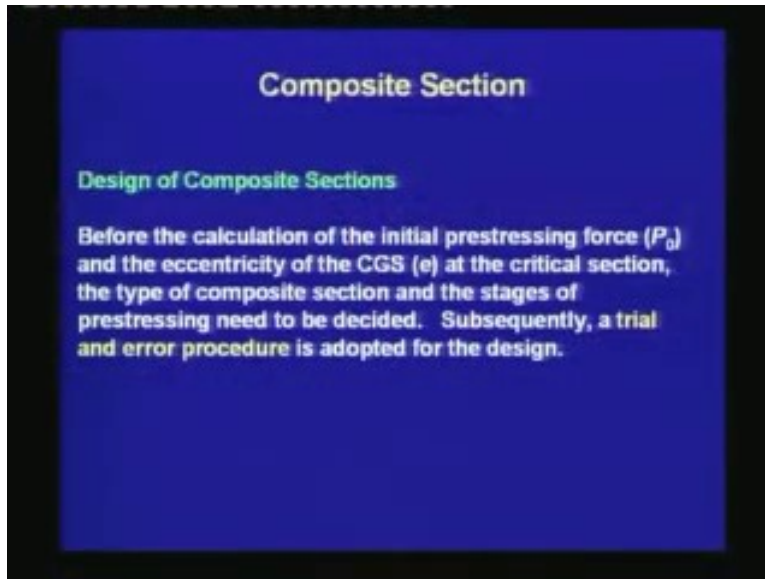
Next we are moving on to the design of the composite sections.

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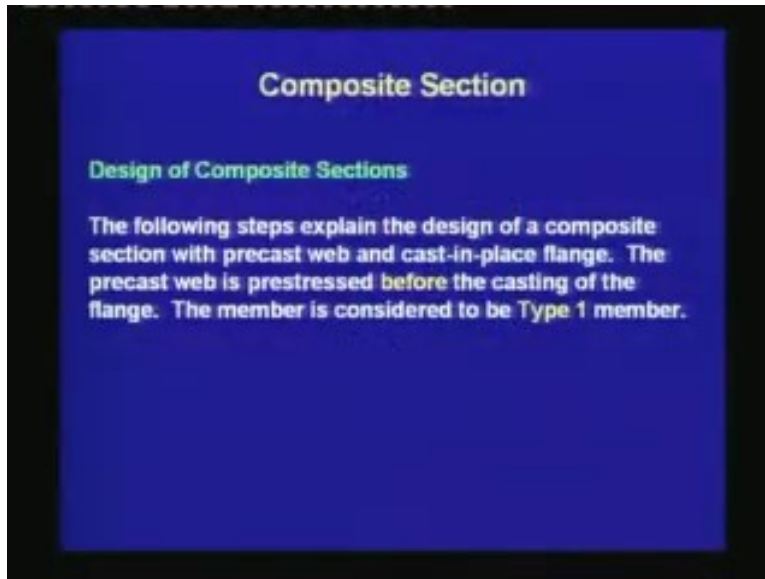
The design is based on satisfying the allowable stresses under service loads and at transfer. The section is then analysed for ultimate loads to satisfy the limit state of collapse. The member is also checked to satisfy the criteria of limit states of serviceability, such as deflection and crack width, for Type 3 members only. Thus, the analysis procedure is similar to that we have seen for conventional sections. But first, the design is based on satisfying the allowable stresses at transfer and at service. Next, we check the moment capacity and make sure that the capacity is greater than the demand under factored loads. We also do the checks for serviceability, the deflection check, if the span to the depth ratio is large. We may also check the crack width, if the member is a Type 3 member and if cracking and the crack width is a concern, then we do the calculations of crack width also.

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Before the calculation of the initial prestressing force which is  $P_0$  and the eccentricity of the CGS  $e$  at the critical section, the type of composite section and the stages of prestressing need to be decided. Subsequently, a trial and error procedure is adopted for the design. That is, before we can evaluate the value of the prestressing force at transfer and the eccentricity, we need to be first clear that what type of composite section we are using and what we will be the method of construction. Once we have decided upon this, then the prestressing force and the eccentricity, they are determined based on the trial and error procedure.

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The following steps explain the design of a composite section with precast web and cast-in-place flange. The precast web is prestressed before the casting of the flange. The member is considered to be Type 1 member. Thus, here we are seeing the design steps for the simple case and the more conventional case. The web is precast, the flange is cast-in-place, the web has been prestressed before the casting of the flange and the member is designed as Type 1 member, where we do not want any tensile stress at transfer or at service.

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**Composite Section**

**Design of Composite Sections**

**Step 1. Compute e.**  
With a trial section of the web, the CGS can be located at the maximum eccentricity ( $e_{max}$ ). The maximum eccentricity is calculated based on zero stress at the top of the precast web. This gives an economical solution.

CGC  $c_t$   
CGS  $c_b$

Web section

Stress profile

$e_{max} = k_b + \frac{M_{sw}}{P_t}$

The step one is after we have decided upon the type of composite section, we are computing the eccentricity  $e$ . With the trial section of the web, the CGS can be located at the maximum eccentricity  $e_{max}$ . The maximum eccentricity is calculated based on zero stress at the top of the precast web. This gives an economical solution. That means, we are pushing down the CGS as low as possible, satisfying the stress condition of zero stress at top during transfer. This gives us an economical solution. Based on this triangular stress block, we can calculate the maximum eccentricity which is  $e_{max}$  is equal to  $k_b$ , which is the depth of the bottom kern point plus  $M_{sw}$  divided by  $P_0$ , where  $M_{sw}$  is the moment due to self weight and  $P_0$  is the prestress at transfer.

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**Composite Section**

**Design of Composite Sections**

Here,

CGC = Centroid of the precast web

$k_b$  = Distance of the bottom kern of the precast web from CGC

$M_{sw}$  = Moment due to self weight of the precast web.

$P_0$  = A trial prestressing force at transfer.

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Remember that the CGC is the centroid of the precast web,  $k_b$  is the distance of the bottom kern of the precast web from CGC,  $M_{sw}$  is the moment due to self weight of the precast web and  $P_0$  is a trial prestressing force at transfer.

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**Composite Section**

**Design of Composite Sections**

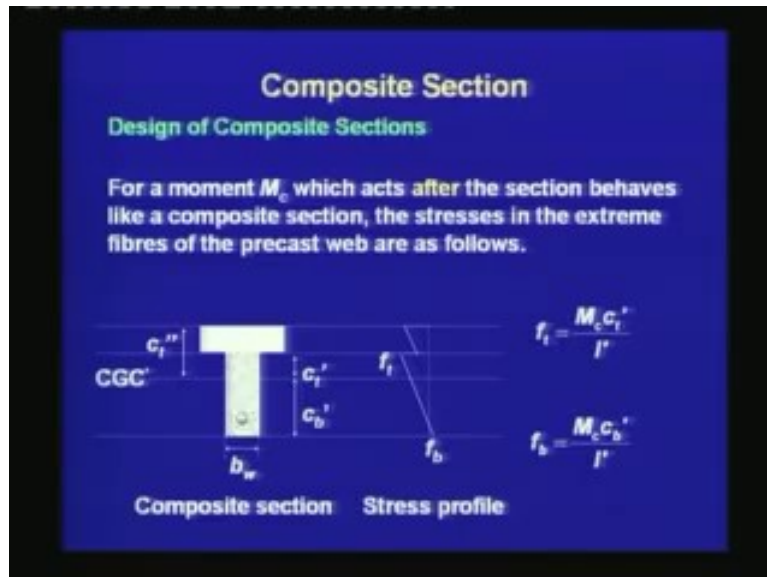
**Step 2. Compute equivalent moment for the precast web.**

A moment acting on the composite section is transformed to an equivalent moment for the precast web. This is done to compute the stresses in the precast web in terms of the properties of the precast web itself and not of the composite section.

Step two is to compute equivalent moment for the precast web. Let us now discuss about the equivalent moment. A moment acting on the composite section is transformed to an

equivalent moment for the precast web. This is done to compute the stresses in the precast web in terms of the properties of the precast web itself and not of the composite section. Thus, the definition of the equivalent moment is for convenience; whatever moment is acting on the composite section we are finding out an equivalent moment, such that we can find out the stresses in the precast web by the properties of the precast web itself and not by the properties of the composite section.

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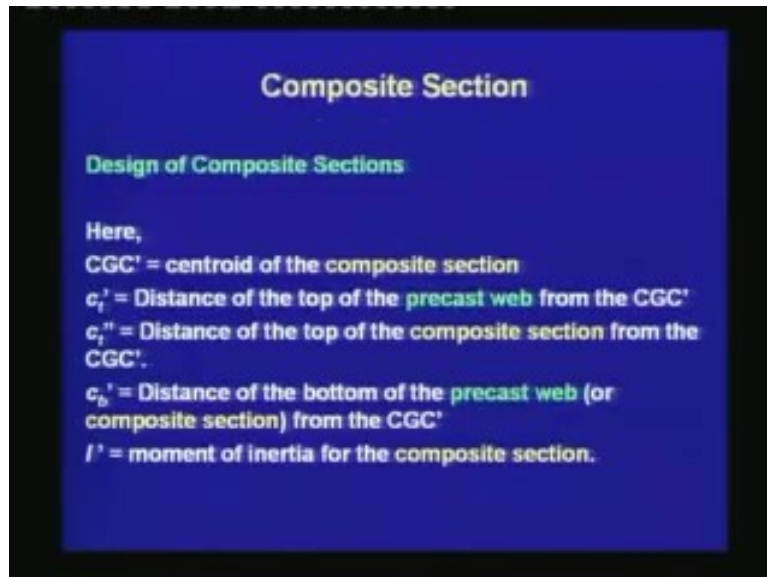


For a moment  $M_c$  which acts after the section behave like a composite section, the stresses in the extreme fibers of the precast web are as follows. For this section which is now behaving like a composite section, let the moment acting be  $M_c$ , where c stands for the composite section. For a composite section, the centroid is different from the centroid of the original precast section. We are referring to the new location of the centroid as CGC prime. The distance of the top fiber of the composite section is denoted as  $C_t$  double prime. The distance of the junction of the precast web and the cast-in-place flange is denoted as  $C_t$  prime and the distance of the bottom fiber of the web from the CGC prime is denoted as  $C_b$  prime. Note that all these distances are been measured from the new centroid of the composite section. The stress block is now more difficult and we have to understand this stress block properly. For the composite section, we have a stress at the top of the web which is higher than the neighboring fiber in the flange. Hence, there is a



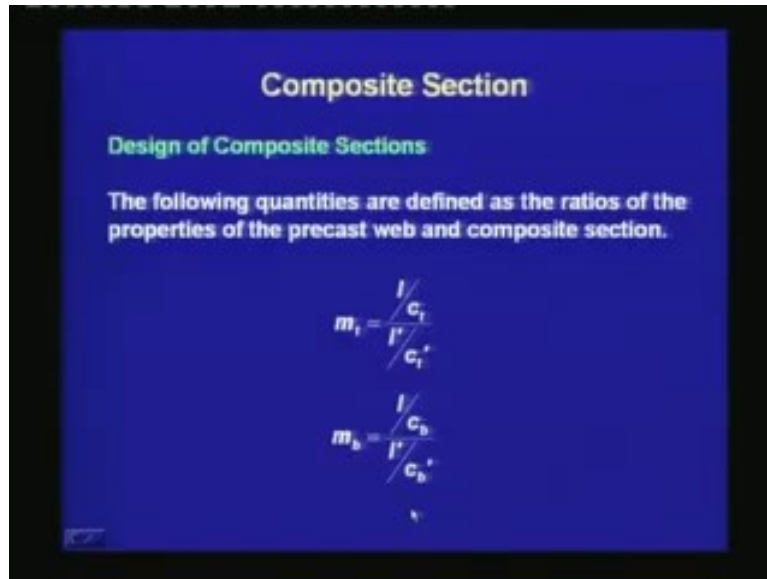
discontinuity in the stress block which we need to consider in the analysis at service loads and  $f_t$  is the stress in the precast web at the top and  $f_b$  is the stress in the precast web at the bottom. For the moment  $M_c$ ,  $f_t$  is given as  $M_c$  times the distance  $C_t$  prime divided by  $I$  prime, where  $I$  prime is the moment of inertia of the composite section, which is different from the  $I$  of the precast web alone and  $f_b$  is given as  $M_c$  times  $C_b$  prime divided by  $I$  prime.

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Again here CGC prime is the centroid of the composite section,  $C_t$  prime is the distance of the top of the precast web from the CGC prime,  $C_t$  double prime is the distance of the top of the composite section from the CGC prime,  $C_b$  prime is the distance of the bottom of the precast web or composite section from the CGC prime and  $I$  prime is the moment of inertia for the composite section.

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The following quantities are defined as the ratios of the properties of the precast web and the composite section. We have two sets of geometric variables: one for the precast web and the other for the composite section. For convenience of calculation, we are taking ratios of this quantity. The first quantity  $m_t$  is the ratio of  $I$  divided by  $C_t$ , for the precast web divided by  $I$  prime divided by  $C_t$  prime; the prime numbers are for the composite section. Thus, it is the ratio of the moment of inertia divided by the distance of the top flange for the precast web and the composite section.

$m_b$  is similarly defined as  $I$  divided by  $C_b$  divided by  $I$  prime divided by  $C_b$  prime. That means it is the ratio of the moment of inertia divided by the distance of the bottom fiber of the web. The numerator values are for the precast section and the denominator values are for the composite section.

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**Composite Section**

**Design of Composite Sections:**

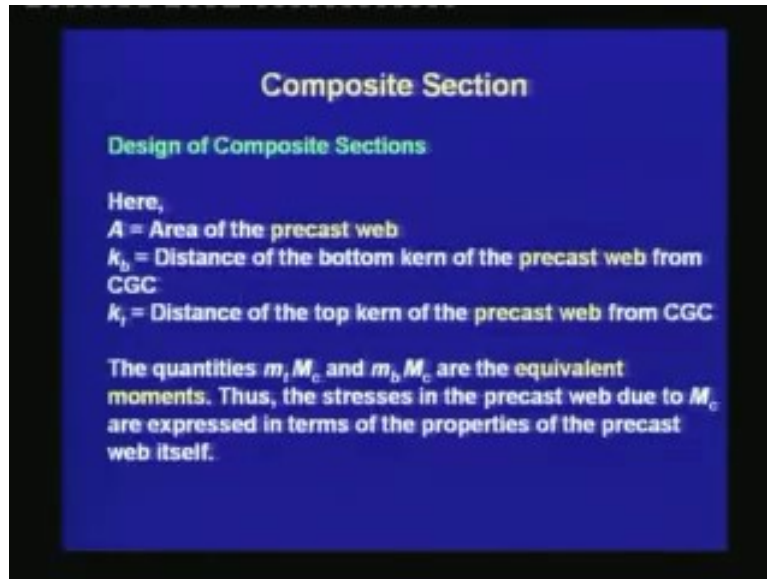
Then the stresses in the extreme fibres of the precast web can be expressed in terms of  $m_t$  and  $m_b$  as follows.

$$f_t = \frac{m_t M_c c_t}{I} = \frac{m_t M_c}{A k_b} \quad (9a-9)$$
$$f_b = \frac{m_b M_c c_b}{I} = \frac{m_b M_c}{A k_t} \quad (9a-10)$$

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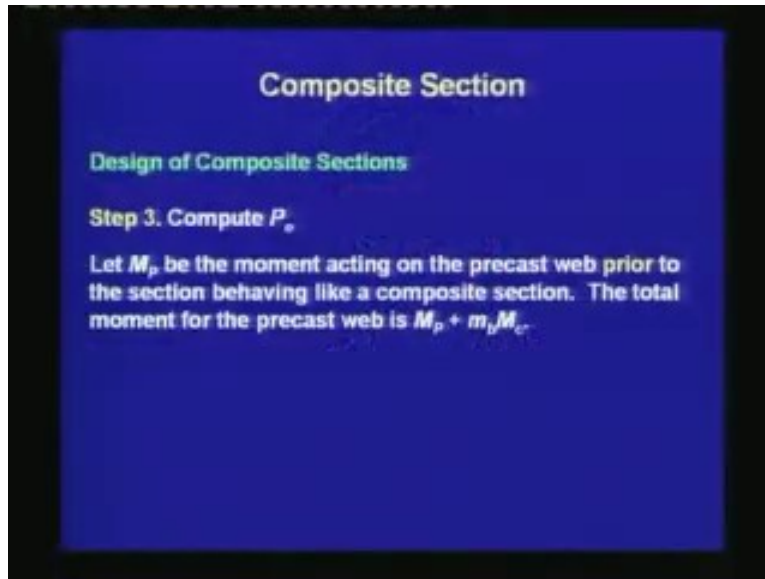
Then the stresses in the extreme fibers of the precast web, under the moment  $m_c$ , can be expressed in terms of  $m_t$  and  $m_b$  as follows.  $f_t$  is equal to  $m_t$  times  $M_c$  times  $C_t$  divided by  $I$  is equal to  $m_t$  times  $M_c$  divided by  $A$  times  $k_b$ . This stresses can be derived once we substitute the definitions of  $m_t$  and  $m_b$ . Similarly  $f_b$  is given as  $m_b$  times  $m_c$  times  $C_b$  divided by  $I$  which is equal to  $m_b$  times  $M_c$  divided by  $A$  times  $k_t$ . Note that the stress in the precast web  $f_t$  and  $f_b$  has been defined in terms of the properties of the precast web only, which is the  $A$ ,  $k_b$  and  $k_t$ , provided we know the values of  $m_t$  and  $m_b$ . So, the purpose of defining small  $m_t$  and  $m_b$  is to express the stress in the precast web in terms of the section properties of the web itself.

(Refer Slide Time 36:14)



Here,  $A$  is the area of the precast web,  $k_b$  is the distance of the bottom kern of the precast web from CGC,  $k_t$  is the distance of the top kern of the precast web from CGC. Remember that we are defining this quantity from the CGC of the precast web itself. The quantities  $m_t M_c$  and  $m_b M_c$  are the equivalent moments. Thus, the stresses in the precast web due to  $M_c$  are expressed in terms of the properties of the precast web itself. The sole purpose of defining the equivalent moment is to have an expression of the stresses in the precast web which involves the properties of the precast web only. The properties of composite sections have already gone into the definitions of the ratios  $m_t$  and  $m_b$ .

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Step three is to compute the effective prestress  $P_e$ . Let  $M_p$  be the moment acting on the precast web prior to the section behaving like a composite section. The total moment for the precast web is  $M_p$  plus  $m_b$  times  $M_c$ . If we know the moment that was acting on the precast web before the section started acting like a composite section, let that moment be denoted as  $M_p$ . Next, we are having a moment  $M_c$  which acts on the composite section and in order to find out the stresses in the precast concrete we are defining an equivalent moment which is called  $m_b$  times  $M_c$  and that is added to  $M_p$ , to calculate the stresses in the precast web. The stress at the bottom for type 1 members due to service loads is zero.

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**Composite Section**

**Design of Composite Sections**

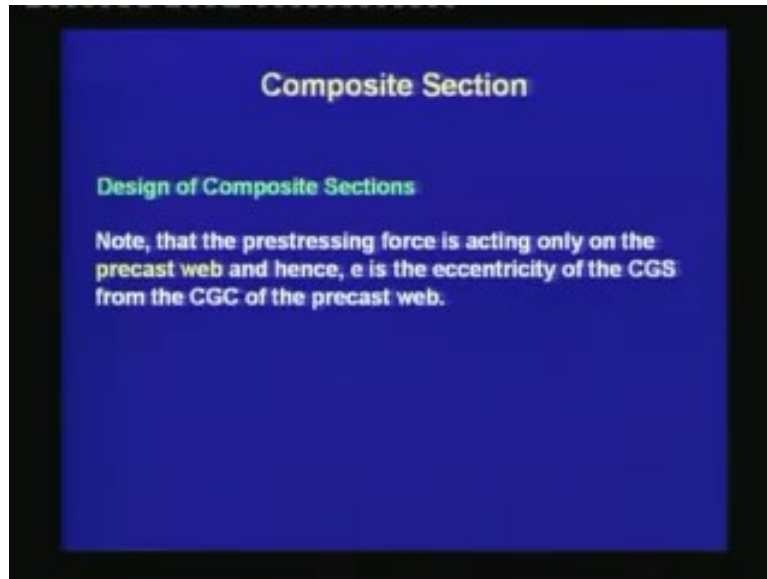
The stress at the bottom for Type 1 member due to service loads is zero.

Therefore, 
$$\frac{P_e}{A} - \frac{P_e e}{Ak_t} + \frac{M_p + m_b M_c}{Ak_t} = 0$$

or, 
$$P_e = \frac{(M_p + m_b M_c)}{e + k_t} \quad (9a-11)$$

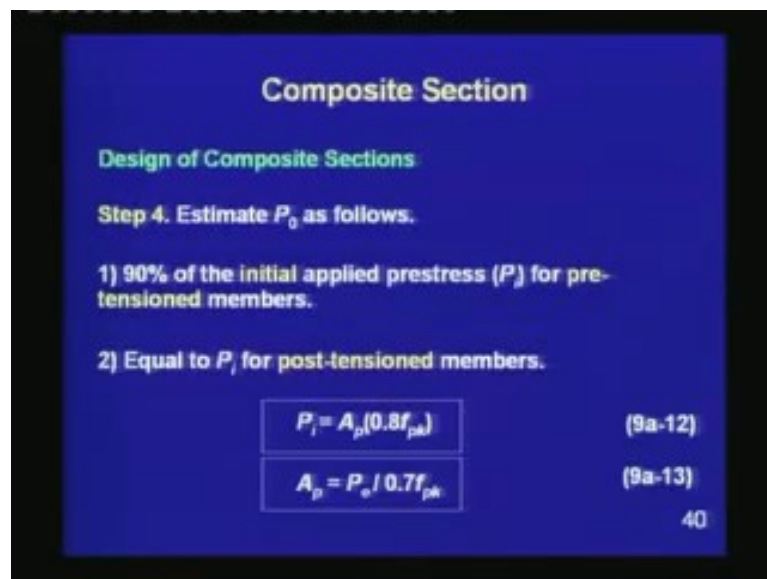
This is the definition of type 1 member. We can have zero stress at the bottom under service loads and therefore, we are substituting the expressions in the stress and we are substituting 0 on the right hand side. That means resultant stress due to the effective prestress force, the eccentricity of the prestressing force, the total moment acting in the web, the final stress resultant at service for a Type 1 member is equal to 0. From this we are calculating the prestress that is required under service loads.  $P_e$  is equal  $M_p$  plus  $m_b$  times  $M_c$  divided by  $e$  plus  $k_t$ . This is the expression that is used to calculate the effective prestress once we know the eccentricity.

(Refer Slide Time 39:15)



Note that the prestressing force is acting only on the precast web and hence,  $e$  is the eccentricity of the CGS from the CGC of the precast web. I said before that if the precast web is prestressed before the top concrete has hardened, then the prestress is carried by the web only. Hence when we are calculating stresses due to the prestress, we are still using only the properties of the web part. The eccentricity is measured from the CGC of the web.

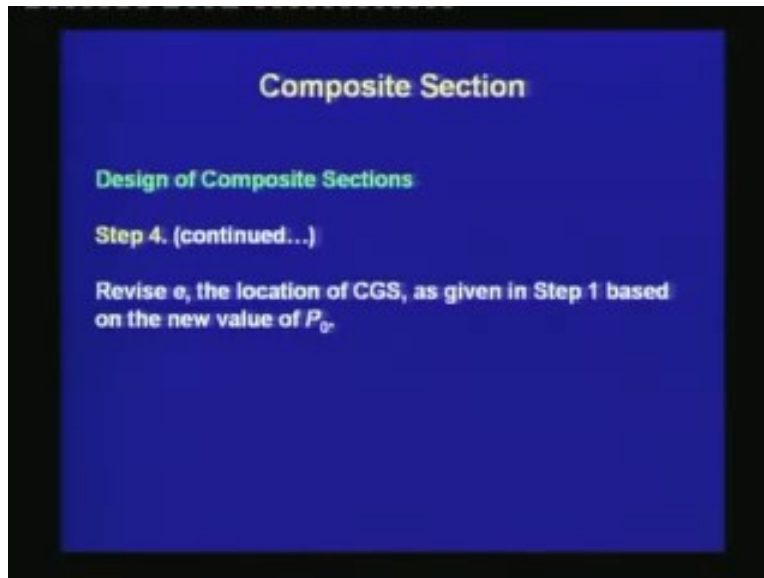
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The fourth step is to estimate  $P_0$  from  $P_e$ . This is done as follows. It is 90% of the initial applied prestress, which is  $P_i$  for a pre-tensioned member; 90% because we are considering the elastic shortening. For a post-tensioned member the elastic shortening occurs during the application of the prestressing force, if the tendons are **stressed or stretched** simultaneously. In that case, we can consider that  $P_0$  is equal to  $P_i$  which we are measuring by the jacks for post-tensioned members.  $P_i$  can be  $A_p$  times 80% of the characteristic strength  $f_{pk}$  and  $A_p$  is calculated from the effective prestress;  $P_e$  divided by about 70% of the characteristic strength, will give us the value of effective prestress.

Thus, once we know the effective prestress from the previous expression, we can calculate the amount of steel that is required by estimating an effective prestress of 70% of the characteristic strength. From the amount of prestressing steel, we are calculating the maximum possible prestressing force at transfer based on the maximum prestress during transfer which is 80% of the characteristic strength. Like this we can estimate  $P_0$  from  $P_e$  that we have calculated in the previous step.

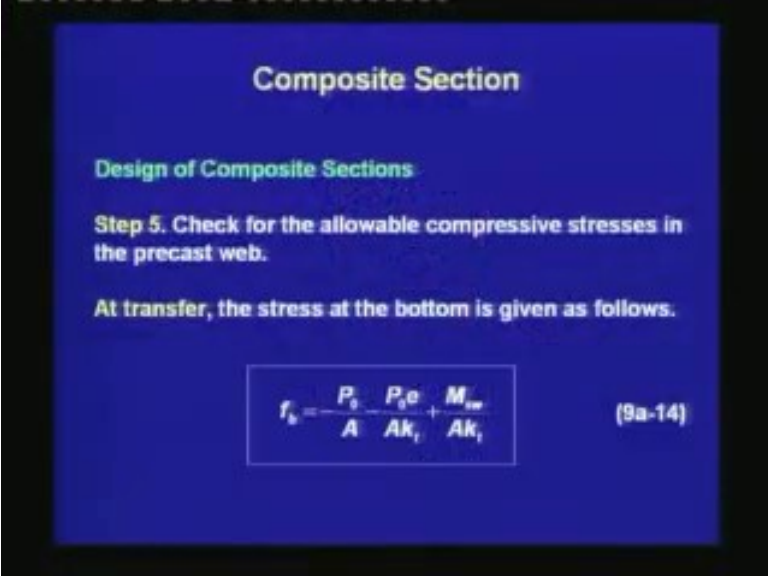
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Step four then continues as revise  $e$ , the location of CGS, as given in step 1 based on new value of  $P_0$ . We have seen that expansion of  $e$  in step one and we can revise the calculation of  $e$  based on the new value of  $P_0$ .



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**Composite Section**

**Design of Composite Sections**

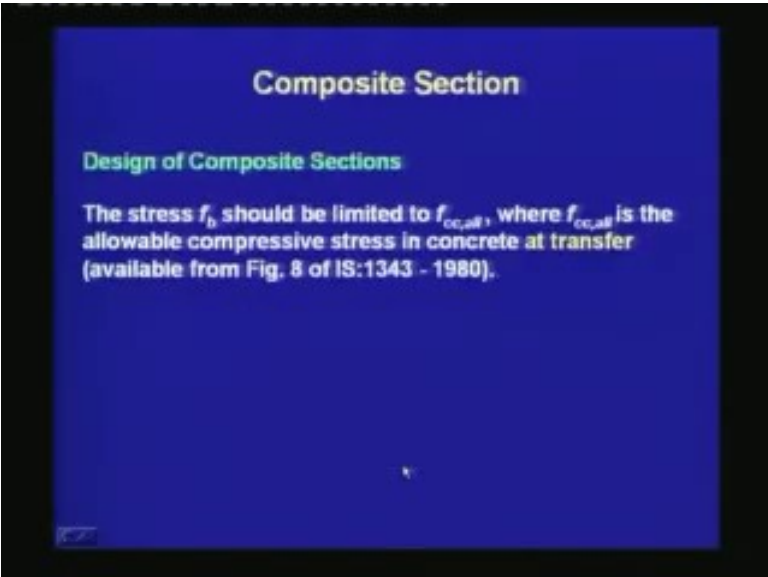
**Step 5. Check for the allowable compressive stresses in the precast web.**

At transfer, the stress at the bottom is given as follows.

$$f_b = \frac{P_s}{A} - \frac{P_s e}{Ak_1} + \frac{M_s}{Ak_1} \quad (9a-14)$$

Step five, checks for the allowable compressive stresses in the precast web. At transfer, the stress at the bottom is given as follows. Again this is the expression that we had seen earlier during the analysis.

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**Composite Section**

**Design of Composite Sections**

The stress  $f_b$  should be limited to  $f_{cc,all}$ , where  $f_{cc,all}$  is the allowable compressive stress in concrete at transfer (available from Fig. 8 of IS:1343 - 1980).

The stress  $f_b$  should be limited to  $f_{cc,allowable}$ , where  $f_{cc,allowable}$  is the allowable compressive stress in concrete at transfer and which is available from figure 8 of IS: 1343.

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**Composite Section**  
Design of Composite Sections

At service,

$$f_t = -\frac{P_e}{A} - \frac{P_e e}{Ak_b} + \frac{(M_p + m_t M_c)}{Ak_b} \quad (9a-15)$$

The stress  $f_t$  should be limited to  $f_{cc,all}$ , where  $f_{cc,all}$  is the allowable compressive stress in concrete under service loads (available from Fig. 7 of IS:1343-1980). If the stress conditions are not satisfied, increase  $A$ .

At service the stress is given as  $f_t$  is equal to minus  $P_e$  by  $A$  minus  $P_e$  times  $e$  divided by  $Ak_b$  plus the total moment  $m_t$  plus equivalent moment of  $M_c$  divided by  $Ak_b$ . The stress  $f_t$  should be limited to  $f_{cc,allowable}$  where  $f_{cc,allowable}$  is the allowable compressive stress in concrete under service loads and this is available from figure 7 of IS: 1343. If the stress conditions are not satisfied, then we have to increase area of the precast web, that is  $A$ .

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**Composite Section**  
Design of Composite Sections

Step 6. Check for the allowable compressive stress in the CIP flange.

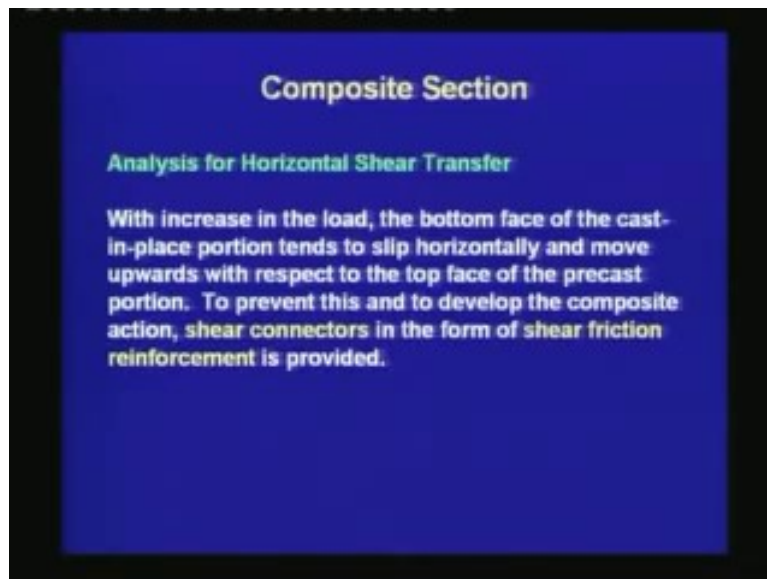
$$f_t = \frac{M_c c_t''}{I'} \quad (9a-16)$$

The stress  $f_t$  should be limited to  $f_{cc,all}$  where  $f_{cc,all}$  is the allowable compressive stress in concrete under service loads.

45

Step six is to check for the allowable compressive stress in the CIP flange. That means, till now we have checked the stresses in the web, but now we are checking the stresses for the flange.  $f_t$  prime which is the maximum compressive stress in the flange is given as  $M_c$ , which is the moment acting in the composite section times  $c_t$  prime divided by  $I$  prime. The stress  $f_t$  prime should be limited to  $f_{cc, allowable}$  where  $f_{cc, allowable}$  is the allowable compressive stress in concrete under service loads for the cast-in-place flange. Another important consideration for a composite section is the analysis for horizontal shear transfer.

(Refer Slide Time 43:56)



With increase in the load, the bottom face of the cast-in-place portion tends to slip horizontally and move upwards with respect to the top face of the precast portion. To prevent this and to develop the composite action, shear connectors in the form of shear friction reinforcement is provided. That means, since the precast web and the cast-in-place flange are trying to deform together, there is a tendency of slip at the interface and the web may differ more than the flange and there is a chance of vertical separation at the interface. In order to check the horizontal slip and the vertical separation, we provide interface reinforcement which is called shear friction reinforcement.

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**Composite Section**

**Analysis for Horizontal Shear Transfer**

The required shear friction reinforcement (per metre) is calculated as follows.

$$A_{sw} = \frac{1000b_v \tau_h}{0.87f_y \mu} \quad (9a-17)$$

The minimum requirements of shear friction reinforcement and spacing are similar to that for shear reinforcement in the web.

The required shear friction reinforcement per meter is calculated as follows:  $A_{sw}$  is equal to 1000 times  $b_v$  times  $\tau_h$  divided by  $0.87f_y$  times  $\mu$ . The minimum requirements of shear friction reinforcement and spacing are similar to that for the shear reinforcement in the web. Thus, this expression gives us the amount of shear friction reinforcement that is required, but the minimum values and the maximum spacing are governed by the web reinforcement.

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**Composite Section**

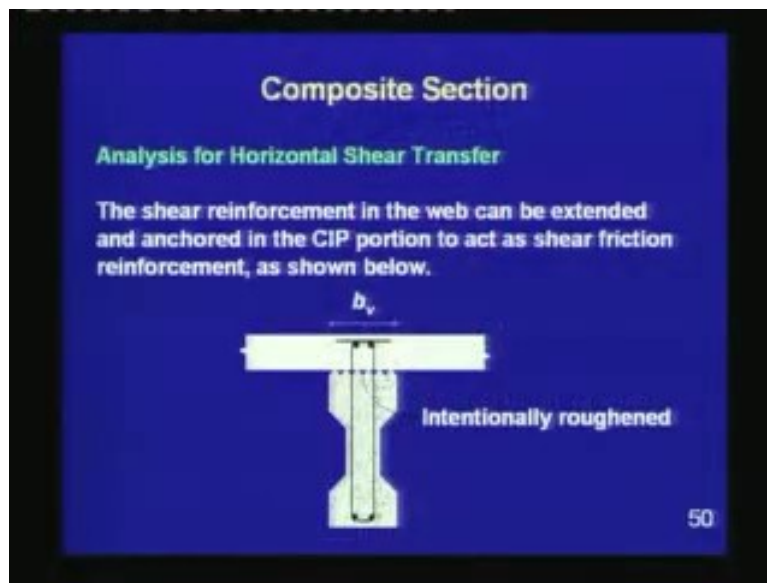
**Analysis for Horizontal Shear Transfer**

In the previous equation,

$A_{sw}$  = area of shear friction reinforcement in  $\text{mm}^2/\text{m}$   
 $b_v$  = width of the interface of precast and CIP portions  
 $\tau_h$  = horizontal shear stress at the interface in  $\text{N}/\text{mm}^2$   
 $f_y$  = yield stress in  $\text{N}/\text{mm}^2$   
 $\mu$  = coefficient of friction  
= 1.0 for intentionally roughened interface with normal weight concrete

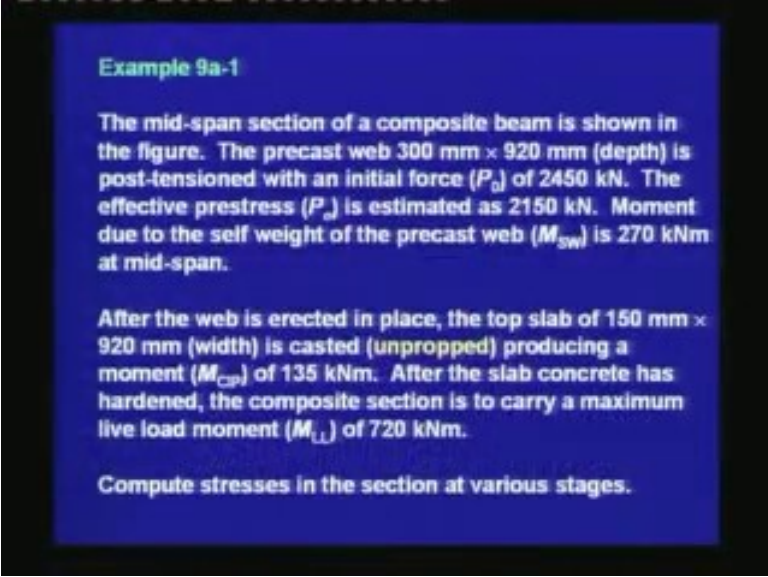
In this expression,  $A_{sv}$  is the area of shear friction reinforcement in millimeter square per meter;  $b_v$  is the width of the interface of precast and CIP portions,  $\tau_h$  is the horizontal shear stress at the interface in Newton per millimeter square,  $f_y$  is the yield stress of shear friction reinforcement in Newton per millimeter square,  $\mu$  is the coefficient of friction at the interface and it is equal to 1 for intentionally roughened interface with normal weight concrete. Thus, before laying the cast-in-place flange, the interface is intentionally roughened, so that there is a good friction between the flange and the web.

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The shear friction reinforcement in the web can be extended and anchored in the cast-in-place portion to act as shear friction reinforcement, as below. That means we do not need additional shear friction reinforcement. The shear reinforcement that is used in the web is extended beyond the web, so that when the concrete in the flange is cast, that shear reinforcement acts as shear friction reinforcement. In this sketch  $b_v$  is the width of the interface and you observe that the shear reinforcement has been extended above the web and it is within the flange and the interface has been intentionally roughened to have a friction between the cast-in-place concrete with the precast concrete.

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**Example 9a-1**

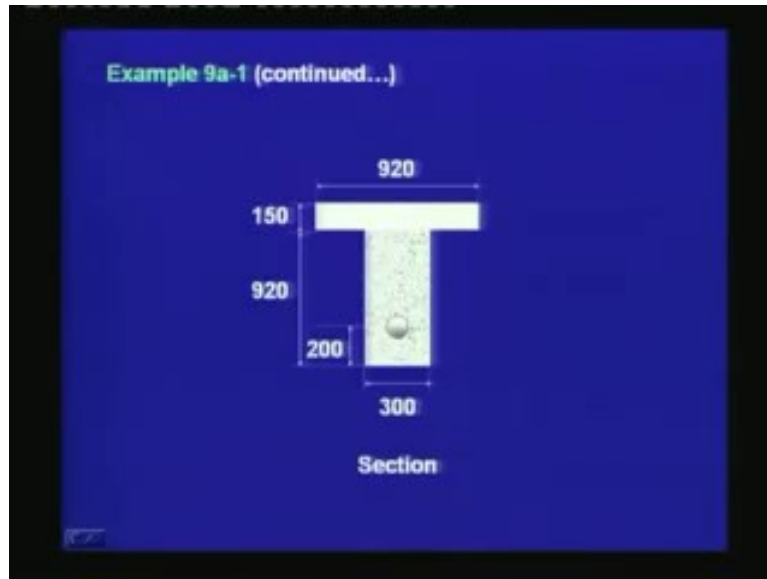
The mid-span section of a composite beam is shown in the figure. The precast web 300 mm × 920 mm (depth) is post-tensioned with an initial force ( $P_0$ ) of 2450 kN. The effective prestress ( $P_e$ ) is estimated as 2150 kN. Moment due to the self weight of the precast web ( $M_{sw}$ ) is 270 kNm at mid-span.

After the web is erected in place, the top slab of 150 mm × 920 mm (width) is casted (unpropped) producing a moment ( $M_{CIP}$ ) of 135 kNm. After the slab concrete has hardened, the composite section is to carry a maximum live load moment ( $M_{LL}$ ) of 720 kNm.

Compute stresses in the section at various stages.

Let us now understand the analysis of a composite section by the help of an example. The mid-span of a composite beam is shown in the figure, which is to follow in the next slide. The precast web 300 millimeter times 920 millimeter in depth is post-tensioned with an initial force  $P_0$  of 2450 Kilonewton. The effective prestress  $P_e$  is estimated as 2150 Kilonewtons, moment due to the self weight of the precast web  $M_{sw}$  is 270 Kilonewton meter at mid-span. After the web is erected in place, the top slab of 150 millimeters times 920 millimeters in width is casted unpropped, producing a moment  $M_{CIP}$  of 135 Kilonewton meter. After the slab concrete has hardened the composite section is to carry a maximum live load moment  $M_{LL}$  of 720 Kilonewton meter. Compute stresses in the section at various stages.

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Thus, for this particular section the web is precast. The width is 300, the depth is 920 and the CGS is located 200 millimeters from the bottom of the web. After the post-tensioning operation has been done, the concrete in the flange has been placed, during which the web was unpropped. The width of the flange is 920 millimeters and depth is 150 millimeters. The initial prestress and the final prestress values are given and we have been asked to find out the stress conditions at the different load stages.

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**Solution**

1) Calculation of geometric properties.

**Precast web**

$A = 2.76 \times 10^5 \text{ mm}^2$

$I = 1.95 \times 10^{10} \text{ mm}^4$

Distance of CGC from bottom = 460 mm.

**Composite section**

$A' = 4.14 \times 10^5 \text{ mm}^2$

$I' = 4.62 \times 10^{10} \text{ mm}^4$

Distance of CGC' from bottom = 638 mm.

638

460

CGC'

CGC



First we are calculating the geometric properties. For the precast web the area is given as 2.76 times 10 to the power of 5 millimeter square. I is equal to 1.95 times 10 to the power 10 millimeters square, distance of CGC from bottom is half of 920 which is 460 millimeters. For the composite section, we have another set of properties where A prime is equal to 4.14 times 10 to the power 5 millimeter square, I prime is equal to 4.62 times 10 to the power of 10 millimeter square, distance of CGC prime from bottom is 638 millimeters. Note that the area of the composite section is larger, the moment of inertia is larger and the CGC prime has moved upwards due to the placement of the flange. Thus, the calculated values make sense with our intuition that after the section behaves like a composite section the moment of inertia will be large; the centroid will also shift up.

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**Solution**

2) Calculation of stresses in web at transfer

$$f = -\frac{P_0}{A} \pm \frac{P_0 e c}{I} \pm \frac{M_{sw} c}{I}$$

$$= -\frac{2450 \times 10^3}{2.76 \times 10^5} \pm \frac{2450 \times 10^3 \times 260 \times 460}{1.95 \times 10^{10}} \pm \frac{270 \times 10^6 \times 460}{1.95 \times 10^{10}}$$

= -0.22 N/mm<sup>2</sup>                      At top fibre

= -17.54 N/mm<sup>2</sup>                      At bottom fibre

Second is calculation of stresses in web at transfer. From the conventional formula we are placing the values of P<sub>0</sub>, area of the precast web, I of the precast web, c is 460 for the top and bottom fiber of the precast web and the self weight of the precast web is also given. Once we substitute these values we get the stress of minus 0.22 Newton per millimeter square at top fiber and minus 17.5 Newton per millimeter square at bottom fiber of the web at transfer. We have now calculated the stress condition of the web when the prestress is transferred to the section.



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**Solution**

3) Calculation of stresses in web after long term losses

$$f = -\frac{P_e}{A} \pm \frac{P_e e c}{I} \pm \frac{M_{sw} c}{I}$$
$$= -\frac{2150 \times 10^3}{2.76 \times 10^5} \pm \frac{2150 \times 10^3 \times 260 \times 460}{1.95 \times 10^{10}} \pm \frac{270 \times 10^6 \times 460}{1.95 \times 10^{10}}$$

    = -0.97 N/mm<sup>2</sup>                      At top fibre

    = -14.61 N/mm<sup>2</sup>                     At bottom fibre

55

The next step is the calculation of the stresses in the web after long term losses. That means, before the cast-in-place concrete has been poured we are recomputing the stresses in the web, considering the drop in the prestress. With the effective prestress is equal to 2150 times 10 to the power 3 Newtons we are having a stress of minus 0.97 Newton per millimeter square at the top and minus 14.61 Newton per millimeter square at the bottom. This is the stress condition in the web just before the casting of the flange.

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**Solution**

4) Calculation of stresses in web after casting of flange

$$f = -\frac{P_e}{A} \pm \frac{P_e e c}{I} \pm \frac{(M_{sw} + M_{cp}) c}{I}$$
$$= -\frac{2150 \times 10^3}{2.76 \times 10^5} \pm \frac{2150 \times 10^3 \times 260 \times 460}{1.95 \times 10^{10}} \pm \frac{(270 + 135) \times 10^6 \times 460}{1.95 \times 10^{10}}$$

    = -4.16 N/mm<sup>2</sup>                      At top fibre

    = -11.42 N/mm<sup>2</sup>                     At bottom fibre

The fourth step is calculation of stresses in web after casting of the flange. Once the flange has been cast, we have to add the moment due to web of the cast-in-place flange. The first two terms remains same as the previous load stage. It is in the third term, where we are making a change we are bringing in  $M_{CIP}$ , which is the moment due to the web of the cast-in-place flange. Once we substitute the value of  $M_{CIP}$  we get the stress of minus 4.16 Newton per millimeters square and minus 11.42 Newton millimeters square at the top and bottom respectively of the precast web.

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**Solution**

5) Calculation of stresses in the composite section at service

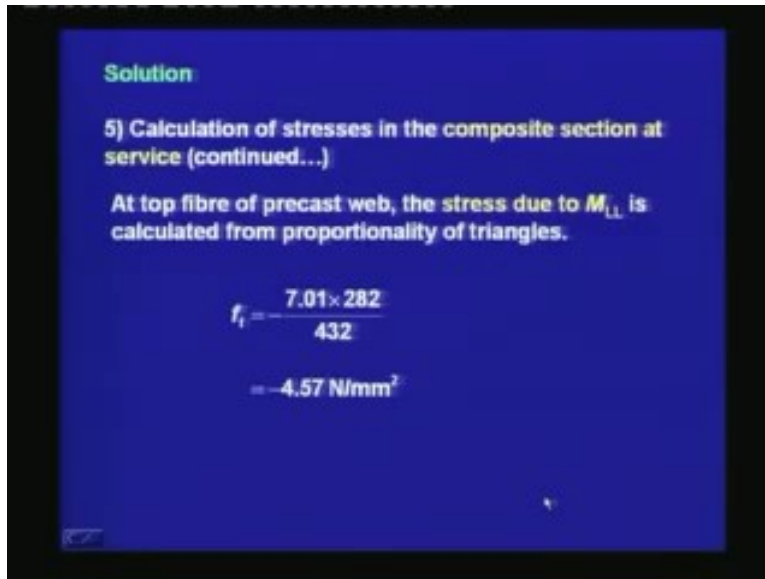
Stress due to  $M_{LL}$

At top fibre	At bottom fibre
$f_t' = -\frac{M_{LL} c_t''}{I'}$	$f_b = \frac{M_{LL} c_b'}{I'}$
$= -\frac{750 \times 10^3 \times 432}{4.62 \times 10^{10}}$	$= \frac{750 \times 10^3 \times 638}{4.62 \times 10^{10}}$
$= -7.01 \text{ N/mm}^2$	$= 10.36 \text{ N/mm}^2$

Next, we are calculating the stresses in the composite section at service loads. Now, the flange has hardened, the live load is acting and we are finding out the stresses under service loads. First, we are calculating the stresses due to moment due to the live load. At the top fiber of the flange  $f_t'$  prime is given as  $M_{LL}$  times  $c_t$  double prime divided by  $I$  prime. Once we substitute the value of  $M_{LL}$ ,  $c_t$  prime we can calculate from the location of CGC prime and  $I$  prime is the property of composite section. We find  $f_t'$  prime is equal to minus 7.01 Newton per millimeter square. This is the stress at the top of the flange.

At the bottom fiber the value is  $f_b$  is equal to  $M_{LL}$  times  $c_b$  prime divided by  $I$  prime. Note, that we are using the properties of the composite section for the live load moment. Once we substitute the values, we find out  $f_b$  is equal to 10.36 Newton per millimeter square.

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**Solution**

5) Calculation of stresses in the composite section at service (continued...)

At top fibre of precast web, the stress due to  $M_{LL}$  is calculated from proportionality of triangles.

$$f_t = -\frac{7.01 \times 282}{432}$$
$$= -4.57 \text{ N/mm}^2$$

Calculation of stresses in the composite section at service loads will now include the calculation at the top fiber of the precast web also. This is calculated from the proportionality of the triangles. We know the stress at the top of the flange; we know the location of the CGC prime. From this we can calculate the stress in the web at the top fiber of the web. That is given as  $f_t$  is equal to the stress at the top of the flange which is 7.01 divided by the depth of CGC prime which is 432 times the distance of the top of the web from CGC prime which is 282. This gives us a stress at the top fiber of the web as minus 4.57 Newton per millimeter square. Once we have calculated the stresses due to the live load we are now adding them with the stresses that has been locked into the web to get the total stresses under service loads.

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**Solution**

5) Calculation of stresses in the composite section at service (continued...)

Total stress in precast web

At top fibre:	At bottom fibre:
$f_t = -4.16 - 4.57$ $= -8.73 \text{ N/mm}^2$	$f_b = -11.42 + 10.36$ $= -1.06 \text{ N/mm}^2$

For the precast web at the top fiber, the total stress is given as minus 4.16 which was there during the casting of the flange, minus 4.57 which has come from the live load moment; the total stress is minus 8.73 Newton per millimeter square. At bottom fiber, it is minus 11.42, which was there during the casting of the flange plus 10.36, which is the tensile stress due to the live load, which gives a resultant compressive stress of minus 1.06 Newton per millimeter square.

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**Solution**

5) Calculation of stresses in the composite section at service (continued...)

Total stress in CIP slab

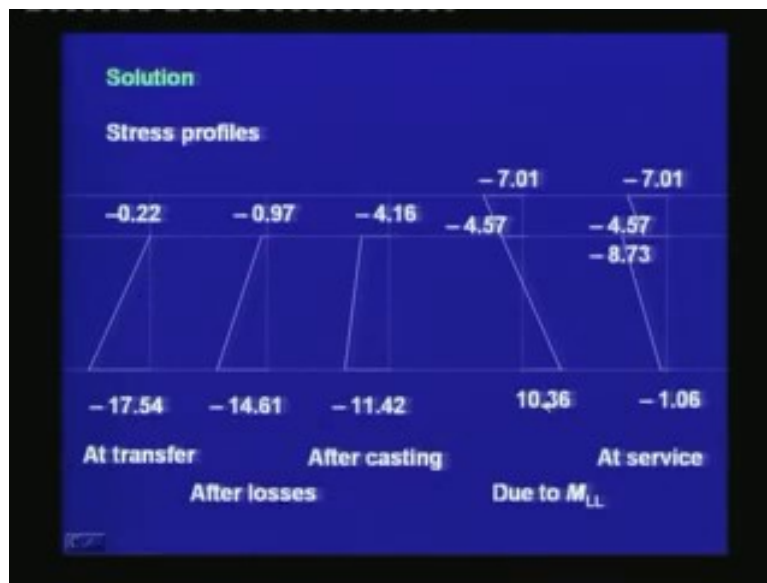
The total stress is due to  $M_{LL}$  only.

At top fibre:	At bottom fibre:
$f_t' = -7.01 \text{ N/mm}^2$	$f_b' = -4.57 \text{ N/mm}^2$

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For total stress in CIP slab, the total stress is due to  $M_{LL}$  only. Note that the CIP slab does not take any stress from the prestressing force or the self weight, because it was an unpropped construction. The only stress that comes in the CIP slab is due to the live load only, when the whole section is behaving like a composite section. Hence, the calculations remain same as that for the live load that  $f_t$  prime at the top is minus 7.01 Newton per millimeter square. The bottom stress is the same that we calculated for the top fiber in the precast web, which is minus 4.57 Newton per millimeter square.

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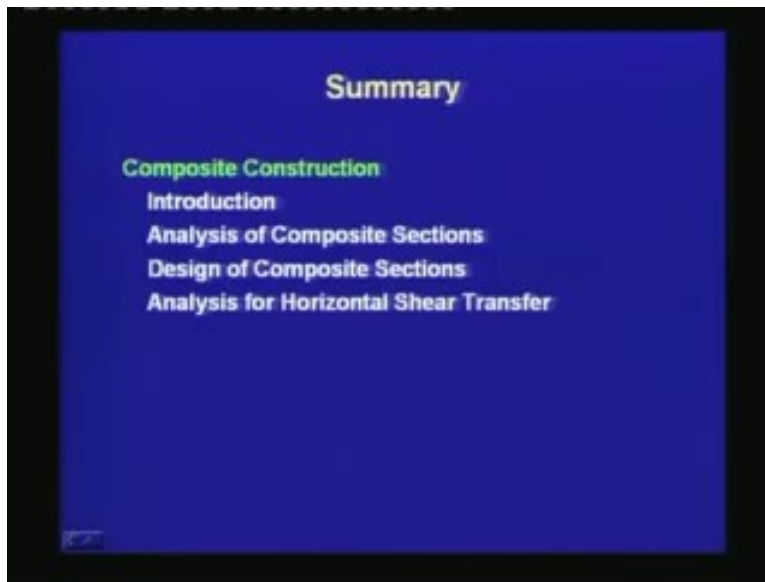


Once we plot stress profiles we understand the effect of composite section. At transfer we had a stress profile like this; there were a substantial compression at the bottom, very little compression at the top, almost negligible. Then after losses the stresses have dropped, the compressive stress at the bottom has reduced, as well as the stress at the top has slightly increased because of the drop in the prestressing force. Now, we are casting the concrete and after the casting of concrete again we can find that the precast web is carrying the weight of the flange and the stresses have increased.

Due to the live load, we have a full composite section resisting and we are able to find out the stress in the top of the flange, at the interface, as well as at the bottom of the composite section. Finally, we get the resultant stress block where we are adding up the

stress that has been locked in the web. Thus, due to the live load we get a stress in the web and the stress in the flange is same as that due to the live load. Thus, the final resultant stress block is minus 7.01 in the flange, minus 4.57 in the flange at the interface, minus 8.73 at the web near the interface and minus 1.06 at the bottom of the web.

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Thus in today's lecture, we covered the composite sections and we first saw that, there can different types of composite sections based on construction, based on stages of prestressing, based on the materials used and the analysis is based on all these factors. We have also studied the principles of analysis of composite sections. We need to calculate two sets of properties; one for the precast section and one for the composite section. We also understood the principles of design where the each steps are similar to that of a conventional section, but we have to be aware of the different properties. Finally, we also learned about the analysis of horizontal shear transfer which is taken care by the shear friction reinforcement and we understood these principles by using an example. With this we are ending the module on composite sections. Thank you.