NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 02 : Snell’s law, Plane wave reflection and transmission
Lecture 02: Seismic spectrum, Seismogram rotation, Spherical Waves

CONCEPTS COVERED

- Recap of previous lecture
- Seismogram rotation
- Seismic Spectrum
- Spherical Waves Solution
- Summary

Recap from previous lecture

- **Stress describes the force/area and is a 3x3 tensor.**
- **We did an overview of parameters that describe harmonic waves.**
- **The traction vector is the surface force per unit area on a plane with given normal.**

$$T_i = \boldsymbol{\sigma}.\hat{\mathbf{n}}$$

$$\sigma_{ji} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{bmatrix} = \begin{bmatrix} T_1^{(1)} & T_2^{(1)} & T_3^{(1)} \\ T_1^{(2)} & T_2^{(2)} & T_3^{(2)} \\ T_1^{(3)} & T_2^{(3)} & T_3^{(3)} \end{bmatrix}$$

The relationship between stress and strain can be expressed as :

$$\sigma_{ij} = \lambda\delta_{ij}\theta + 2\mu e_{ij}$$

Recap from previous lecture

Equation of motion

$$\frac{\partial \sigma_{ij}(x,t)}{\partial x_j} + f_i(x,t) = \rho \frac{\partial^2 u_i(x,t)}{\partial t^2}$$

Elastodynamic equation

$$(\lambda + 2\mu)\nabla(\nabla . u(x,t)) - \mu\nabla \times (\nabla \times u(x,t)) = \rho \frac{\partial^2 u(x,t)}{\partial t^2}$$

P-waves

$$\nabla^2 \phi(x,t) = \frac{1}{\alpha^2} \frac{\partial^2 \phi(x,t)}{\partial t^2}$$

With the velocity $\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}$

S-waves

$$\nabla^2 \gamma(x,t) = \frac{1}{\beta^2} \frac{\partial^2 \gamma(x,t)}{\partial t^2}$$

With the velocity $\beta = \sqrt{\frac{\mu}{\rho}}$

Recap from previous lecture

wave number vector shows the direction of wave propagation
$$\overrightarrow{k} = \left\|\overrightarrow{k}\right\|\hat{k} = \frac{\omega}{\alpha}\hat{k}$$

Slowness vector has direction same as wavenumber vector
$$\vec{s} = \frac{\hat{s}}{c} = \frac{\hat{s}}{\text{velocity}} = \frac{\hat{s}}{\alpha}$$

and incorporated slowness vector into wavenumber vector
$$\vec{k} = \omega\vec{s}$$

Particle motion of P-wave is parallel to wavenumber vector
$$\overrightarrow{u} = (-ik_x\phi, 0, 0)$$

Particle motion of S-wave is perpendicular to wavenumber vector
$$\vec{u} = \left(0, -\frac{\partial\psi_z}{\partial x}, \frac{\partial\psi_y}{\partial x}\right)$$

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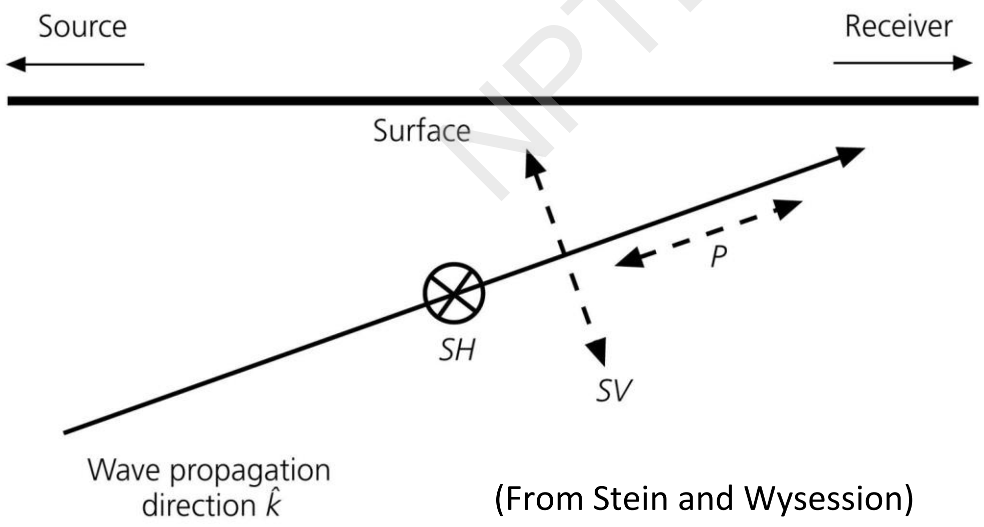
- In real applications, we often define the Z-axis as the vertical direction and orient the X-Z plane along the great circle connecting a seismic source and a receiver.

- Plane waves travelling on the direct path between the source and the receiver thus propagate in the X-Z plane.

- The shear wave polarization directions are defined as SV, for shear waves with displacement in the vertical (X-Z) plane, and SH, for horizontally polarized shear waves with displacement in the Y-direction, parallel to the earth's surface.

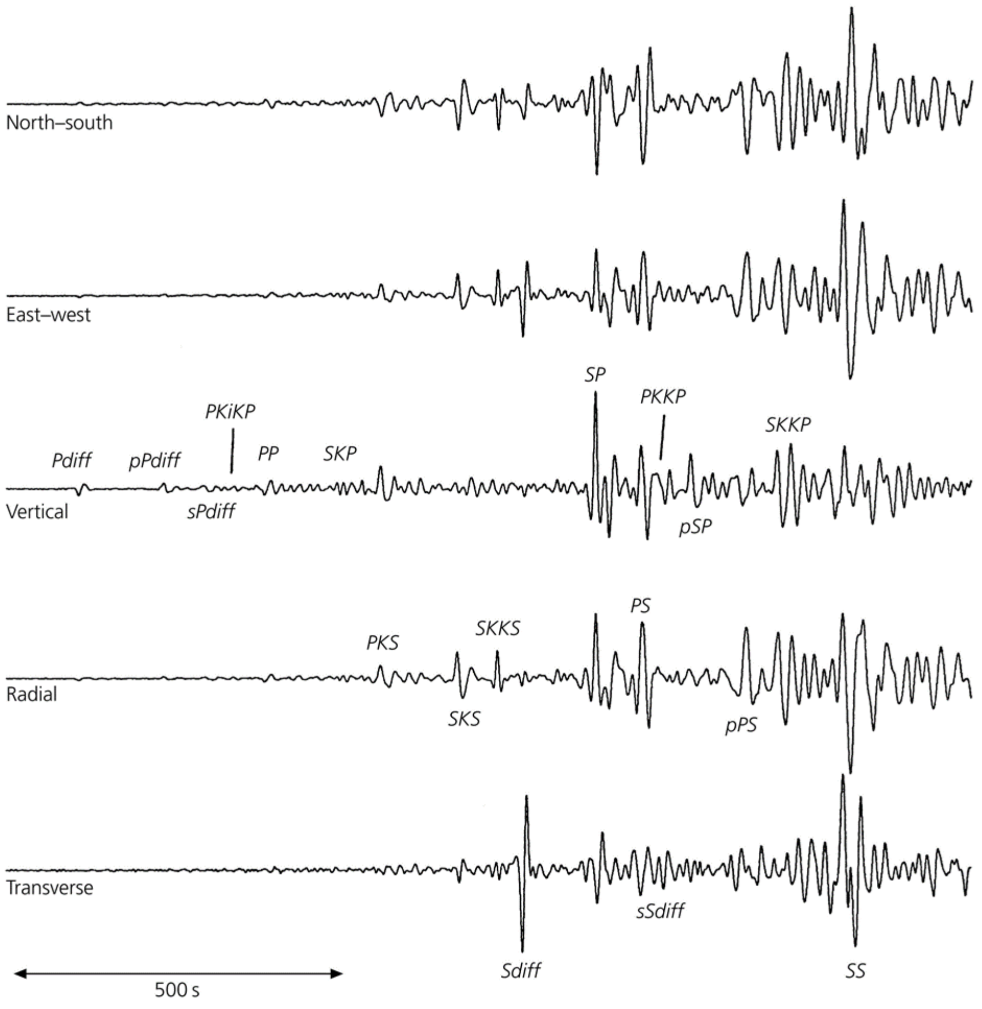
- **Seismometers record horizontal motions in north-south and east-west directions, which rarely correspond exactly to the SH and SV polarizations.**

- **Hence, horizontal components of seismograms are rotated in transverse (SH-motions) and radial directions (SV-motion).**



(From Stein and Wysession)

Figure 2.4-5: Seismograms for a deep earthquake recorded at a distance of 110°.



$$\begin{pmatrix} u_R \\ u_T \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_{EW} \\ u_{NS} \end{pmatrix}$$

with $\theta = 3\pi/2 - \zeta'$

Distance = 110°

Depth = 597 km

Earthquake = Mariana trench

Station = HRV

Seismic Spectrum

1. **It shows seismic waves of various frequencies and types.**

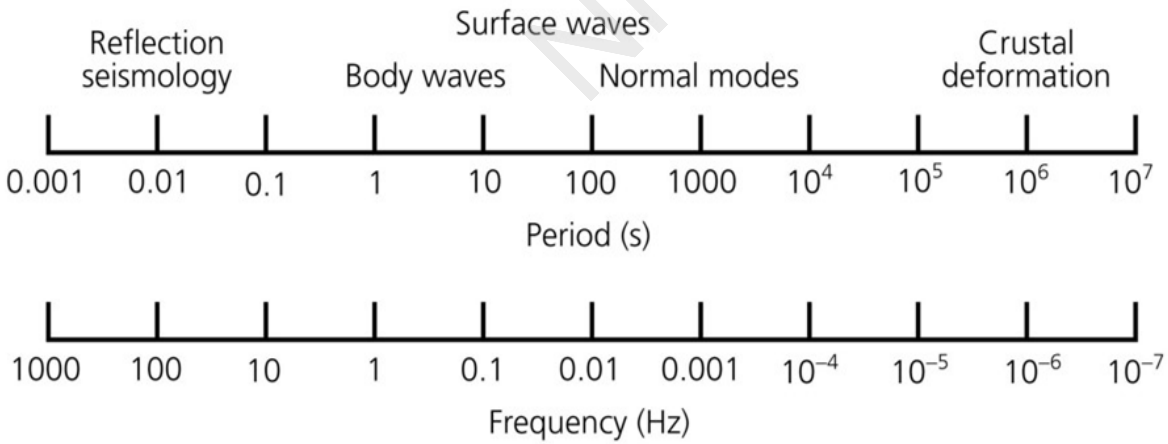
   Earthquakes: 01s to more than 3000s OR 3×10^{-4} to 10 Hz

   Explosions and Artificial sources in Reflection Seismology: 20-80 Hz

   Marine geophysicists to map the seafloor: 3-12 kHz

   Crust deformations: less than 10^{-4} Hz

Figure 2.4-7: Seismic spectrum for various studies.



Spherical Waves Solution

A second solution to the three dimensional scalar wave equation comes from spherical waves rather than planar wavefronts. If we consider a spherically symmetric solution, that means solution will depend only on "r" and ∂_θ, ∂_ϕ will be zero.

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$

This is "Laplacian" in spherical coordinates. As mentioned earlier, our solution will depend only on "r" and ∂_θ, ∂_ϕ will be zero. So,

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right)$$

Spherical Waves Solution

We will convert the P-wave equation in cartesian coordinates to spherical coordinates.

If we use it into P-wave potential equation

$$\nabla^2\Phi - \frac{1}{\alpha^2}\ddot{\Phi} = 0$$

then we will get,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) - \frac{1}{\alpha^2}\frac{\partial^2\Phi}{\partial t^2} = 0$$

$$\frac{1}{r^2}\left[2r\frac{\partial \Phi}{\partial r} + r^2\frac{\partial^2 \Phi}{\partial r^2}\right] - \frac{1}{\alpha^2}\frac{\partial^2 \Phi}{\partial t^2} = 0$$

$$\frac{2}{r}\frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Phi}{\partial r^2} - \frac{1}{\alpha^2}\frac{\partial^2 \Phi}{\partial t^2} = 0 \qquad \text{… equ. 1}$$

To solve it, let us assume $R = r\Phi$ **; this is because we are assuming the solution just dependent or radial distance 'r'.**

$$R = r\Phi$$

$$\frac{\partial R}{\partial r} = \Phi + r\frac{\partial \Phi}{\partial r}$$

$$\frac{\partial^2 R}{\partial r^2} = \frac{\partial \Phi}{\partial r} + r\frac{\partial^2 \Phi}{\partial r^2} + \frac{\partial \Phi}{\partial r} = r\frac{\partial^2 \Phi}{\partial r^2} + 2\frac{\partial \Phi}{\partial r}$$

Similarly,

$$\frac{\partial R}{\partial t} = r\frac{\partial \Phi}{\partial t} \quad \& \quad \frac{\partial^2 R}{\partial t^2} = r\frac{\partial^2 \Phi}{\partial t^2}$$

If we multiply equ. 1 by ‘r’, then

$$2\frac{\partial \Phi}{\partial r} + r\frac{\partial^2 \Phi}{\partial r^2} - \frac{r}{\alpha^2}\frac{\partial^2 \Phi}{\partial t^2} = 0$$

$$\frac{\partial^2 R}{\partial r^2} \qquad -\frac{1}{\alpha^2}\frac{\partial^2 R}{\partial t^2}$$

This will turn into a wave equation,

$$\frac{\partial^2 R}{\partial r^2} - \frac{1}{\alpha^2}\frac{\partial^2 R}{\partial t^2} = 0$$

And it can be solved with a solution of the form:

$$R = f\left(t \pm \frac{r}{\alpha}\right) \quad or \quad r\Phi = f\left(t \pm \frac{r}{\alpha}\right)$$

$$or \;\; \Phi(r,t) = \frac{f\left(t \pm \frac{r}{\alpha}\right)}{r} = \frac{f(r \pm \alpha t)}{r}$$

It's a spherically symmetric solution to the scalar wave equation.

Let us assume that the Φ is sine wave that is

$$\Phi = \frac{1}{r}\sin\left(t - \frac{r}{\alpha}\right)$$

sin **will be peaked at $\frac{\pi}{2}$. Lets trackdown the movement of peak by solving for “t”**

So we get,

$$t - \frac{r}{\alpha} = \frac{\pi}{2}$$

$$t = \frac{\pi}{2} + \frac{r}{\alpha}$$

For increasing values of “t” there is increase in “r” as well

t	r
2π	$3\pi\frac{\alpha}{2}$
$3\frac{\pi}{2}$	$2\pi\alpha$
3π	$5\pi\frac{\alpha}{2}$

As ‘t’ increases
‘r’ also increases

Important points

1. " -ve" sign indicates , wavefront is diverging outward from a source at origin with the amplitude decaying as 1/r.

1. The "+" sign yields an incoming spherical wave, growing in amplitude as 1/r and converging at the origin. Generally, we consider the wavefronts are propagating outwards.

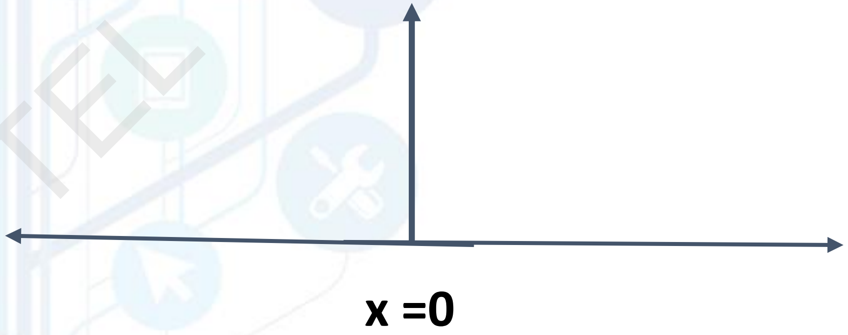
1. The term ' $\frac{1}{r}$ ' indicates geometrical factor. That means, as "r" increases the amplitude decreases. In other words, as the spherical wavefront gets larger with distance, it's amplitude decreases.

One thing to note that, at r =0, the solution explodes

$$\Phi = \frac{f\left(t - \frac{r}{\alpha}\right)}{r}$$

We will understand this using Dirac-Delta function.

Dirac-Delta Function



- Infinite peak & unit area
- It is '∞' at x =0 and zero otherwise.
- It is a 'impulse function'

$$\delta(r) = \begin{cases} 0\,, & x \neq 0 \\ \infty\,, & x = 0 \end{cases} \qquad \int_{-\infty}^{+\infty} \delta(r)dr = 1$$

This allows us to tell you that the solution to the P-wave potential equation in spherical geometry contains a source term for inhomogeneous solution.

$$\nabla^2\Phi(r) - \frac{1}{\alpha^2}\frac{\partial^2\Phi}{\partial t^2} = \underbrace{-4\pi\overbrace{\delta(r)}^{\text{Dirac delta function}}\overbrace{f(t)}^{\text{Source time function of Earthquake}}}_{\text{Source term}}$$

Dirac delta function

Source time function of Earthquake

Source term

This allows us to start thinking about the earthquake source and use that to understand how seismic waves propagate outward from earthquake source

Summary

- **Solution of Elastodynamic equation in spherical coordinates**

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$

$$\Phi(r,t) = \frac{f\left(t \pm \frac{r}{\alpha}\right)}{r} = \frac{f(r \pm \alpha t)}{r}$$

- **Dirac delta function**

$$\delta(r) = \begin{cases} 0\,, & x \neq 0 \\ \infty\,, & x = 0 \end{cases} \qquad \int_{-\infty}^{+\infty}\delta(r)dr = 1$$

- **Consideration of source term in inhomogeneous solution to the P wave potential equation**

$$\nabla^2\Phi(r) - \frac{1}{\alpha^2}\frac{\partial^2\Phi}{\partial t^2} = -4\pi\delta(r)f(t)$$

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THANK YOU !