Structural Reliability
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Lecture –81
Monte Carlo Simulations (Part - 13)

Now one situation that is often encountered in problems involving dependent random variables is that we know the marginal distributions of the random variables we know the covariance structure between them or the correlation coefficients between them. But we do not have any further information about their dependence. Obviously this would be enough if those random variables were jointly normal but they are often not.

So, how do we use this partial information to generate dependent samples and for that we actually go back to generating correlated normals.

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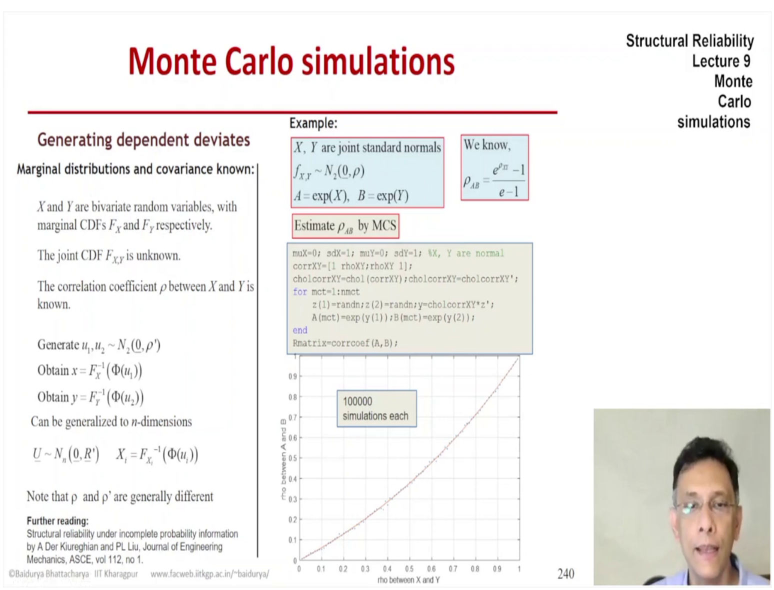

Let us state the problem in terms of two random variables X and Y. So, we know their marginal CDFs F x and F y but what we do not know is the joint CDF or the conditional CDF equality x given y or y given x. So, we cannot use what we did before in that wave height and we have period example for example that we did but what we know is we know the correlation coefficient between x and y.

So, we have this bit of information about their dependence. So, the steps are we generate u 1 and u 2 two dependent standard normals 0 mean unit variance but a correlation coefficient rho prime between them. And there is a reason that we have rho prime here instead of rho which is the correlation coefficient between x and y and we will come to that reason in a minute. So, once we have u 1 and u 2 we obtain x from u 1 through the normal CDF transformation and y from u 2.

Let us take a minute to understand this transformation what we do is that we equate the two CDFs F x and phi of u1, phi being the standard normal CDF and then we invert that relationship. So, this way what we have done is we have generated x and y true to their marginal distributions and we have introduced some dependence among them because u 1 and u 2 are correlated this can be generalized to n dimensions we would simulate the n dimensional standard normal with 0 mean and the correlation matrix r prime and invert that point wise to obtain the x vector.

Now the reason is this nonlinear transformation between u and x as we have alluded to before does not preserve the correlation coefficient. So, the rho that we start off between x and y and the rho dash that we have between u1 and u2 they are not necessarily the same. In fact if you want a certain row between x and y you have to start with a desired row prime between u1 and u2 and there is a a very nice paper by Professor Der Kiurekghian came out in engineering mechanics you could read that for further information.

Let us solve one problem involving such a transformation to show what is happening and all the steps involved. So, we have seen this problem before X and Y are joint standard normals and we exponentiate them to get two dependent log normals. We do not have the entire joint distribution of A and B but we know the correlation between x and y. So, what do we have in terms of the correlation between A and B.

In fact if we wanted a certain correlation between A and B what is the correlation rho between X and Y that we would start with. We know we have already determined this a few lectures back what the correlation between A and B is in terms of that between X and Y but here we are going to solve that by simulation. So we will find rho of AB by multiple trials here is the code. So, we would define the random variable parameters in the beginning x is standard normal.

So, is Y and X and y are correlated uh. So, we define the correlation matrix and we take the JavaScript factor of that and then we generate two independent random normals rand n z 1 and z 2 and then combine them linearly to get the dependent y 1 and y 2 and variables y 1 and y 2 are normal random variables and then we exponentiate y 1 to get A and exponentiate y 2 to get B and we do this repeatedly.

And in the end we can find the correlation coefficient between A and B and here is the result of 10000 such simulations at a range of values of rho. So, you see the non-linear relationship the red line is the true known theoretical relationship which we have on the top and the blue dots are the simulation results. So, for example if we want to have or we know the correlation coefficient between A and B is 0.5 then we would have to start with a correlation coefficient of about 0.63 between X and Y.