

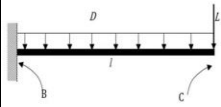
Structural Reliability
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Lecture –95
Reliability Problem Formulation (Part - 07)

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Structural reliability problem formulation

Example: Cantilevered beam with three failure modes



The structural system, its loads and failure modes:

The point load, L , and the uniform load, D , are acting together.

Beam can fail in three modes:

- Flexure (at B)
- Shear (at B)
- Excessive deflection (at C)

Bending failure: Maximum applied bending moment occurs at point B. $M_{app} = LI + D L^2 / 2$

The bending moment capacity of the beam is: $M_{cap} = S_z Y$

Bending failure occurs when M_{app} exceeds M_{cap} .

Shear failure: The maximum applied shear force also occurs at point B. $V_{app} = L + DL$

The shear force capacity of the beam is: $V_{cap} = 0.6 A_{sh} Y$

Shear failure occurs when V_{app} exceeds V_{cap} .

Excessive deformation: The maximum vertical deformation occurs at point C. $\Delta = D L^4 / (8EI) + L^3 / (3EI)$

The maximum allowable deformation is $d_{max} = l/100$.


Deflection failure occurs when Δ exceeds d_{max} .

$$F_{sys} = \{ S_z Y - LI - D L^2 / 2 < 0 \} \cup$$

$$\{ 0.6 A_{sh} Y - L - DL < 0 \} \cup$$

$$\{ l/100 - D L^4 / (8EI) - L^3 / (3EI) < 0 \}$$

Structural Reliability
Lecture 11
Reliability
problem
formulation



We next look at a determinate structure a cantilevered beam as you see on the screen supported at point B and subjected to a uniform dead load D and a point load a live load a point live load L at the free end C. So, our understanding of the mechanics of the beam tells us that it can fail in three modes flexure shear and excessive deflection. We are ignoring other failure modes for example lateral, torsional, buckling.

So, which failure mode to be considered in a reliable analysis has to come from our understanding of the mechanics and our experience with similar situations? So, our understanding of the mechanics of the problem also tells us that in order to evaluate or consider flexural failure we do not need to look at the entire length of the beam at all possible cross sections but the most critical cross section is the support B.

Likewise for shear failure it is the same support at B which is most likely to fail in shear if it

does. Likewise the point at which maximum deflection has to be considered is the tip at C. Thus although this is a distributed structure we have reduced the scope of the problem considerably by using our understanding our experience and focusing on just two critical locations of point B for flexural failure point B again for shear failure and point C for deflection failure.

So, we are not going to look at all the other cross sections throughout the length of the B. Now let us take a look at these three limit states one by one for bending the applied moment at B is L times the length plus the contribution of the dead load Dl square by 2 and the bending moment capacity of the beam is its section modulus as S_z times the yield strength. Now the S_z depending on our understanding our preference and also the mechanical properties could be the elastic section modulus or the plastic section modulus.

Now whichever that is bending failure would occur when the applied moment exceeds the moment capacity. So, that would be setting up the failure description for the B in lecture. Let us look at shear next the applied shear force is $L + D$ small l on at B and the shear force capacity we choose to define it at 0.6 times their resistance shift times the yield strength of the material and shear failure occurs when the applied force exceeds the shear force capacity.

Finally for the deflection limit state we choose to do an elastic analysis. So, we see that well known result the tip deflection δ is given is the sum of the contribution of dead load and the live load where we have used the bending rigidity EI for the B. Now the maximum allowable deformation at point C let us say it is 1 over 100 again it depends on the application of the structure what sort of purpose it is going to serve.

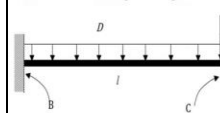
So, let us say it is the length of 100 and deflection failure occurs when the tip deflection δ exceeds that limit 1 over 100 and. So, this way we have defined the failure of the beam in terms of a series representation the system failure is the bending failure or the shear failure or the deflection phase. So, now we are ready to take up the next stage of the analysis which we will look at in a week or two in analyzing the reliability in computing the reliability of this structure.

(Refer Slide Time: 05:54)

Structural reliability problem formulation

Structural Reliability
Lecture 11
Reliability
problem
formulation

Example: Cantilevered beam with three failure modes - time varying live load and deteriorating strength



The structural system, its loads and failure modes:

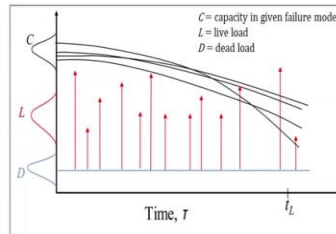
The point load, L (time-varying), and the uniform load, D (time-invariant), are acting together.

Beam can fail in three modes:

Flexure (at B)

Shear (at B)

Excessive deflection (at C)



Take flexural failure mode:

$$M_{cap} = S_y(t) \bar{I}$$

$$M_{req} = L(t)l + Dl^2 / 2$$

$$\text{Failure: } M_{req}(\tau) < M_{cap}(\tau) \text{ at any } \tau \in (0, l]$$



Let us see what would happen if we bring in an explicit time aspect of the problem we have the same mean the cantilever beam supported at B subjected to dead load the distributed dead load D and the point live load L is no longer a time invariant quantity it changes with time and let us say that it occurs as pulses as short pulses during the life of the structure for very small durations.

It could be the occurrence of vehicles on part of the bridge structure or other loads caused by human activities. The dead load D is still timing variant. So, if we were to draw a sample function of what is happening this is what it would look like? So, the dead load D it does not change with time it is a random quantity but it does not change with time example from the distribution here that you see on the left.

The live load L as I said occurs as random pulse load. So, it is added to the dead load but it is not invariant in time it is sampled from the distribution that you see on the left but the peaks are all random. The locations the instances time where these pulses occur are also random in nature. Now something else could also vary in time and that is the capacity. Let us say we are talking just about moment capacity. So, the moment capacity c is also not invariant in time but it decreases maybe randomly in time I say due to processes like corrosion or fitting crack growth.

So, here we have drawn a few sample functions of that deteriorating capacity C. Now if you in a

very nice manner it will just add DML. So, the points in time that the red tip crosses those black capacity lines would constitute to be affected. So, now we just cannot have a very compact description of failure but failure would look something like this. We need to take into account the moment capacity as it changes with time.

So, it is the S_z the section modulus which is now a time dependent function maybe can depend on random function and the applied moment which is now it depends on the time dependent like load. So, now that that checking of whether m applied is more than capacity or not it cannot be done just once but it has to be done at theoretically at all instances of time. So, obviously it is an infinite dimensional problem.

But we can shorten that we can reduce the dimensionality if we just look at the instances of time that L occurs and then we look at all such instances of time then we would have a proper description of flexural failure of this B we should do that something similar for the shear limit state as well as for the deflection. So, this if we have such a time dependent problem this would be a way to set down and then simplifications and computations would come next.