Structural Reliability
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Lecture –99
Representation of Systems (Part -03)

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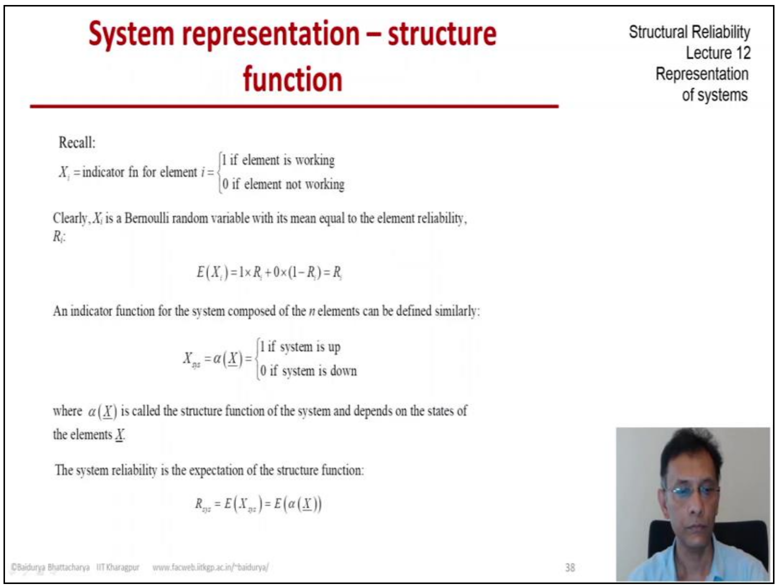

We next look at system representation through the structure function. Recall what we just discussed that the indicator function for the i^{th} element is the binary in fact it is a binary random variable one if the element is working and zero otherwise. So the expected value of this random variable is its reliability because if it is up it is reliable it is the probability is that it is its reliability and it gets down its probability is one's complement strenuous probability.

So, the expected value of X i is R i, R i being the element reliability for element i. Now can we actually have a similar representation for the system which would be equal to 1 when the system is up and would be zero when the system is down and not only that could that function be given in terms of the element states the element indicator function the axis it turns out that we can and that function is actually called the structure function of the system.

So, it is a function of the element states and it is equal to 1 when the system is up and it is equal to 0 and the system is done just like for the elements and just as it was for the elements the system reliability is the expected value of the structure function.

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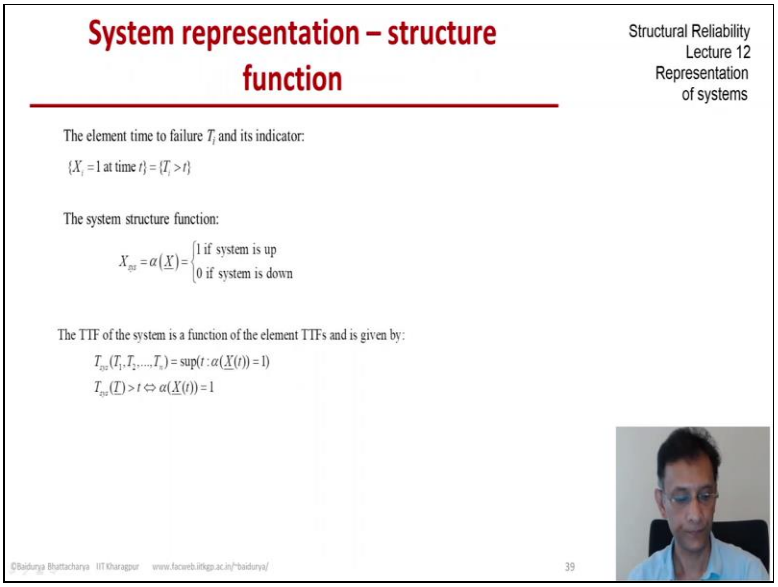

Just as we were able to relate the state of the system to its time to failure in terms of X i being equal to one it stayed at time t was equivalent to saying that the time to failure of element i is greater than small t we can do the similar thing for the system through its structure function. We will look at it later on the system time to failure T subscript cis which is a function of all the element times to failure.

And is the least upper bound of all times when the structure function of the system is equal to 1. So, that would give me a valid definition of the time to failure of the system in terms of its structure function.

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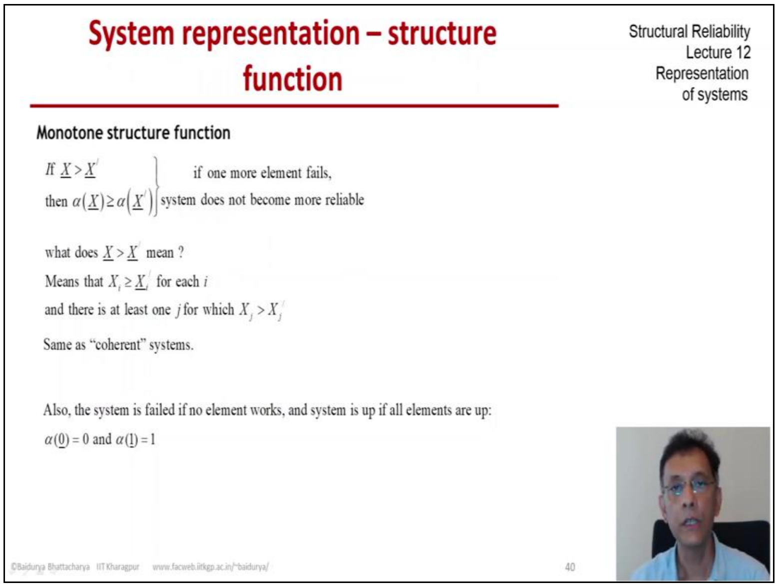

The monotone structure function it is a very useful property and also a rather reasonable property for a system to have simple language it means that if the system is in downstate if the system has failed; if by failing an additional element it is not going to come back up. So, the system doesn't become stronger or safer by failure of additional or new elements because one could argue that if an element makes a system better by failing then why have the element in the first place at all.

Obviously not all systems work that way in all situations but it is a very useful property to have it is a very helpful property to have because it simplifies computations in many situations. So in terms of the formal definition if two state vectors X and X prime are such that that X is strictly greater than X prime and we have defined it on on the screen then alpha of X will at least be equal to alpha of X prime and this is the same thing as the system being a coherent system.

So, a coherent system has a monotone structure function and then we can put the boundary conditions. So, if all the elements are down the system is obviously down and if all the elements are up then the system is obviously up.

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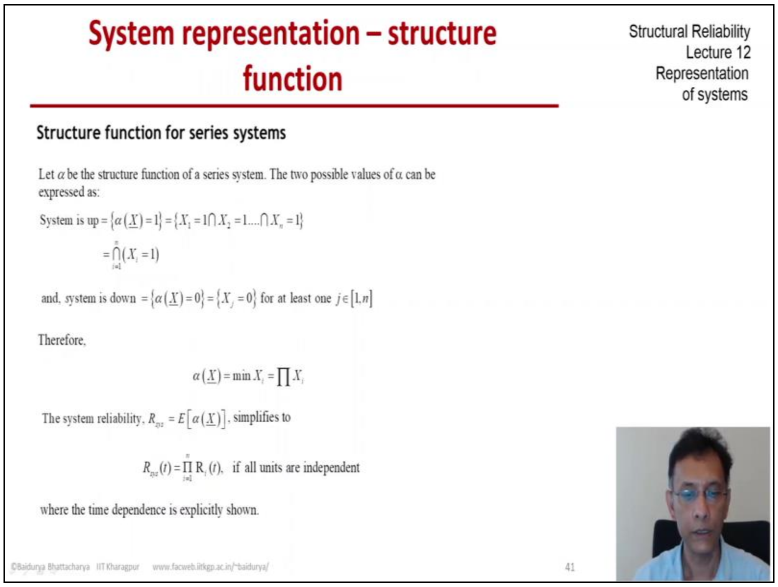

Let us define the structure function for the pure series system which we have looked at in the past. So, the system as we know that it is up only when all the elements are up. So, we can express the event of alpha equals one in terms of all the X's as you see on the screen and likewise alpha is zero when at least one of the element states is 0. So, then we could express alpha in terms of X's in 2 equivalent ways.

One is that it is the minimum of all the X's or it is the product of all the X's. And the expected value of alpha would be expected value of the product of the axis and if the X's are mutually independent then the reliability of the system as we know as we have discussed equals the product of the element reliabilities.

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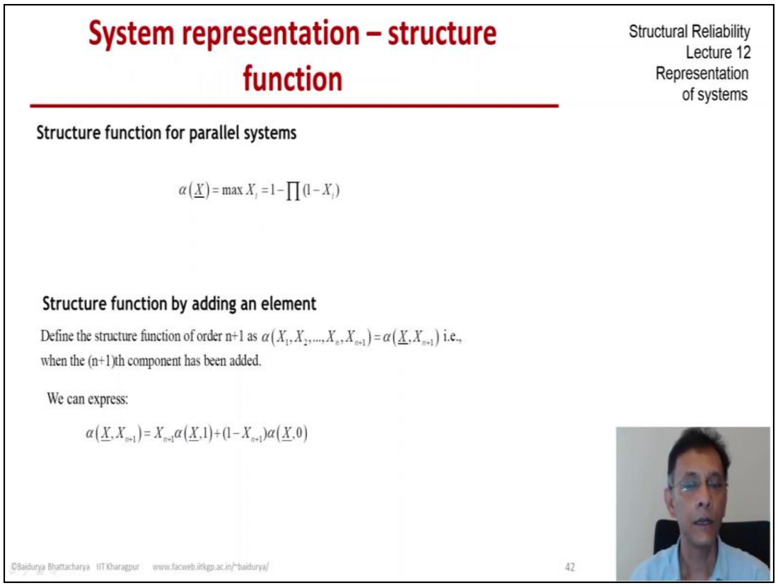

The structure function for the parallel system would going through a similar logic would be the maximum of all the X’s even if one of them is one then the system is up and which could be given in terms of one's complement of the product of 1 – X is there is one useful way of expanding the structure function of a system when you add one element. So, it is basically using a partition approach that we have done before.

So, if there are n elements and we add one more as X n + 1. So, which we explicitly write as alpha of X vector, X n + 1 then we can express this very conveniently in terms of the sum that for one case the system is up. So, which is alpha of X vector, 1 and the other case the element is down. So, alpha of X vector, 0. So the first term on the right corresponds to the fact that that new element is up and the second term on the right corresponds to the case where the new element is down and you can express the structure function of the augmented system in terms of the sum of these two.