

Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –99
Representation of Systems (Part -03)

(Refer Slide Time: 00:27)

System representation – structure function

Structural Reliability
Lecture 12
Representation of systems

Recall:

$$X_i = \text{indicator fn for element } i = \begin{cases} 1 & \text{if element is working} \\ 0 & \text{if element not working} \end{cases}$$

Clearly, X_i is a Bernoulli random variable with its mean equal to the element reliability, R_i :

$$E(X_i) = 1 \times R_i + 0 \times (1 - R_i) = R_i$$

An indicator function for the system composed of the n elements can be defined similarly:


$$X_{sys} = \alpha(\underline{X}) = \begin{cases} 1 & \text{if system is up} \\ 0 & \text{if system is down} \end{cases}$$

where $\alpha(\underline{X})$ is called the structure function of the system and depends on the states of the elements \underline{X} .

The system reliability is the expectation of the structure function:

$$R_{sys} = E(X_{sys}) = E(\alpha(\underline{X}))$$

©Baidurya Bhattacharya IIT Kharagpur www.facweb@iitkgp.ac.in/baidurya/
38



We next look at system representation through the structure function. Recall what we just discussed that the indicator function for the i^{th} element is the binary in fact it is a binary random variable one if the element is working and zero otherwise. So the expected value of this random variable is its reliability because if it is up it is reliable it is the probability is that it is its reliability and it gets down its probability is one's complement strenuous probability.

So, the expected value of X_i is R_i , R_i being the element reliability for element i . Now can we actually have a similar representation for the system which would be equal to 1 when the system is up and would be zero when the system is down and not only that could that function be given in terms of the element states the element indicator function the axis it turns out that we can and that function is actually called the structure function of the system.

So, it is a function of the element states and it is equal to 1 when the system is up and it is equal

to 0 and the system is done just like for the elements and just as it was for the elements the system reliability is the expected value of the structure function.

(Refer Slide Time: 03:30)

System representation – structure function

Structural Reliability
Lecture 12
Representation
of systems

The element time to failure T_i and its indicator:

$$\{X_i = 1 \text{ at time } t\} = \{T_i > t\}$$


The system structure function:

$$X_{cs} = \alpha(\underline{X}) = \begin{cases} 1 & \text{if system is up} \\ 0 & \text{if system is down} \end{cases}$$

The TTF of the system is a function of the element TTFs and is given by:

$$T_{cs}(T_1, T_2, \dots, T_n) = \sup\{t : \alpha(\underline{X}(t)) = 1\}$$

$$T_{cs}(\underline{T}) > t \Leftrightarrow \alpha(\underline{X}(t)) = 1$$



©Baidurya Bhattacharya IIT Kharagpur www.facweb@iitkgp.ac.in/~baidurya/ 39

Just as we were able to relate the state of the system to its time to failure in terms of X_i being equal to one it stayed at time t was equivalent to saying that the time to failure of element i is greater than small t we can do the similar thing for the system through its structure function. We will look at it later on the system time to failure T_{cs} which is a function of all the element times to failure.

And is the least upper bound of all times when the structure function of the system is equal to 1. So, that would give me a valid definition of the time to failure of the system in terms of its structure function.

(Refer Slide Time: 03:38)

System representation – structure function

Structural Reliability
Lecture 12
Representation
of systems

Monotone structure function

If $\underline{X} > \underline{X}'$ } if one more element fails,
then $\alpha(\underline{X}) \geq \alpha(\underline{X}')$ system does not become more reliable

what does $\underline{X} > \underline{X}'$ mean ?

Means that $X_i \geq X'_i$ for each i

and there is at least one j for which $X_j > X'_j$

Same as “coherent” systems.

Also, the system is failed if no element works, and system is up if all elements are up:

$\alpha(\underline{0}) = 0$ and $\alpha(\underline{1}) = 1$



The monotone structure function it is a very useful property and also a rather reasonable property for a system to have simple language it means that if the system is in downstate if the system has failed; if by failing an additional element it is not going to come back up. So, the system doesn't become stronger or safer by failure of additional or new elements because one could argue that if an element makes a system better by failing then why have the element in the first place at all.

Obviously not all systems work that way in all situations but it is a very useful property to have it is a very helpful property to have because it simplifies computations in many situations. So in terms of the formal definition if two state vectors X and X' are such that that X is strictly greater than X' and we have defined it on on the screen then alpha of X will at least be equal to alpha of X' and this is the same thing as the system being a coherent system.

So, a coherent system has a monotone structure function and then we can put the boundary conditions. So, if all the elements are down the system is obviously down and if all the elements are up then the system is obviously up.

(Refer Slide Time: 05:33)

System representation – structure function

Structural Reliability
Lecture 12
Representation
of systems

Structure function for series systems

Let α be the structure function of a series system. The two possible values of α can be expressed as:

$$\text{System is up} = \{\alpha(\underline{X}) = 1\} = \{X_1 = 1 \cap X_2 = 1 \dots \cap X_n = 1\}$$
$$= \prod_{i=1}^n (X_i = 1)$$

and, system is down = $\{\alpha(\underline{X}) = 0\} = \{X_j = 0\}$ for at least one $j \in [1, n]$

Therefore,

$$\alpha(\underline{X}) = \min X_i = \prod X_i$$

The system reliability, $R_{sys} = E[\alpha(\underline{X})]$, simplifies to

$$R_{sys}(t) = \prod_{i=1}^n R_i(t), \text{ if all units are independent}$$

where the time dependence is explicitly shown.



Let us define the structure function for the pure series system which we have looked at in the past. So, the system as we know that it is up only when all the elements are up. So, we can express the event of alpha equals one in terms of all the X's as you see on the screen and likewise alpha is zero when at least one of the element states is 0. So, then we could express alpha in terms of X's in 2 equivalent ways.

One is that it is the minimum of all the X's or it is the product of all the X's. And the expected value of alpha would be expected value of the product of the axis and if the X's are mutually independent then the reliability of the system as we know as we have discussed equals the product of the element reliabilities.

(Refer Slide Time: 06:53)

System representation – structure function

Structural Reliability
Lecture 12
Representation
of systems

Structure function for parallel systems

$$\alpha(\underline{X}) = \max X_i = 1 - \prod (1 - X_i)$$

Structure function by adding an element

Define the structure function of order n+1 as $\alpha(X_1, X_2, \dots, X_n, X_{n+1}) = \alpha(\underline{X}, X_{n+1})$ i.e., when the (n+1)th component has been added.

We can express:

$$\alpha(\underline{X}, X_{n+1}) = X_{n+1} \alpha(\underline{X}, 1) + (1 - X_{n+1}) \alpha(\underline{X}, 0)$$



The structure function for the parallel system would go through a similar logic. It would be the maximum of all the X 's even if one of them is one then the system is up and which could be given in terms of one's complement of the product of $1 - X$. There is one useful way of expanding the structure function of a system when you add one element. So, it is basically using a partition approach that we have done before.

So, if there are n elements and we add one more as X_{n+1} . So, which we explicitly write as $\alpha(\underline{X}, X_{n+1})$ then we can express this very conveniently in terms of the sum that for one case the system is up. So, which is $\alpha(\underline{X}, 1)$ and the other case the element is down. So, $\alpha(\underline{X}, 0)$. So the first term on the right corresponds to the fact that that new element is up and the second term on the right corresponds to the case where the new element is down and you can express the structure function of the augmented system in terms of the sum of these two.