NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 02 : Snell's law, Plane wave reflection and transmission
Lecture 03: Energy in the Plane Wave, Potentials at an Interface

CONCEPTS COVERED

- Energy in the plane wave
- Potentials at an Interface

Energy in a plane wave

- **Similar to the string, seismic waves also transport energy both as K.E. and as strain , or P.E.**
- **To find this energy, consider harmonic plane S-and P-wave travelling in Z-direction, so the SH-wave with displacement in the Y-direction is**

$$u_y(z,t) = B\cos\left(wt - kz\right)$$

- **This expression is in displacement rather than potential.**
- **K.E. in a volume 'V' is**

$$K.E = \frac{1}{2}\int_v \rho\left(\frac{\partial u_i}{\partial t}\right)^2 dv$$

The K.E per unit wavefront average over a wavelength 'λ' is

$$K.E = \frac{1}{2\lambda}\rho B^2\omega^2 \int_0^{\lambda} \sin^2(\omega t - kz)dz$$

$$= \frac{1}{2\lambda}\rho B^2\omega^2 \frac{\lambda}{2} = \frac{B^2\omega^2\rho}{4}$$

The strain energy (P.E) is

$$W = \frac{1}{2}\int_v \sigma_{ij}e_{ij}dv = \frac{1}{2}\int_v c_{ijkl}e_{kl}e_{ij}dv$$

When we solve it

$$W = \frac{B^2\omega^2\rho}{4}$$

Hence, the total energy averaged over a wavelength is

$$E = K.E + W = \frac{B^2\omega^2\rho}{2}$$

The average energy flux in the propagation direction is found by multiplying by the velocity

$$\dot{E} = \frac{B^2\omega^2\rho\beta}{2}$$

The total energy and flux are proportional to the square of the amplitude and the frequency, so for waves of the same amplitude, the higher frequency transports more energy.

Similarly for P-wave travelling in the Z-direction is described by scalar potential

The displacement would be $\phi(z,t) = Ae^{i(\omega t \pm kz)}$

$$u(z,t) = \nabla\phi(z,t) = (0, 0, -ik)Ae^{i(\omega t - kz)}$$

The real part is:

$$u_z(z,t) = Ak\sin(\omega t - kz)$$

$$K.E = \frac{\rho A^2 k^2 \omega^2}{2\lambda}\int_0^\lambda \cos^2(\omega t - kz)dz = \frac{A^2\omega^2 k^2\rho}{4}$$

The strain energy is

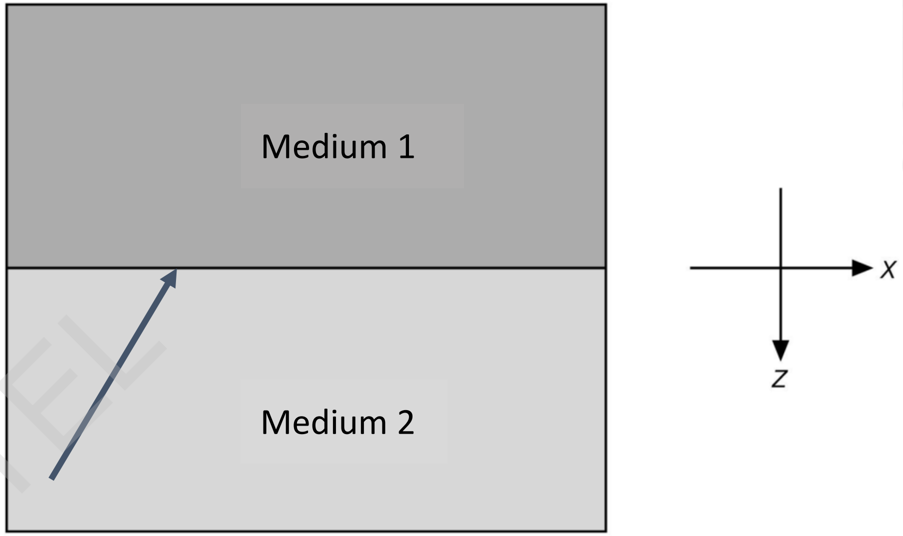
$$W = \frac{1}{2\lambda}\int_0^\lambda \rho\alpha^2 A^2 k^4 \cos^2(\omega t - kz)dz = \frac{A^2\omega^2 k^2\rho}{4}$$

Details are provided in section 2.4.5, Stein & Wysession

These expressions differ from those for the energy of the SH-wave by a factor of k^2, because A is the amplitude of the potential while B is the amplitude of displacement.

Potentials at an Interface

The geometry of the problem is shown in the figure where two half spaces of different materials are in contact along a boundary that is the X-Y plane, and the Z-axis, the normal to the interface, is positive downward.



Let's consider a plane wave in X-Z plane. To separate the SV and SH waves, we split the vector potential γ into two terms

$$\gamma(x,z,t) = \psi(x,z,t) + \nabla \times \chi(x,z,t)$$

The displacement vector can now be written using the scalar potential $\Phi(x, z, t)$ and two vector potentials.

$$
\begin{aligned}
u(x, z, t) &= \nabla\Phi(x, z, t) + \nabla \times \gamma(x, z, t) \\
&= \underbrace{\nabla\Phi(x, z, t)}_{\text{P}} + \underbrace{\nabla \times \psi(x, z, t)}_{\text{SV}} + \underbrace{\nabla \times \nabla \times \chi(x, z, t)}_{\text{SH}}
\end{aligned}
$$

To get $\nabla \times \vec{\Psi}$ to produce SV vibration in the X-Z plane and $\nabla \times \nabla \times \vec{\chi}$ to produce SH vibrations in the Y-plane, we choose

$$\vec{\Psi}(x, z, t) = (0, \Psi(x, z, t), 0)$$

$$\vec{\chi}(x, z, t) = (0, \chi(x, z, t), 0)$$

When plug these into the equation for 'u' gives

$$u_x(x,z,t) = \frac{\partial \phi(x,z,t)}{\partial x} - \frac{\partial \Psi(x,z,t)}{\partial z}$$

$$u_z(x,z,t) = \frac{\partial \phi(x,z,t)}{\partial z} - \frac{\partial \Psi(x,z,t)}{\partial x}$$

Displacement in the X-Z plane involves both P- and SV-wave. That means, P- and SV waves are coupled.

$$u_y(x,z,t) = -\left[\frac{\partial^2 \chi(x,z,t)}{\partial x^2} + \frac{\partial^2 \chi(x,z,t)}{\partial z^2}\right] = -\nabla^2 \chi(x,z,t)$$

Displacement in the Y direction only involves SH-wave

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THANK YOU !