

PRESTRESSED CONCRETE STRUCTURES

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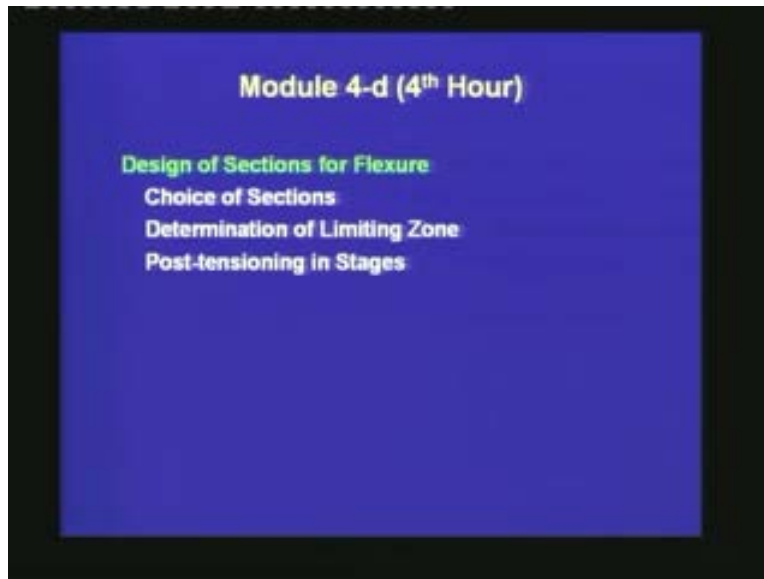
Indian Institute of Technology Madras

Module - 4: Design of Members

Lecture – 20: Choice of Sections and Determination of Limiting Zone

Welcome back to prestressed concrete structures. This is the fourth lecture of module four on design of members.

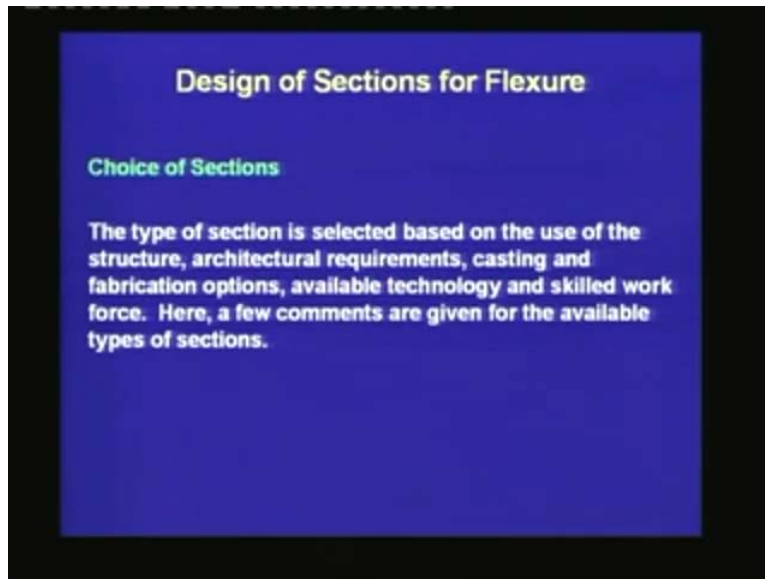
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In this lecture, we shall study the choice of different sections; then we shall move on to the determination of limiting zone and finally, we shall talk a little bit on post-tensioning in stages.

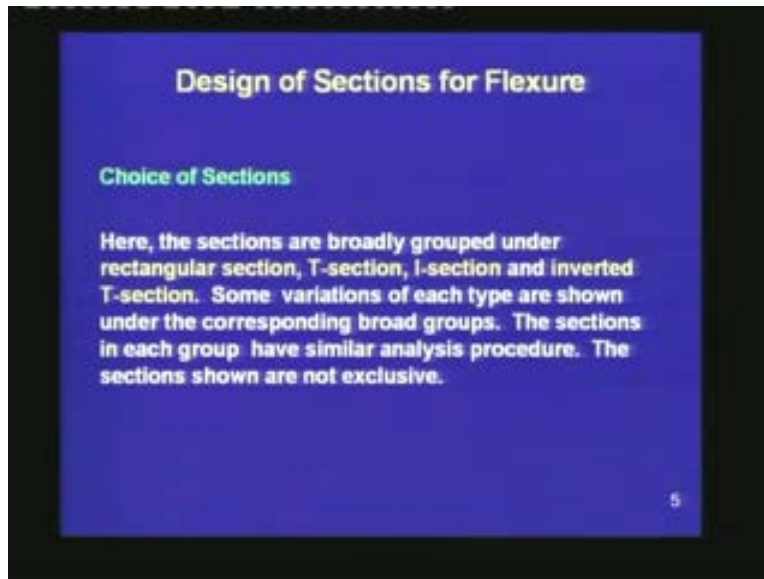
First is the choice of sections.

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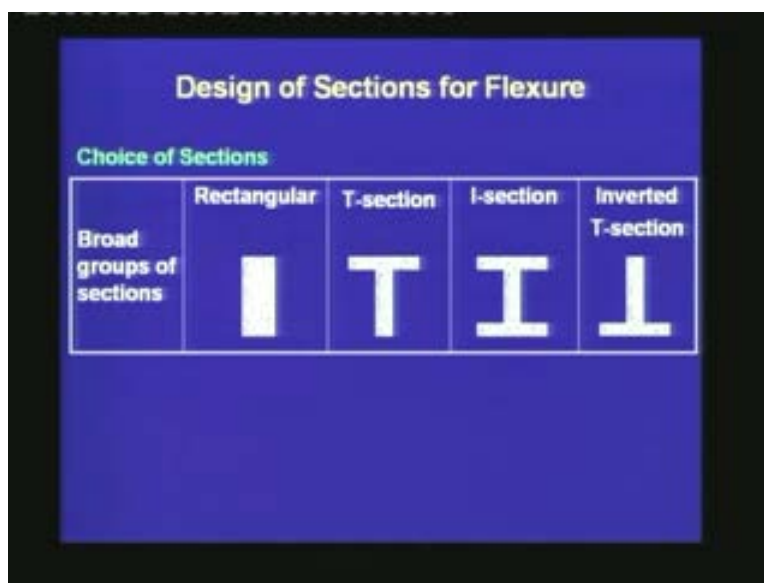
The type of section is selected based on the use of the structure, architectural requirements, casting and fabrication options, available technology and skilled work force. Here, a few comments are given for the available types of section. The prestressed applications are quite varied in nature. The type of section satisfies the structural strength, the architectural requirement and the aesthetic consideration. The choice of sections is a wide ranging topic. In this lecture, we shall cover the different sections in a summarized form by stating the basic points for each type of section.

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The sections here are broadly classified under rectangular section, T-section, I-section and inverted T-section. Some variations of each type are shown under the corresponding broad groups. The sections in each group have similar analysis procedure for the primary flexural reinforcement. The sections shown are not exclusive.

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The broad groups that we have selected are: first, in the rectangular type of section where there is no outstanding flange; the second is the T-section, where the outstanding flange is at the top; next the I-section, where there are flanges at the top and the bottom, and finally, the inverted T-section, where the flange is at the bottom.

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Design of Sections for Flexure					
Choice of Sections					
Variations					
Remarks on the sections		(a) More room for tendons in lower flange. (b) Better stability during erection.	Torsionally stiff and strong.		





Regarding variations, again we are showing some typical types of sections with schematic sketches. A rectangular section can be trapezoidal in nature, where the width of the web varies along the depth. A T-section can have a bulbed flange at the bottom. The flange need not always be outstanding. There can be a flange between two webs. For the Type (a) section under this category, there is more room for tendons in the lower flange. The purpose of enlarging the bottom of the web is to place the tendons properly. For (b) there is better stability during erection. After placing the members adjacently, a layer of topping concrete can be cast.

For the third group, we can have a box type of section which is more common in bridges. There can be a void in a section to reduce weight and to create a conduit. For a box type of section, the analysis is similar to a flanged section. For a voided section, if the size of the void is smaller than a certain value, then the section can be treated as a solid section with equivalent properties. The box sections are torsionally stiff and strong. The inverted T-section can have variations like U-section (channel section).

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Design of Sections for Flexure

Choice of Sections





Broad groups of sections				
Fabrication	Easy	Easy	Expensive form work	Difficult
Space for reinforcement	Adequate	Less than adequate	Good	Good

Let us study some of the aspects of these different types of sections. The fabrication is easy for a rectangular section. It is easy for even a T-section. But it can be expensive for an I-section. The fabrication for the inverted T-section can also be difficult. Regarding the space of reinforcement, the rectangular section has adequate space. A T-section will have less than adequate space unless the bottom of the web is enlarged as a bulb. There is good space for the tendons in an I-section as well as in an inverted T-section.

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Design of Sections for Flexure

Choice of Sections

Broad groups of sections				
For non-composite sections	Efficiency	Poor, $z = 0.4h$	Good, $z = 0.5h$	Very good, $z = 0.7h$
		1) Very inefficient. 2) Small ultimate moment capacity		





h : total depth, z : lever arm

Let us now study some structural properties for these sections, when they are used as non-composite sections. Regarding flexural efficiency, the rectangular section is poor. The lever arm which is represented by z , is about 40% of the total depth which is represented by h . Compared to a rectangular section, a T-section is flexurally more efficient and the lever arm is about 50% of the total depth. The I-section is flexurally very good and the lever arm is 70% of the total depth. The inverted T-section is very inefficient under sagging. However, the efficiency can be increased with a topping slab, which we shall mention under the category of composite sections. The ultimate moment capacity for an inverted T-section is small as compared to the other types of sections.

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Design of Sections for Flexure





Choice of Sections

Broad groups of sections					
For non-composite sections	Application	1) Light load and short span. 2) M_{sw}/M_T large.	1) Especially good for long span roofs, when $LL \ll DL$. 2) M_{sw}/M_T large.	1) Good for long span and heavy loads. 2) M_{sw}/M_T small.	M_{sw}/M_T large

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If we look into the applications of this type of sections as non-composite sections, then the rectangular sections are used for light load and short span. The self-weight moment which is represented as M_{sw} is quite large compared to the total moment carried by the section, which is represented as M_T . The T-section is good for long span roofs, when the live load is much smaller compared to the dead load. For a T-section also, the ratio M_{sw}/M_T is large. The I-section is good for long span and heavy loads. Here, the M_{sw}/M_T ratio is small compared to the rectangular and T-sections. For inverted T-sections, the applications are limited. If such a section is used, the M_{sw}/M_T ratio will be large.

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Broad Groups					
For composite sections	Efficiency	Very good particularly when section is shored.	Increases load capacity only slightly.	Very good	Very good with cast-in-place flange.

Let us see the properties of the sections, when they are used as composite sections. Here, there will be topping slabs above the sections. Regarding efficiency, the rectangular section is very good, when the section is shored during the casting of the slab. For the T-section, the increase in load capacity is marginal, if the topping slab is of small depth. For the I-section, the increase in the flexural capacity can be very good. For an inverted T-section, the flexural capacity increases substantially with a topping slab. Thus, the inverted T-section is flexurally efficient, only in presence of a topping slab in composite construction.

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Broad Groups					
For composite sections	Application	Good for building construction	1) Topping serves to tie all sections together. 2) No form required for composite pour.	Long span buildings and bridges.	Bridges

Regarding application, the rectangular section is good for building construction, such as in joists. For the T-section, the topping concrete ties the units and allows the members to deflect together. For this type of section, additional formwork is not required for the topping concrete. The I-section is used in long span applications in buildings and bridges. The inverted T-section can be used in bridges with cast-in-place flange at the top.

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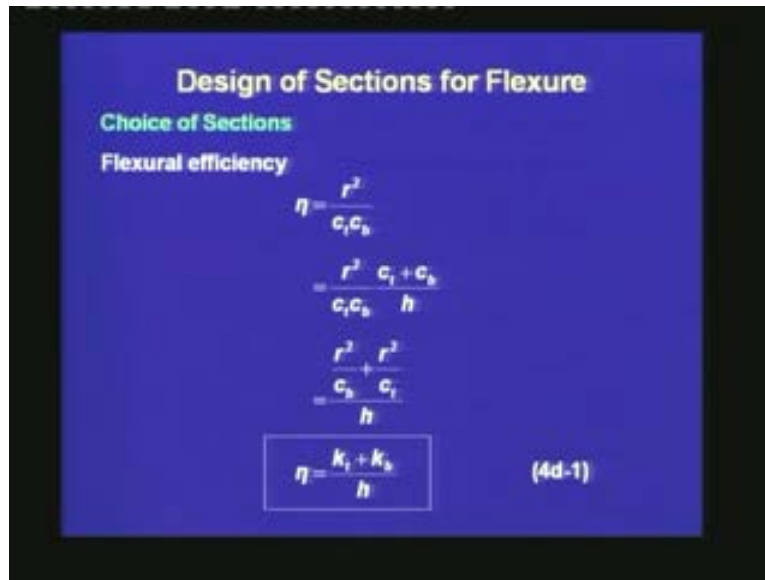
Design of Sections for Flexure

Choice of Sections

The different types of sections can be compared by a measure of flexural efficiency η . The flexural efficiency is defined in terms of the radius of gyration r as follows.

The different types of sections can be compared by a quantity called flexural efficiency, which is represented by the symbol η . The flexural efficiency is defined in terms of the radius of gyration, r as follows. Note, $r^2 = I/A$, where I is the moment of inertia and A is the sectional area.

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Design of Sections for Flexure

Choice of Sections

Flexural efficiency

$$\eta = \frac{r^2}{c_t c_b}$$

$$= \frac{r^2}{c_t c_b} \frac{c_t + c_b}{h}$$

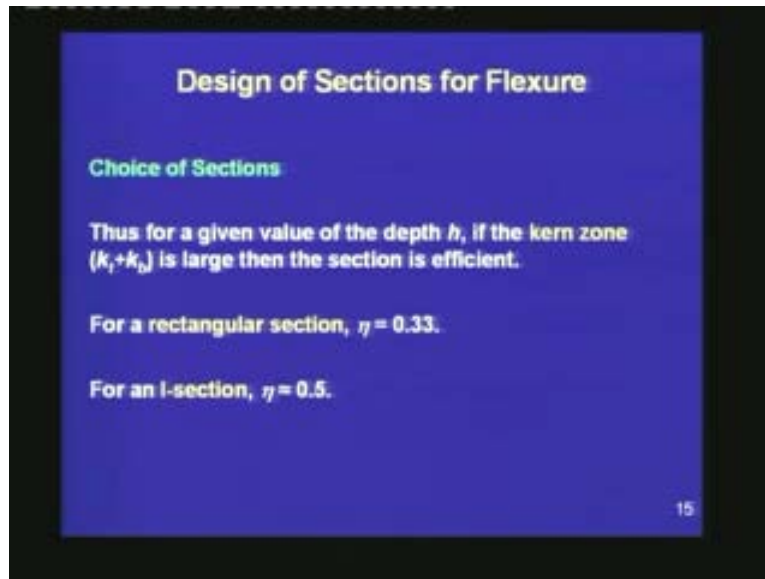
$$= \frac{r^2}{c_b} + \frac{r^2}{c_t}$$

$$\eta = \frac{k_t + k_b}{h} \quad (4d-1)$$

The flexural efficiency is defined as $\eta = r^2/c_t c_b$, where c_t is the distance of the top of the section from the CGC, and c_b is the distance of the bottom from the CGC. To this expression, we are multiplying the factor $(c_t + c_b)/h$. This factor is equal to 1, because $c_t + c_b = h$. Then, splitting the numerator of this factor, $\eta = (r^2/c_b + r^2/c_t)/h$, which leads to the final expression $\eta = (k_t + k_b)/h$. The quantity $k_t + k_b$ represents the depth of the kern zone.

Thus, the above quantity compares different types of sections in terms of their flexural efficiency. For a section of a certain height h , if the depth of the kern zone can be increased, then the flexural efficiency goes up.

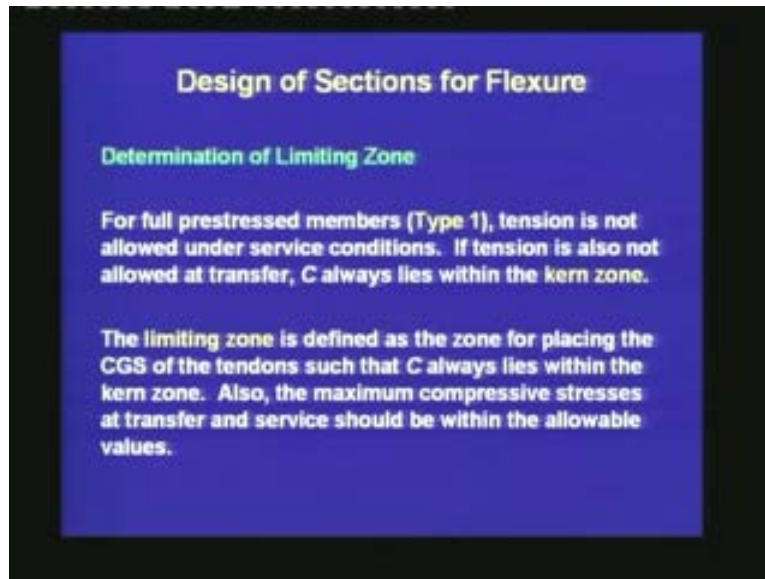
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For a rectangular section, $\eta = 0.33$. For an I-section, η can be around 0.5. Thus, we see that the flexural efficiency of an I-section is greater than that of a rectangular section.

Next, we move on to another important aspect which is the determination of limiting zone. This helps us to place the CGS of the tendons, as well as the distribution of the tendons along the span of the beam.

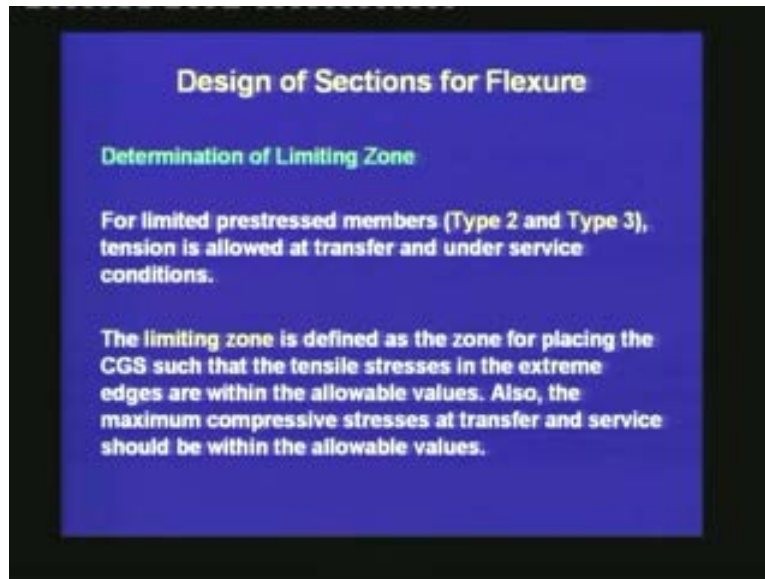
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In the flexural design, we had studied how to calculate the required eccentricity of the CGS at the critical section. Now we shall study, how to place the CGS throughout the span of the beam. For full prestressed members which are represented as Type 1, tension is not allowed under service conditions. If tension is also not allowed at transfer, the compression in the concrete which is represented as C , always lies within the kern zone. The limiting zone is defined as the zone for placing the CGS such that C always lies within the kern zone. Also, the maximum compressive stresses at transfer and service should be within the allowable values.

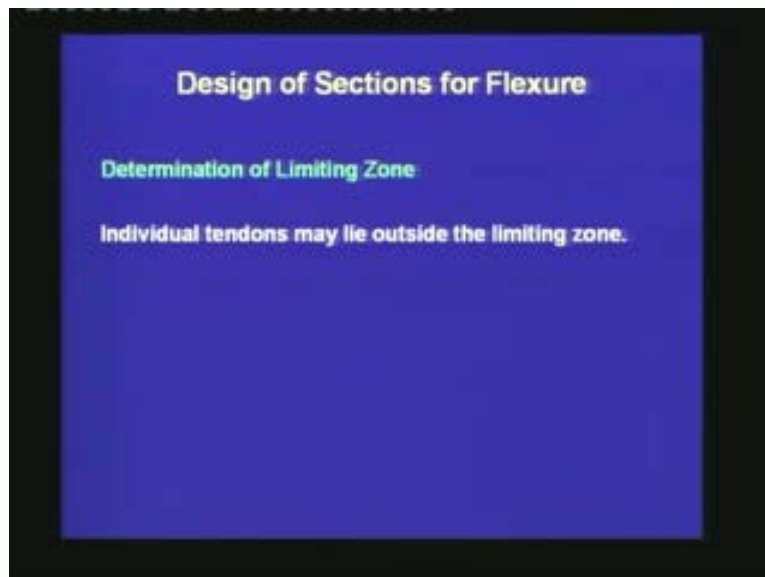
For Type 2 and Type 3 members, tensile stresses are allowed and hence, the compression in concrete can lie outside the kern zone, but it has to be within a certain limit such that the tensile stresses at the bottom or at the top are within the allowable values.

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The limiting zone is defined as the zone for placing the CGS such that the tensile stresses in the extreme edges are within the allowable values. Also, the maximum compressive stresses at transfer and service should be within the allowable values.

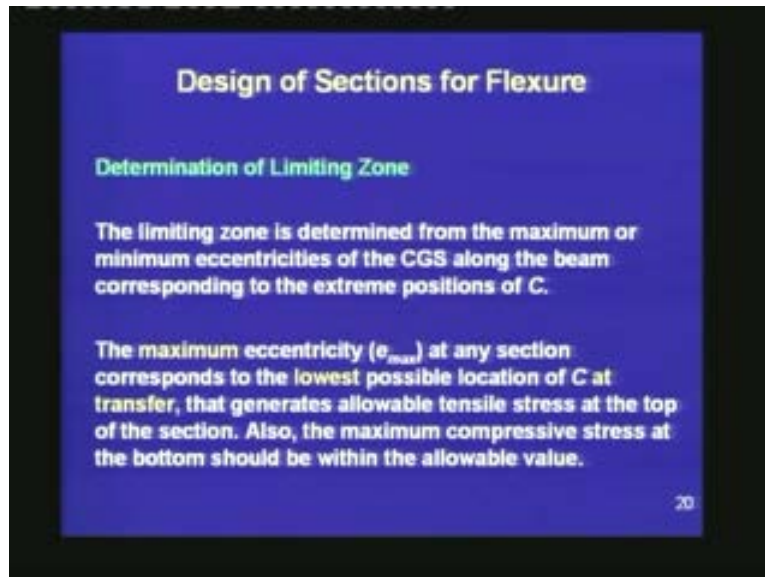
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Thus, the definition of the limiting zone is more general for Type 2 and Type 3 members as compared to Type 1 members because for Type 2 and Type 3 members, we are

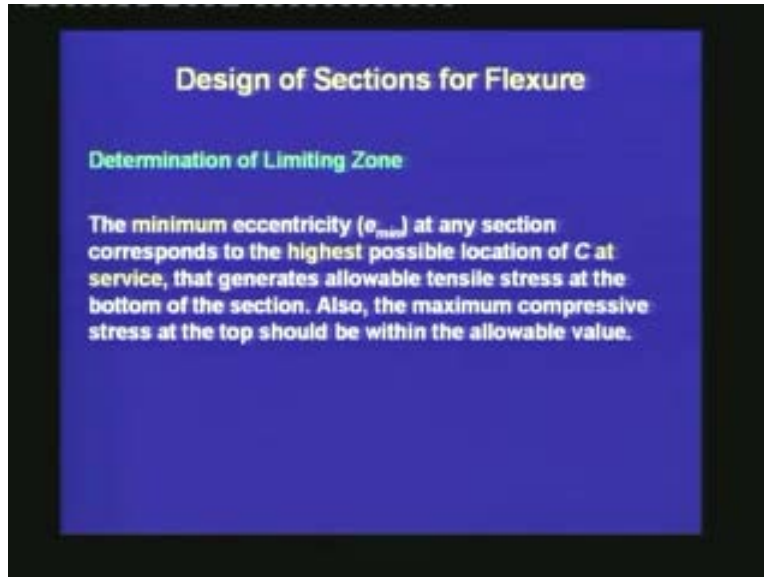
allowing tension in the concrete and hence, the compression can lie outside the kern zone. Remember that, the limiting zone is defined for the CGS. The individual tendons may lie outside the limiting zone provided the CGS is within the limiting zone.

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The limiting zone is determined from the maximum or minimum eccentricities of the CGS along the beam corresponding to the extreme positions of C. The maximum eccentricity (which is represented as e_{max}) at any section corresponds to the lowest possible location of C at transfer that generates allowable tensile stress at the top of the section. Also, the maximum compressive stress at the bottom should be within the allowable value.

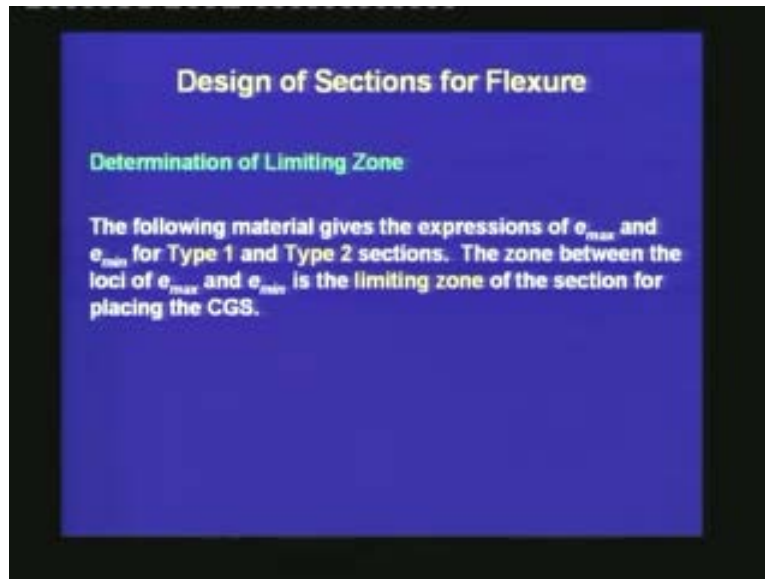
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The minimum eccentricity of the CGS (which is represented as e_{min}) corresponds to the highest possible location of C at service, that generates allowable tensile stress at the bottom of the section. Also, the compressive stress at the top should be within the allowable value.

Thus, to determine the minimum eccentricity of the CGS, we are allowing the compressive force to rise as much as possible at a particular section, such that the compressive stress generated at the top and the tensile stress at the bottom are within the allowable values. The purpose of allowing the compression to rise as much distance as possible is to have an efficient design. With that, we are able to reduce the amounts of prestressing force and prestressing steel.

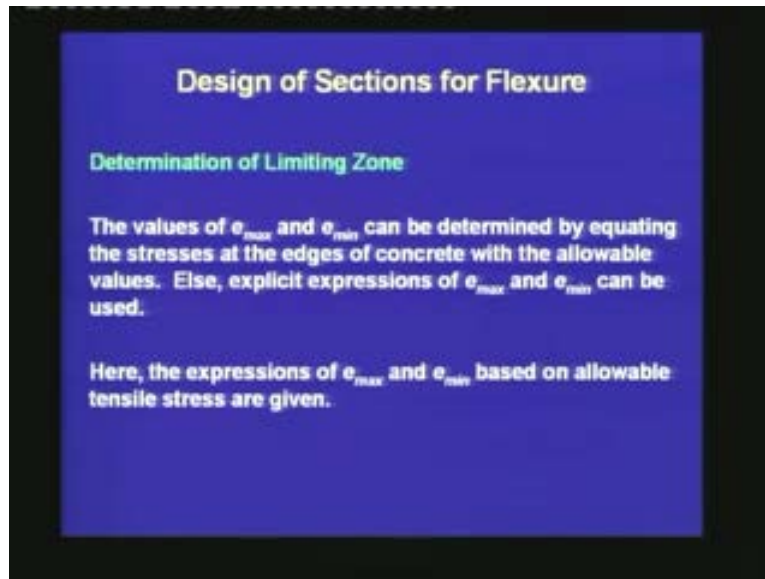
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The following material gives the expressions of e_{max} and e_{min} for Type 1 and Type 2 sections. The difference between Type 2 and Type 3 sections is only in the value of the allowable tensile stress. If we have the expressions for Type 2 section, we can use that for Type 3 section, provided we change the value of the allowable tensile stress. The zone between the loci of e_{max} and e_{min} is the limiting zone of the member for placing the CGS.

We calculate e_{min} and e_{max} at different sections throughout the length of the beam. Next, we join them by lines, one line for the e_{min} values, and another line for the e_{max} values. The space in between these two lines is termed as the limiting zone for placing the CGS of the tendons.

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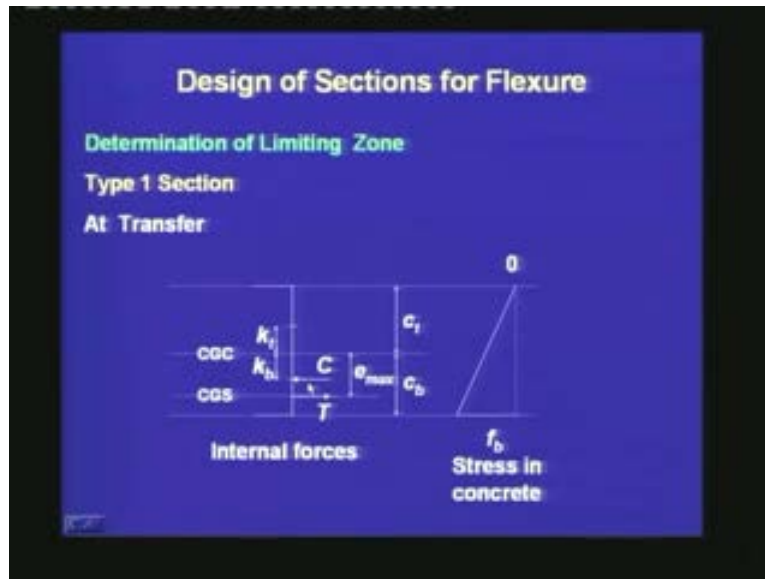


The values of e_{max} and e_{min} can be determined by equating the stresses at the edges of the concrete with the allowable values. Else, explicit expressions of e_{max} and e_{min} can be used.

There are two ways to determine the values of e_{max} and e_{min} at a particular section. One way is to write the expression of the stresses at the top and the bottom, equate the stresses to the allowable values; from there, we can find out the e_{max} or e_{min} values. Another approach is to use explicit expressions of e_{max} and e_{min} so that, we can directly calculate their values without calculating the stresses. Both the approaches will give the same results.

Here, the expressions of e_{max} and e_{min} based on allowable tensile stress are given.

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For Type 1 section, at transfer we are pushing down the CGS as low as possible to have the lowest possible location of C which is at the bottom kern point. In this figure, we can see that the C is located at a distance k_b from CGC, and the stress block in concrete is triangular with values zero at the top and f_b at the bottom.

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The slide presents the equation for the limiting zone of a Type 1 section at transfer:

$$e_{max} - k_b = \frac{M_{transfer}}{P_0}$$

or,

$$e_{max} = \frac{M_{transfer}}{P_0} + k_b \quad (4d-2)$$

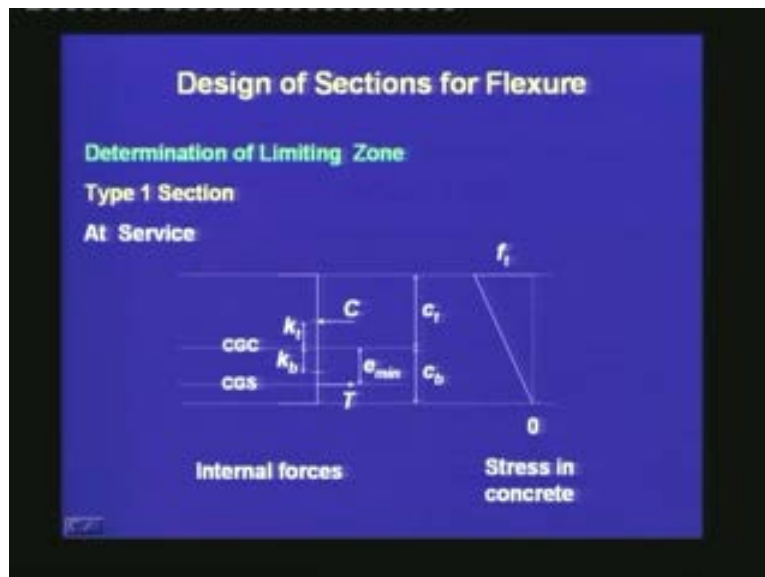
Also, $|f_b| \leq f_{cc,all}$

The lever arm (distance between T and C) at transfer is governed by the self-weight moment (M_{sw}). It is equal to $e_{max} - k_b = M_{sw}/P_0$, where P_0 is the value of C at transfer. From this, we find the expression of e_{max} .

$$e_{max} = M_{sw}/P_0 + k_b$$

This expression is based on satisfying the allowable tensile stress at the top, which is zero for a Type 1 member. We should also check e_{max} to satisfy that, the stress at the bottom f_b should be less than the allowable compressive stress at transfer, which is represented as $f_{cc,all}$.

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Next, for Type 1 section, we are determining e_{min} . At service, C is at the top of the kern zone which is at a distance k_t from CGC. The stress block in concrete is triangular with zero stress at the bottom and a certain value f_t at the top.

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Design of Sections for Flexure

Determination of Limiting Zone

Type 1 Section

At Service

$$e_{\min} + k_t = \frac{M_T}{P_e}$$

or, $e_{\min} = \frac{M_T}{P_e} - k_t$ (4d-3)

Also, $f_t \leq f_{cc,sl}$

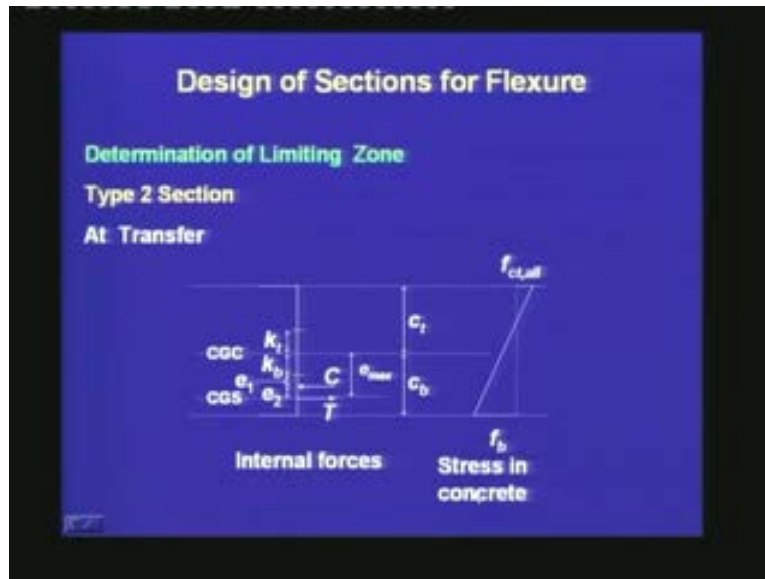
If for a particular section e_{\min} is negative, it implies that the CGS can be placed above CGC. This happens near the supports.

The lever arm at service is governed by the total moment (M_T). It is equal to $e_{\min} + k_t = M_T/P_e$, where P_e is the value of C at service. Thus,

$$e_{\min} = M_T/P_e - k_t$$

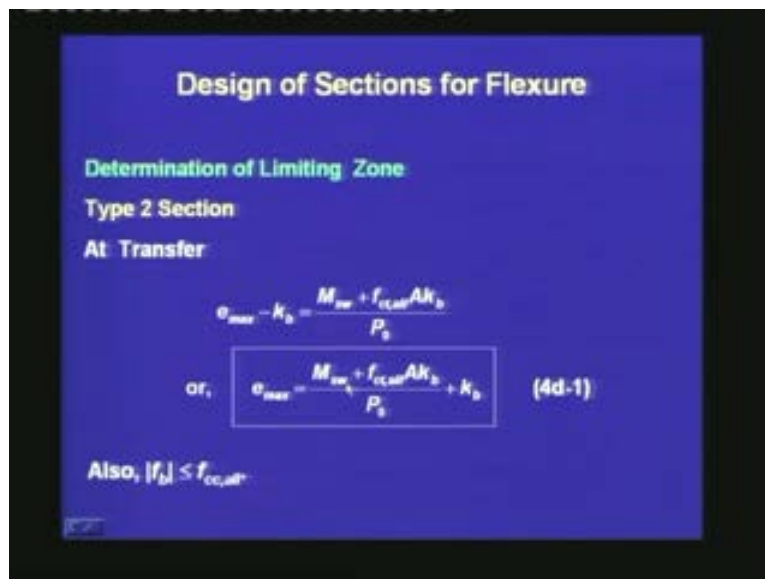
Also, the compressive stress at the top f_t should be less than the allowable compressive stress at service. If for a particular section e_{\min} is negative, it implies that the CGS can be placed above CGC. This happens near the supports.

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For Type 2 section at transfer, we can push the CGS even below. C can lie outside the kern zone so that, the tensile stress generated at the top is equal to the allowable value. The stress block in the concrete has some tension at the top. The distance of C from the bottom most kern point is represented as e_1 and the distance between C and T is represented as e_2 .

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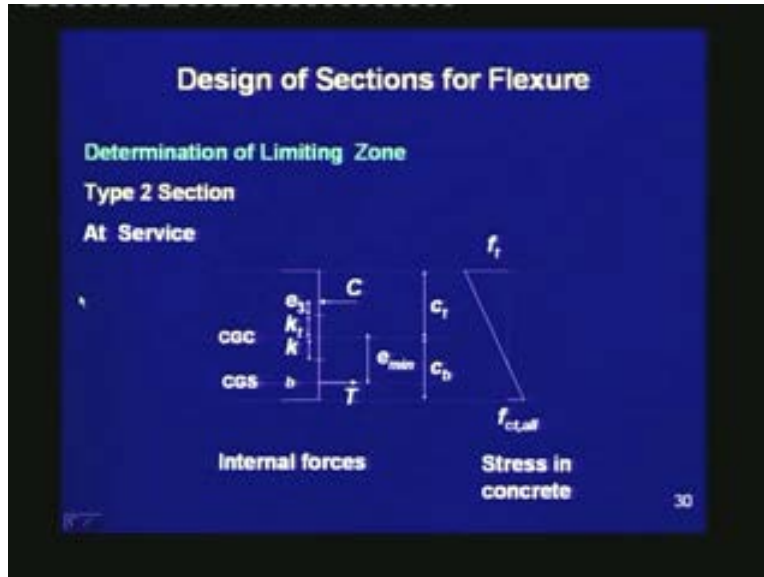


We can write that the distance between the CGS and the bottom most kern point as $e_{\max} - k_b = (M_{sw} + f_{ct,all} A k_b) / P_0$. From this we find an explicit expression of e_{\max} .

$$e_{\max} = (M_{sw} + f_{ct,all} A k_b) / P_0 + k_b$$

This expression is the general form of the previous expression that we had seen for Type 1 section. This expression has an extra term involving the allowable tensile stress at the top ($f_{ct,all}$). Also, the compressive stress at the bottom should be less than the allowable compressive stress at transfer.

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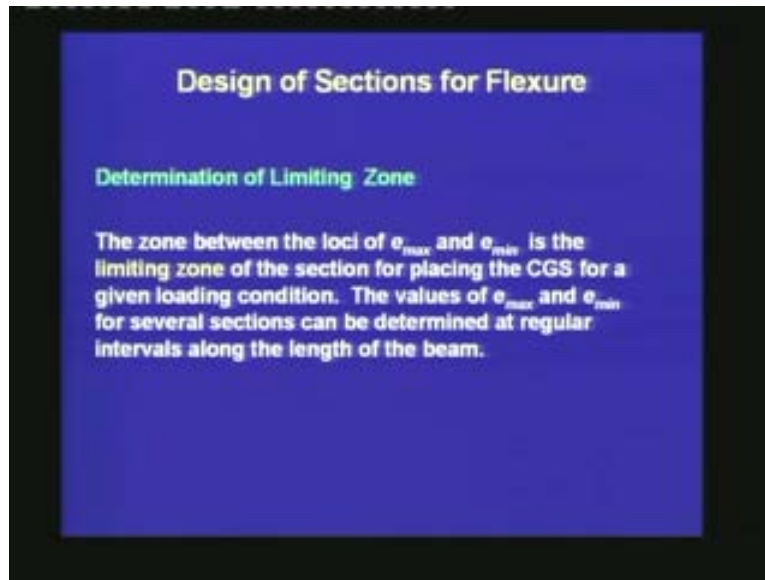
For Type 2 section at service, we can have C going above the kern region by a distance equal to e_3 , so that we have a tensile stress at the bottom which is equal to $f_{ct,all}$ for service.

The minimum eccentricity plus the distance of the kern point from CGC is equal to $(M_T - f_{ct,all} A k_t) / P_e$. Thus,

$$e_{\min} = (M_T - f_{ct,all} A k_t) / P_e - k_t$$

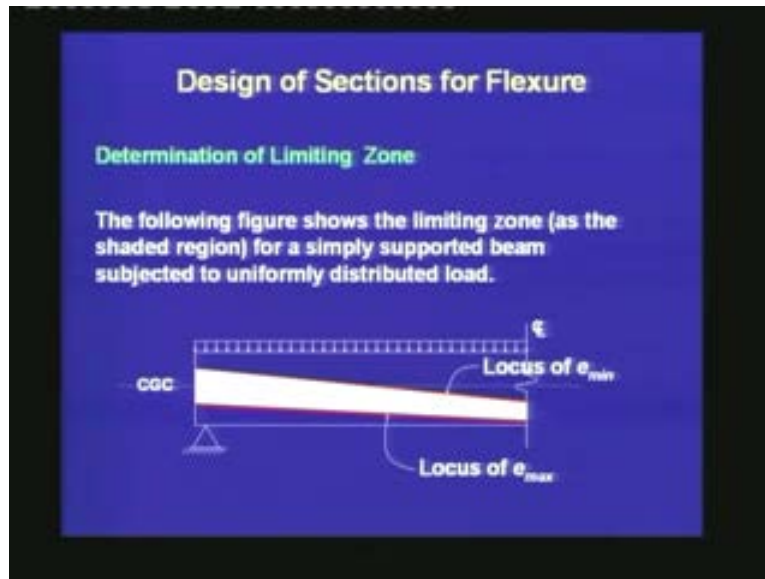
The expression of e_{\min} for a Type 2 section is similar to that for a Type 1 section. It has an additional term involving the allowable tensile stress in the concrete. If we substitute $f_{ct,all} = 0$, then we get back the expression for a Type 1 section. Also, we need to check the stress at the top to be within the allowable compressive stress under service conditions.

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The zone between the loci of e_{\max} and e_{\min} is the limiting zone of the member for placing the CGS for a given loading condition. The values of e_{\max} and e_{\min} for several sections can be determined at regular intervals along the length of the beam. For example, an interval of one-tenth of the span can be selected. Depending on the moment due to self-weight and the moment due to service loads, we can find out the e_{\max} and e_{\min} values for each section. Once we join these points, we get the boundaries of the limiting zone and we can place our CGS within this limiting zone.

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The following figure shows the limiting zone as the shaded region for a simply supported beam subjected to uniformly distributed load. Here, the locus of e_{\min} and the locus of e_{\max} are the upper and the lower boundaries of the limiting zone, respectively. The depth of the limiting zone is wide at the end. As we proceed towards the centre of the span, the depth of the limiting zone gets reduced.

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Example 4d-1

For the post-tensioned beam with a flanged section as shown, the span is 18 m. For uniform loads, the profile of the CGS is parabolic. The live load moment at mid-span (M_{LL}) is 648 kNm. The prestress after transfer (P_d) is 1600 kN. Assume 15% loss at service.

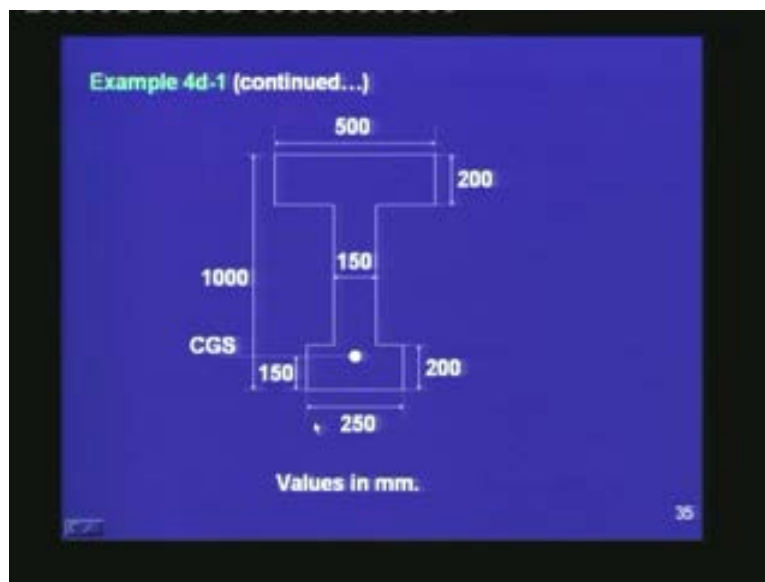
Evaluate the limiting zone of CGS, if the allowable stresses at transfer and at service are as follows.

For compression, $f_{cc,all} = 18.0 \text{ N/mm}^2$
For tension, $f_{ct,all} = 1.5 \text{ N/mm}^2$.

Let us now try to understand the concept of limiting zone by solving a problem. For the post-tensioned beam with a flanged section as shown in the next slide, the span is 18 m. For uniform loads, the profile of the CGS is parabolic. The live load moment at mid-span (M_{LL}) is 648 kNm. The prestress after transfer (P_0) is 1600 kN. Assume 15% loss at service.

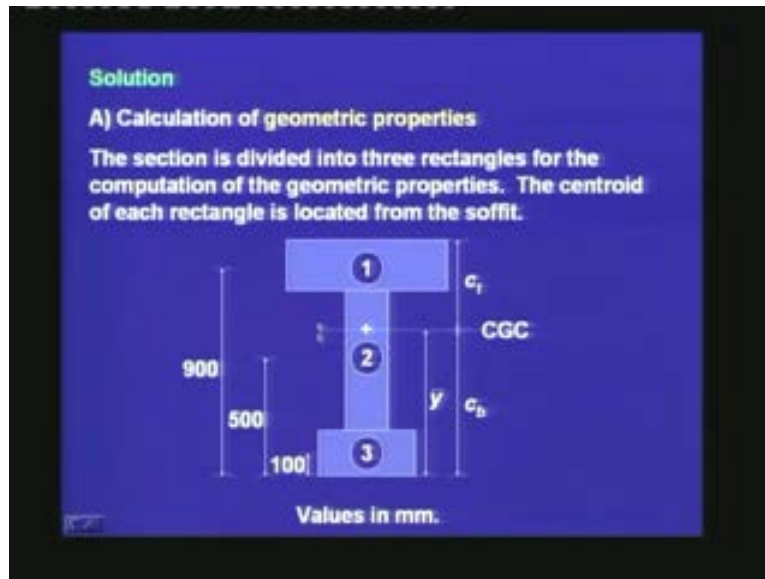
Evaluate the limiting zone of CGS, if the allowable stresses at transfer and at service are as follows. For compression, the allowable stress ($f_{cc,all}$) is 18.0 N/mm^2 . For tension, the allowable stress ($f_{ct,all}$) is 1.5 N/mm^2 . In this problem, the allowable values are same for transfer and service. But in general, these values will be different because the strength of the concrete will be different at transfer and at service.

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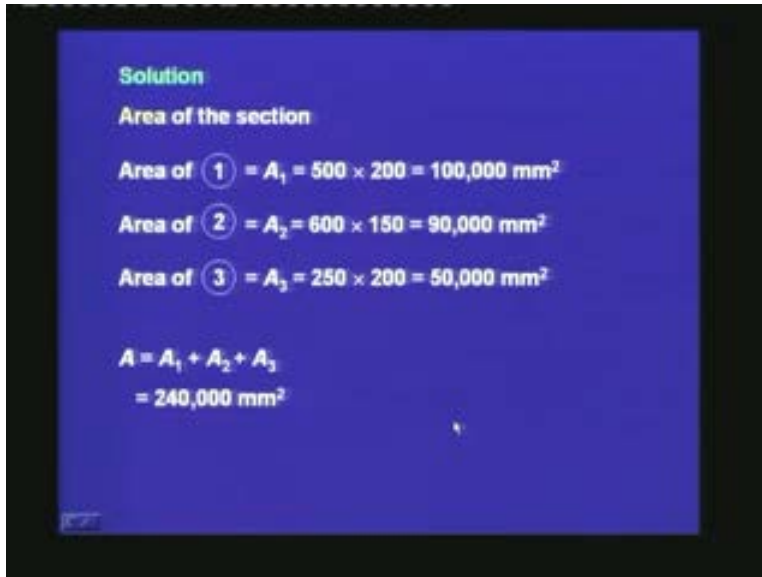
The section is an unsymmetric I-section, where the top flange width is 500 mm and the bottom flange width is 250 mm. The total depth is 1000 mm. The depth of both the flanges is 200 mm, and the width of the web is 150 mm. The CGS is intended to be located at a distance of 150 mm from the soffit of the beam at mid-span.

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First, we are calculating the geometric properties. Since the section is unsymmetric, we need to locate the CGC which is not at the mid-depth any more. The section is divided into three rectangles for the computation of the geometric properties. The centroid of each rectangle is located from the soffit. For the first rectangle, the centroid is 900 mm from the soffit. For the second rectangle, the centroid is 500 mm from the soffit and for the third rectangle it is 100 mm from the soffit. We intend to find out \bar{y} , the distance of the CGC from the soffit. Once we have determined \bar{y} , we can calculate c_t , the distance of the top from the CGC and c_b , the distance of the bottom from the CGC. Of course, $c_b = \bar{y}$.

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Solution

Area of the section

Area of ① = $A_1 = 500 \times 200 = 100,000 \text{ mm}^2$

Area of ② = $A_2 = 600 \times 150 = 90,000 \text{ mm}^2$

Area of ③ = $A_3 = 250 \times 200 = 50,000 \text{ mm}^2$

$A = A_1 + A_2 + A_3$
 $= 240,000 \text{ mm}^2$

The area of the section is given as the summation of the areas of the three rectangles.

Area of Rectangle 1 is $A_1 = 500 \times 200 = 100,000 \text{ mm}^2$

Area of Rectangle 2 is $A_2 = 600 \times 150 = 90,000 \text{ mm}^2$

Area of Rectangle 3 is $A_3 = 250 \times 200 = 50,000 \text{ mm}^2$

Thus, the total area $A = A_1 + A_2 + A_3 = 240,000 \text{ mm}^2$.

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Solution

Distance of CGC from the soffit:

$$\bar{y} = \frac{A_1 \times 900 + A_2 \times 500 + A_3 \times 100}{A}$$
$$= 583.3 \text{ mm}$$

Therefore,

$$c_b = 583.3 \text{ mm}$$
$$c_t = 1000.0 - 583.3$$
$$= 416.7 \text{ mm}$$

The distance of CGC from the soffit is given as the summation of the first moments of the individual areas divided by the total area. The moment of the first area is $A_1 \times 900$, the moment of the second area is $A_2 \times 500$, and the moment of the third area is $A_3 \times 100$. All these three distances are the distances of the centroids of the individual rectangles from the soffit of the beam. The sum of the moments divided by the total area is $\bar{y} = 583.3$ mm. Thus, the CGC is located at a distance of 583.3 mm from the soffit of the beam. Therefore, $c_b = \bar{y} = 583.3$ mm, $c_t = 1000.0 - 583.3 = 416.7$ mm.

(Refer Slide Time: 37:46)

Solution

Moment of inertia of ① about axis through CGC

$$I_1 = \frac{1}{12} \times 500 \times 200^3 + A_1 \times (900 - 583.3)^2$$
$$= 1.036 \times 10^{10} \text{ mm}^4$$

Moment of inertia of ②

$$I_2 = \frac{1}{12} \times 150 \times 600^3 + A_2 \times (583.3 - 500)^2$$
$$= 3.32 \times 10^9 \text{ mm}^4$$

Moment of inertia of Rectangle 1 about axis through CGC can be computed by the parallel axis theorem. I_1 is equal to the moment of inertia about the centroid of the rectangle plus the area of the rectangle times the distance squared. Thus,

$$I_1 = 500 \times 200^3/12 + A_1 \times (900 - 583.3)^2 = 1.036 \times 10^{10} \text{ mm}^4.$$

Similarly, we can calculate the moments of inertia of Rectangles 2 and 3.

$$I_2 = 3.32 \times 10^9 \text{ mm}^4.$$

(Refer Slide Time: 39:07)

Solution

Moment of inertia of ③

$$I_3 = \frac{1}{12} \times 250 \times 200^3 + A_3 \times (583.3 - 100)^2$$
$$= 1.184 \times 10^{10} \text{ mm}^4$$

Moment of inertia of the section

$$I = I_1 + I_2 + I_3$$
$$= (1.036 + 0.336 + 1.184) \times 10^{10}$$
$$= 2.552 \times 10^{10} \text{ mm}^4$$

40

$$I_3 = 1.184 \times 10^{10} \text{ mm}^4.$$

The total moment of inertia of the section $I = I_1 + I_2 + I_3 = 2.552 \times 10^{10} \text{ mm}^4$.

(Refer Slide Time: 39:49)

Solution

Calculation of moment due to self weight.

$$W_{sw} = 24 \text{ kN/m}^3 \times 240,000 \text{ mm}^2 \times \left(\frac{1}{10^3}\right)^2 \frac{\text{m}^2}{\text{mm}^2}$$
$$= 5.76 \text{ kN/m}$$
$$M_{sw} = \frac{W_{sw} l^2}{8}$$
$$= \frac{5.76 \times 18^2}{8}$$
$$= 233.3 \text{ kNm}$$

Next, we are calculating the moment due to self-weight. The distributed self-weight is equal to the unit weight of concrete which is taken as 24 kN/m^3 times the area which is $240,000 \text{ mm}^2$ times a factor which converts mm^2 to m^2 . This gives a self-weight of 5.76

kN/m. The moment due to the self-weight M_{sw} is equal to the weight times the span squared divided by 8. After substituting the values, $M_{sw} = 233.3$ kNm.

(Refer Slide Time: 40:42)

Solution

B) Determination of limiting zone

The values of e_{max} and e_{min} are determined by equating the stresses at the edges of concrete with the allowable values. The expression of stress is given below.

$$f = -\frac{P}{A} \pm \frac{Pey}{I} \pm \frac{My}{I}$$

CGC

M

P

$-P/A$ $\pm Pey/I$ $\pm My/I$ **Resultant stress profile**

Now, we move on to calculating the limiting zone. The values of e_{max} and e_{min} are determined by equating the stresses at the edges of concrete with the allowable values. The expression of stress at depth y is given below.

$$f = -P/A \pm Pey/I \pm My/I$$

The first term is the uniform component. The second term is due to the eccentricity of the prestressing force. The third term is due to the moment. The summation of the three components gives the resultant stress profile.

(Refer Slide Time: 41:49)

Solution

Limiting position at mid-span

For e_{\max} , consider the load stage at transfer.

i) Calculate e based on $f_b = -18.0 \text{ N/mm}^2$.

$$\frac{P_0}{A} = \frac{1600 \times 10^3}{240 \times 10^3}$$
$$= -6.67 \text{ N/mm}^2$$
$$\frac{P_0ec_b}{I} = \frac{1600 \times 10^3 \times e \times 583.3}{2.552 \times 10^{10}}$$
$$= -0.0366e$$

At mid-span, for e_{\max} , consider the load stage at transfer. First, calculate e based on the stress at the bottom $f_b = -18.0 \text{ N/mm}^2$, the allowable compressive stress. The individual components of the stress are as follows.

$$-P_0/A = -1600 \times 10^3 \text{ N} / 240 \times 10^3 \text{ mm}^2 = -6.67 \text{ N/mm}^2.$$

$P_0ec_b/I = -1600 \times 10^3 \times e \times 583.3 / 2.552 \times 10^{10} = -0.0366e$. Thus, this component of the stress is in terms of the eccentricity e .

(Refer Slide Time: 43:22)

Solution

$$\frac{M_{sw}c_b}{I} = \frac{233.3 \times 10^6 \times 583.3}{2.552 \times 10^{10}}$$
$$= 5.33 \text{ N/mm}^2$$
$$f_b = -6.67 - 0.0366e + 5.33$$
$$= -18.0 \text{ N/mm}^2$$

Solving, $e = \frac{18.0 - 6.67 + 5.33}{0.0366}$

$$= 455.2 \text{ mm}$$

$$M_{sw}c_b/I = 233.3 \times 10^6 \times 583.3 / 2.552 \times 10^{10} = 5.33 \text{ N/mm}^2.$$

Thus, the total stress at the bottom is $f_b = -6.67 - 0.0366e + 5.33$, which is equated to -18.0 N/mm^2 . Solving, $e = 455.2 \text{ mm}$. This value of e is based on satisfying the compressive stress at the bottom.

(Refer Slide Time: 44:31)

Solution

ii) Calculate e based on $f_t = 1.5 \text{ N/mm}^2$.

$$\frac{P_e c_t}{I} = \frac{1600 \times 10^3 \times e \times 416.7}{2.552 \times 10^{10}}$$
$$= 0.0261 e$$
$$\frac{M_{sw}c_t}{I} = \frac{233.3 \times 10^6 \times 416.7}{2.552 \times 10^{10}}$$
$$= -3.81 \text{ N/mm}^2$$

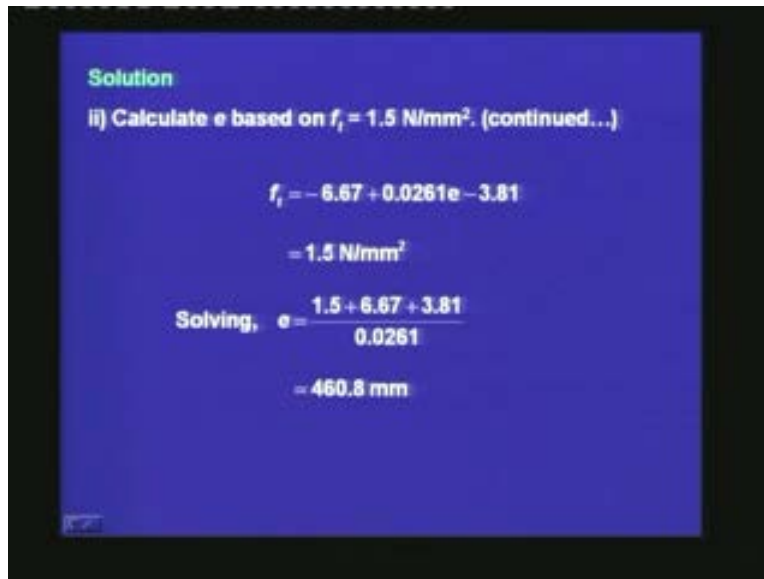
45

Let us now calculate e based on the allowable tensile stress at the top $f_t = 1.5 \text{ N/mm}^2$. The uniform component of the stress is already known to be -6.67 N/mm^2 . The other components are as follows.

$$P_0 e c_t / I = 1600 \times 10^3 \times e \times 416.7 / 2.552 \times 10^{10} = 0.0261e$$

$$M_{sw} c_t / I = -233.3 \times 10^6 \times 416.7 / 2.552 \times 10^{10} = -3.81 \text{ N/mm}^2$$

(Refer Slide Time: 45:28)



Solution
ii) Calculate e based on $f_t = 1.5 \text{ N/mm}^2$. (continued...)

$$f_t = -6.67 + 0.0261e - 3.81$$
$$= 1.5 \text{ N/mm}^2$$

Solving, $e = \frac{1.5 + 6.67 + 3.81}{0.0261}$

$$= 460.8 \text{ mm}$$

Thus, the total stress at the top is $f_t = -6.67 + 0.0261e - 3.81$, which is equated to 1.5 N/mm^2 . Solving, $e = 460.8 \text{ mm}$.

(Refer Slide Time: 45:55)

Solution
Out of the two values of e , the lower value 455.2 mm governs.

$$\therefore e_{\max} = 455.2 \text{ mm}$$

Out of the two values, the lower value of 455.2 mm governs. This is selected as the maximum eccentricity for the CGS at mid-span.

(Refer Slide Time: 46:20)

Solution
For e_{\min} , consider the load stage at service.

i) Calculate e based on $f_t = -18.0 \text{ N/mm}^2$.

$$\frac{P_s}{A} = 0.85 \frac{P_o}{A}$$
$$= -5.67 \text{ N/mm}^2$$
$$\frac{P_s e c_t}{I} = \frac{0.85 \times 1600 \times 10^3 \times e \times 416.7}{2.552 \times 10^{10}}$$
$$= 0.022 e$$

For e_{\min} , consider the load stage at service. We are proceeding with a similar set of calculations.

The stress at the top (f_t) is equated to -18.0 N/mm^2 . The effective prestress (P_e) is equal to 85% of the prestress at transfer (P_0). The individual components of the stress are as follows.

$$P_e/A = 0.85 \times P_0/A = -0.85 \times 6.67 = -5.67 \text{ N/mm}^2$$

$$P_e e c_t / I = 0.85 \times 1600 \times 10^3 \times e \times 416.7 / 2.552 \times 10^{10} = 0.022 e.$$

(Refer Slide Time: 47:26)

Solution

i) Calculate e based on $f_t = -18.0 \text{ N/mm}^2$. (continued...)

$$\frac{M_{LL} c_t}{I} = \frac{648.0 \times 10^6 \times 416.7}{2.552 \times 10^{10}}$$

$$= -10.58 \text{ N/mm}^2$$

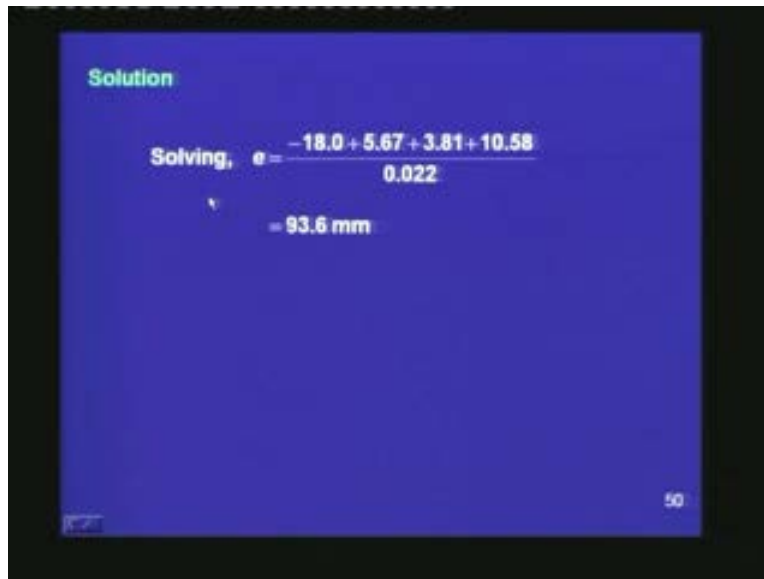
$$f_t = -5.67 + 0.022e - 3.81 - 10.58$$

$$= -18.0 \text{ N/mm}^2$$

$$M_{LL} c_t / I = -648.0 \times 10^6 \times 416.7 / 2.552 \times 10^{10} = -10.58 \text{ N/mm}^2$$

Thus, the total stress at the top is $f_t = -5.67 + 0.022e - 3.81 - 10.58$, which is equated to -18.0 N/mm^2 .

(Refer Slide Time: 48:28)



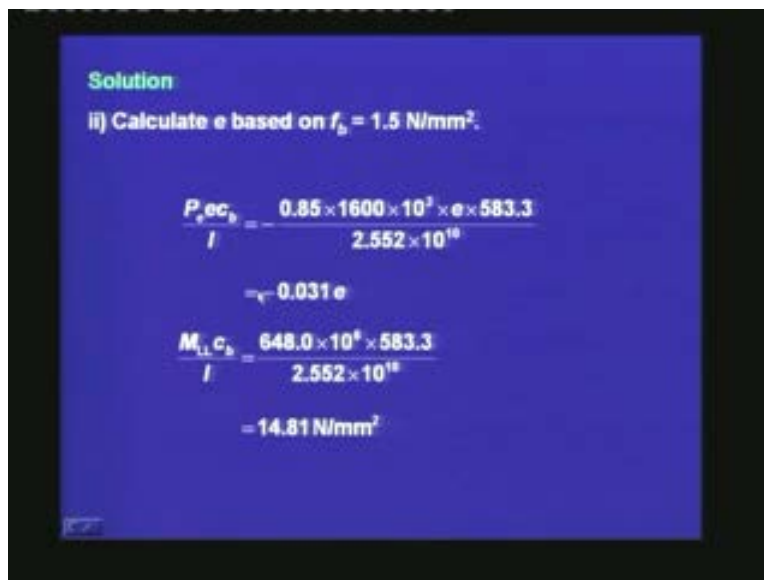
A blue slide with the word "Solution" in green at the top left. Below it, the text "Solving, e = ..." is followed by a fraction: the numerator is $-18.0 + 5.67 + 3.81 + 10.58$ and the denominator is 0.022 . Below the fraction, the result is $= 93.6 \text{ mm}$. In the bottom right corner, the number "50" is visible.

$$\text{Solving, } e = \frac{-18.0 + 5.67 + 3.81 + 10.58}{0.022}$$
$$= 93.6 \text{ mm}$$

Solving, $e = 93.6 \text{ mm}$.

Next, e is to be calculated based on the allowable tensile stress at the bottom $f_b = 1.5 \text{ N/mm}^2$.

(Refer Slide Time: 48:47)



A blue slide with the word "Solution" in green at the top left. Below it, the text "ii) Calculate e based on $f_b = 1.5 \text{ N/mm}^2$." is shown. The next part shows a fraction: the numerator is $P_e e c_b = 0.85 \times 1600 \times 10^3 \times e \times 583.3$ and the denominator is 2.552×10^{18} . Below this, the result is $= -0.031 e$. The next part shows another fraction: the numerator is $M_{11} c_b = 648.0 \times 10^4 \times 583.3$ and the denominator is 2.552×10^{18} . Below this, the result is $= 14.81 \text{ N/mm}^2$. In the bottom left corner, there is a small logo.

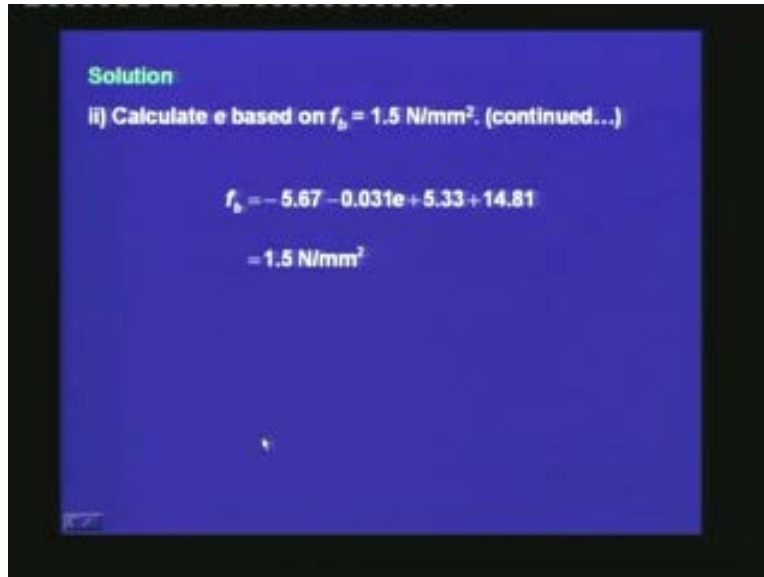
ii) Calculate e based on $f_b = 1.5 \text{ N/mm}^2$.

$$\frac{P_e e c_b}{I} = \frac{0.85 \times 1600 \times 10^3 \times e \times 583.3}{2.552 \times 10^{18}}$$
$$= -0.031 e$$
$$\frac{M_{11} c_b}{I} = \frac{648.0 \times 10^4 \times 583.3}{2.552 \times 10^{18}}$$
$$= 14.81 \text{ N/mm}^2$$

$$P_e e c_b / I = -0.031 e$$

$$M_{LL}c_b/I = 14.81 \text{ N/mm}^2.$$

(Refer Slide Time: 49:22)

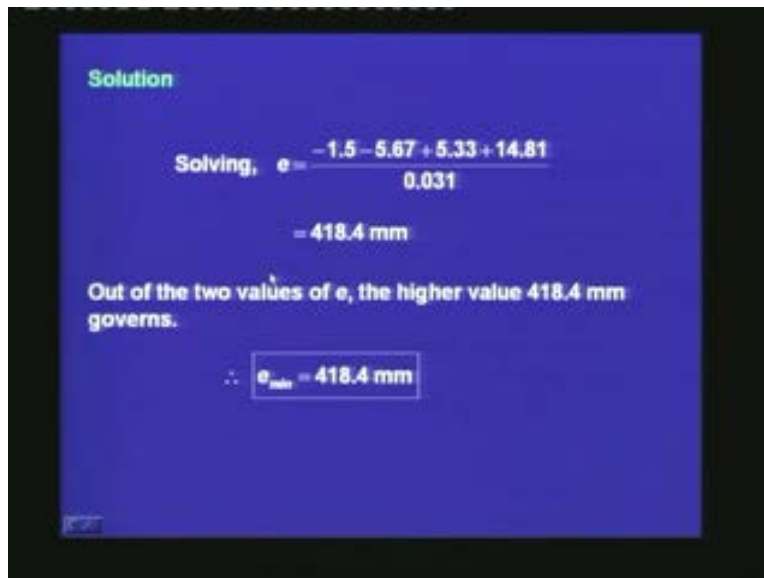


Solution
ii) Calculate e based on $f_b = 1.5 \text{ N/mm}^2$. (continued...)

$$f_b = -5.67 - 0.031e + 5.33 + 14.81$$
$$= 1.5 \text{ N/mm}^2$$

Thus, the total stress at the bottom is $f_b = -5.67 - 0.031e + 5.33 + 14.81$, which is equated to 1.5 N/mm^2 .

(Refer Slide Time: 49:45)



Solution

Solving, $e = \frac{-1.5 - 5.67 + 5.33 + 14.81}{0.031}$

$$= 418.4 \text{ mm}$$

Out of the two values of e , the higher value 418.4 mm governs.

$\therefore e_{\text{max}} = 418.4 \text{ mm}$

Solving, $e = 418.4$ mm. Out of the two values of e , the higher value of 418.4 mm governs because if we still reduce e then, we shall have tensile stress larger than the allowable value at the bottom.

Thus, we have found out the two extreme values of the CGS at the mid-span of the section.

(Refer Slide Time: 50:38)

Solution
Limiting position at end

$$M_{sw} = M_{LL} = 0.0$$

At transfer
For $f_b = -18.0 \text{ N/mm}^2$

$$f_b = -6.67 - 0.0366e$$
$$= -18.0 \text{ N/mm}^2$$

Solving, $e = \frac{18.0 - 6.67}{0.0366}$

$$= 309.6 \text{ mm}$$

Similarly, we can find out the limiting position at the end. Here, the moments due to self-weight and live load are equal to zero. Going through a similar procedure, at transfer for $f_b = -18.0 \text{ N/mm}^2$, $e = 309.6$ mm.

(Refer Slide Time: 51:05)

Solution

For $f_t = 1.5 \text{ N/mm}^2$

$$f_t = -6.67 + 0.026e$$
$$= 1.5 \text{ N/mm}^2$$

Solving, $e = \frac{1.5 + 6.67}{0.026}$

$$= 314.2 \text{ mm}$$

Selecting the lower value

$e_{\max} = 309.6 \text{ mm}$

55

For $f_t = 1.5 \text{ N/mm}^2$, $e = 314.2 \text{ mm}$. Selecting the lower value of the two, $e_{\max} = 309.6 \text{ mm}$.

(Refer Slide Time: 51:17)

Solution

At service

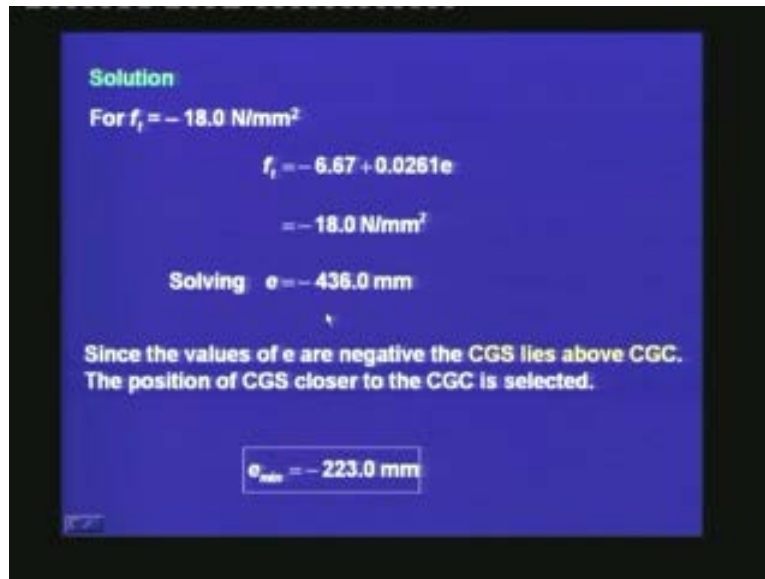
For $f_b = 1.5 \text{ N/mm}^2$

$$f_b = -6.67 - 0.0366e$$
$$= 1.5 \text{ N/mm}^2$$

Solving, $e = -223.0 \text{ mm}$

At service, for $f_b = 1.5 \text{ N/mm}^2$, $e = -223.0 \text{ mm}$.

(Refer Slide Time: 51:35)



Solution

For $f_t = -18.0 \text{ N/mm}^2$

$$f_t = -6.67 + 0.0261e$$
$$= -18.0 \text{ N/mm}^2$$

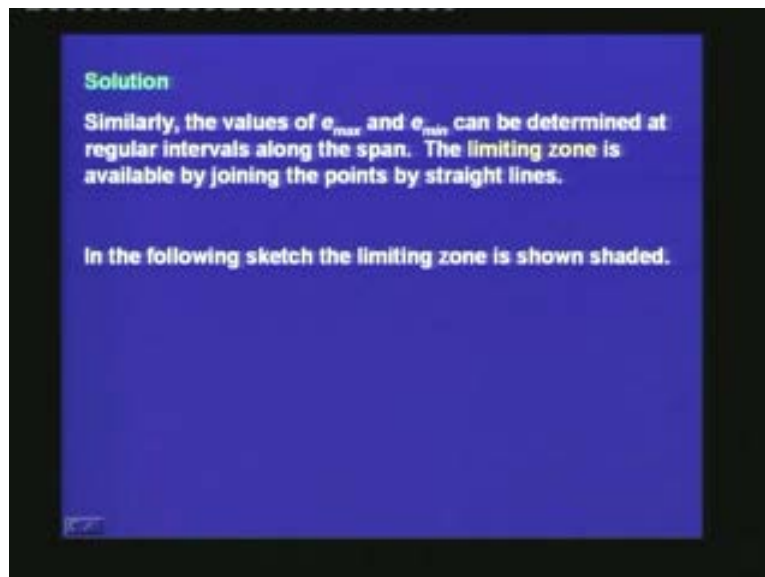
Solving $e = -436.0 \text{ mm}$

Since the values of e are negative the CGS lies above CGC.
The position of CGS closer to the CGC is selected.

$e_{\min} = -223.0 \text{ mm}$

For $f_t = -18.0 \text{ N/mm}^2$, $e = -436.0 \text{ mm}$. Since, the values of e are negative, the CGS can lie above the CGC. The position of CGS closer to the CGC is selected. Thus, out of the two values, the numerically smaller one is selected. Therefore, $e_{\min} = -223.0 \text{ mm}$.

(Refer Slide Time: 52:15)



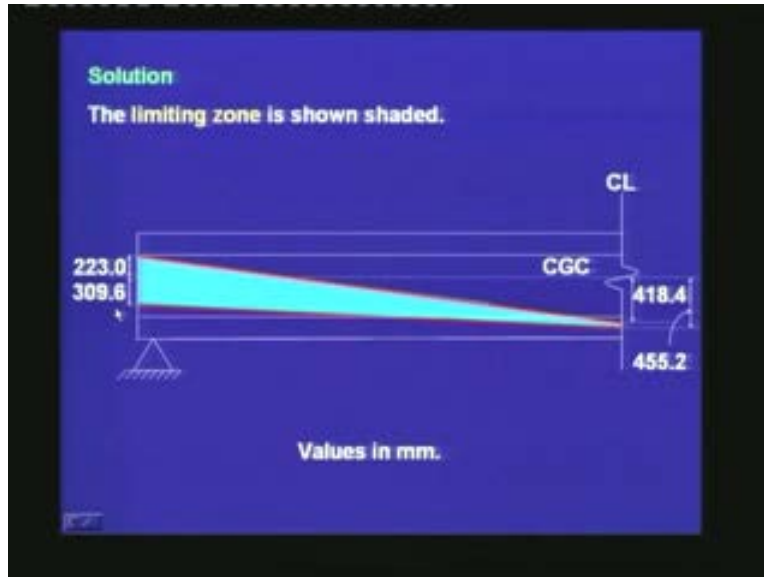
Solution

Similarly, the values of e_{\max} and e_{\min} can be determined at regular intervals along the span. The limiting zone is available by joining the points by straight lines.

In the following sketch the limiting zone is shown shaded.

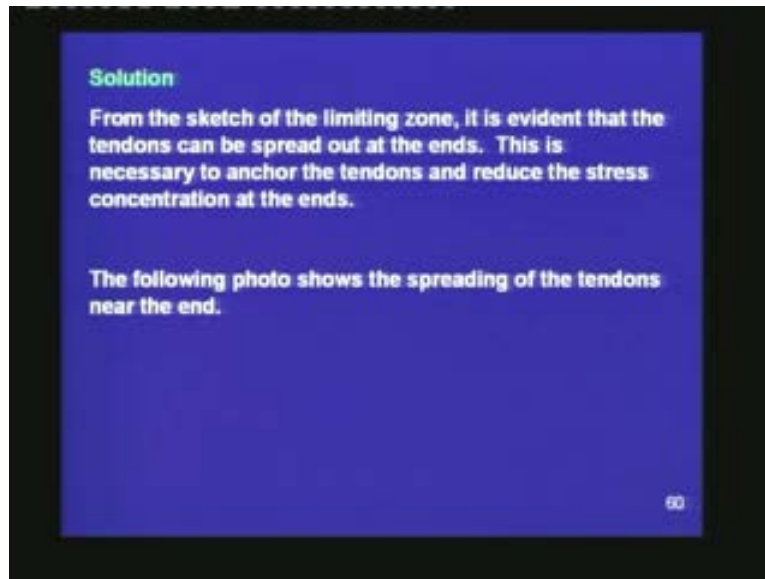
Similarly, the values of e_{\max} and e_{\min} can be determined at regular intervals along the span. The limiting zone is available by joining the points by straight lines. In the following sketch the limiting zone is shown shaded.

(Refer Slide Time: 52:29)



Thus, at the centre, the depth of the limiting zone is small which is between 418.4 and 455.2 mm. At the end, the depth of the limiting zone is large. Here, the CGS can lie above the CGC, up to a height of 223 mm, and it can lie below the CGC up to a depth of 309.6 mm.

(Refer Slide Time: 53:17)



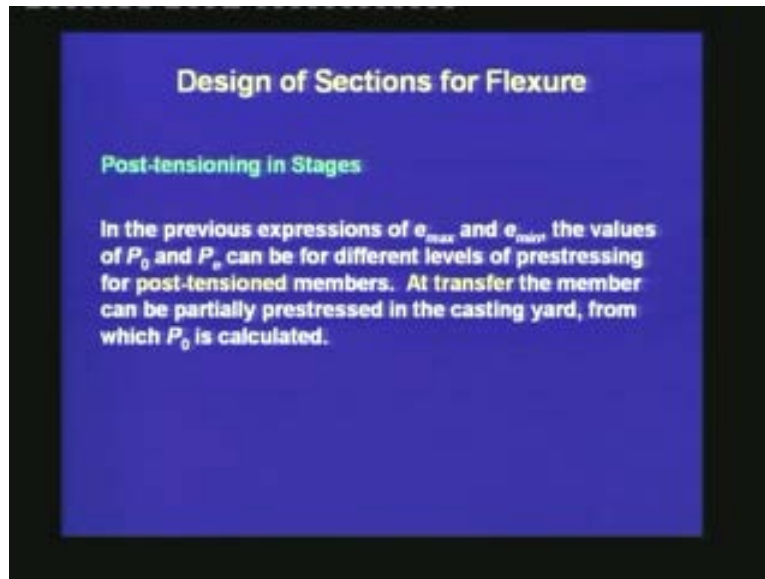
From the sketch of the limiting zone, it is evident that the tendons can be spread out at the ends. This is necessary to anchor the tendons and reduce the stress concentration at the ends. The following photo shows the spreading of the tendons near the end.

(Refer Slide Time: 53:38)



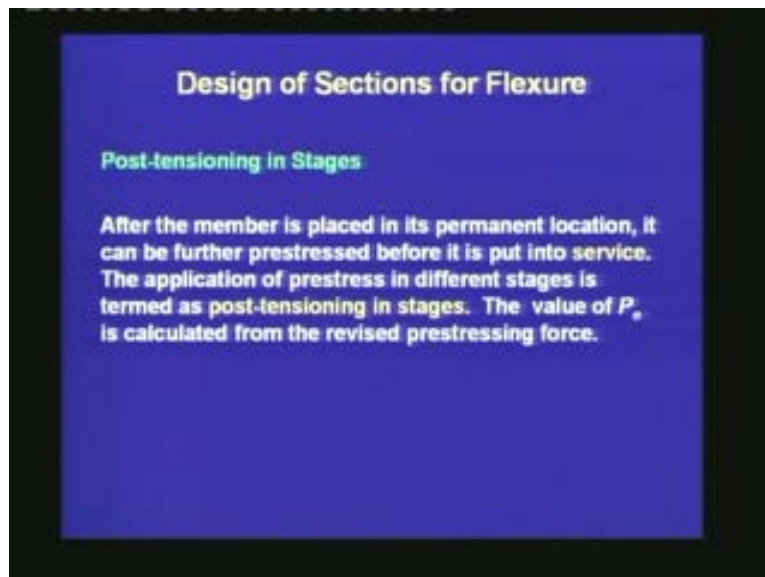
We are learning another concept which is helpful in avoiding stresses going beyond the allowable values, and this concept is called post-tensioning in stages.

(Refer Slide Time: 54:46)



In the previous expressions of e_{max} and e_{min} , the values of P_0 and P_e can be for different levels of prestressing for post-tensioned members. At transfer, the member can be partially prestressed in the casting yard, from which P_0 is calculated.

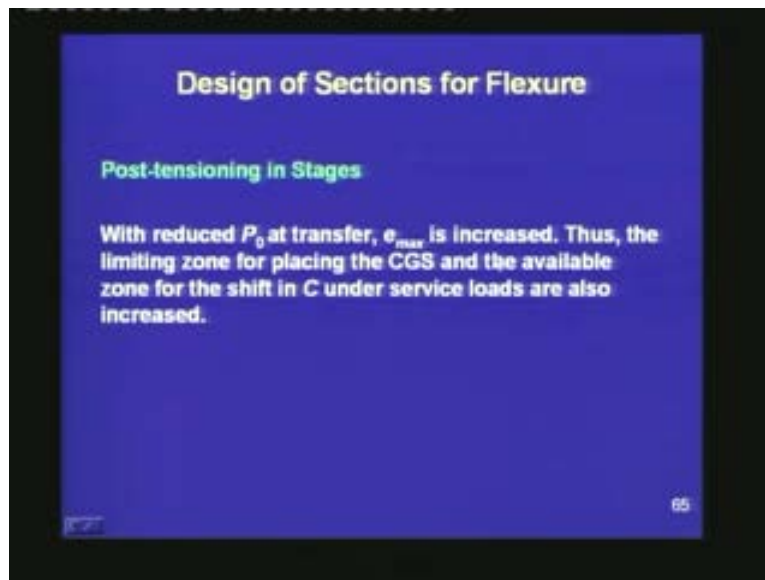
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After the member is placed in its permanent location, it can be further prestressed before it is put into service. The application of prestress in different stages is termed as post-tensioning in stages. The value of P_e is calculated from the revised prestressing force.

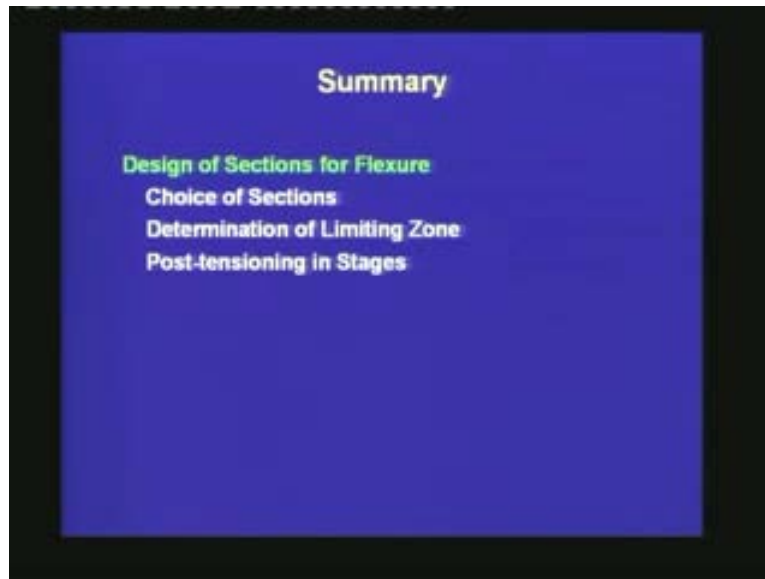
In the previous example, both P_0 and P_e were due to the same prestressing force at transfer. But, the prestress can be applied in stages. First in the casting yard, the prestress is partial, for which P_0 is low and e_{max} is increased. Next, after the member is moved to its permanent location, the prestress can be increased. By this time, the concrete has attained more strength and the allowable stresses go up. Also, the loss in the prestress is reduced. The value of P_e is calculated from the new value of the prestressing force.

(Refer Slide Time: 57:37)



Thus, the limiting zone for placing the CGS and the available zone for the shift in C under service loads are also increased.

(Refer Slide Time: 57:53)



In today's lecture, we studied three important concepts of design of members for flexure. First, we studied the choice of sections. We know that the sections can be of varied types depending on the type of application, the available technology and money for the construction. We have divided the sections into four broad groups: the rectangular section, the T-section, the I-section and the inverted T-section. We saw certain variations of these sections. We can have a tapered section, a bulbed T-section, a box section and U-type channel section. For long span structures, the I-section or the T-section are preferred.

Next, we moved on to the determination of the limiting zone. The limiting zone helps to place the tendons throughout the span of the beam. For a simply supported beam, the limiting zone is deeper at the ends and shallower towards the mid-span. This helps to spread out the tendons near the ends for anchorage, and to reduce stress concentration. Post-tensioning in stages can increase the depth of the limiting zone. With this, we are ending the design of members.

Thank you.