Structural Reliability

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Lecture –80
Monte Carlo Simulations (Part - 12)

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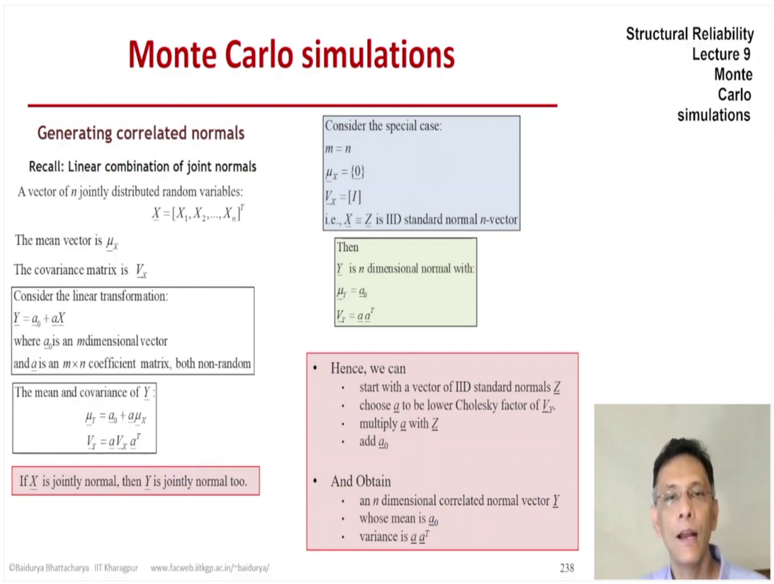

We next look at generating correlated normals the joint normal distribution has this very convenient and very powerful property that the entire dependent structure is captured in the covalence matrix. So, if we just know the mean vector and the variance and the correlation coefficients then we know the entire dependence information. So, that lets us generate correlated normals very conveniently and then with that we can generate other dependent normal deviates.

So, let us walk to the steps. We have seen that the a linear combination of joint normal random variables is again a set of joint normal random variables. So, if we have a vector of n joint normals and whose mean vector is mu x and whose covariance matrix is V x then if we undertake a linear transformation of X and obtain Y where a is a rectangular matrix and a 0 is a column vector and both a and a 0 are non-random then Y is a joint normal random vector with mean and covariance matrix defined in terms of those of X and the matrix a and the column a 0.

Now the under the special case where m is equal to n, so, a is a square matrix then we have the y also to be an n dimensional joint norm. Now if the mean of x is 0 and the covariance matrix of x is the identity matrix which basically means that x is a vector of independent standard normals. Then Y has its mean as the vector a 0 itself and the covariance of Y is a, a transpose. So, this actually gives us a means to generate any correlated normal starting from a set of independent standard normals.

So, the steps would be we generate the IID standard normals z and then we choose a the a matrix to be the lower Cholesky factor of the desired covariance matrix Vy and then multiply that a with z add a 0 which is the mean of Y that we want and obtain the n dimensional joint normal Y whose mean is a 0 because that is what we did and whose variance is a a transpose. So, that is how we engineered it.

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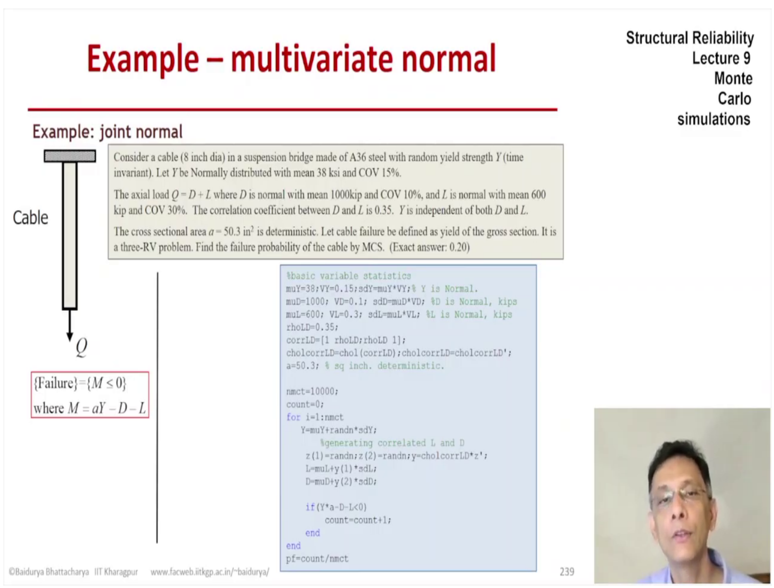

Let us look at an example we have encountered this problem already and we have solved this analytically but it would be a good idea to repeat this and solve it through Monte Carlo simulations and see if the answer is matching to the already known answer. And so, here we have Y and D and L the three normal random variables, two of them are correlated D and L and we need to find the probability that m is less than or equal to zero.

And we go through the steps we define all the basic variable parameters which means the mean of standard deviation of Y and D and L and also define the correlation information between D and L and the non-random quantity a the cross-sectional area and then we enter the loop and for each value of i for each rung through the loop we generate one value of Y using rand n which is the standard normal random variable.

Then we generate z1 and z2 which are the independent standard normals and then use the the Cholesky factor of the correlation matrix between L and D which is what we have already determined before the loop and then we use that to multiply with the independent z's and obtain Y. Now Y have the desired dependent structure and now we multiply Y with the desired standard deviation of L and the add to it the mean of else to get one value of L.

Likewise we do it for D and the correlation coefficient between L and D is now what we want it to be and then do the if block if there is if Y times a - D – L is negative then we have failure. So, we increase the count by one and when we exit the loop the failure probability is the ratio of the account to the total number of trials. So, this would be a nice way to make sure that your coding is right and you're able to generate dependent normals and solve more challenging problems later on