Structural Reliability
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Lecture –76
Monte Carlo Simulations (Part - 08)

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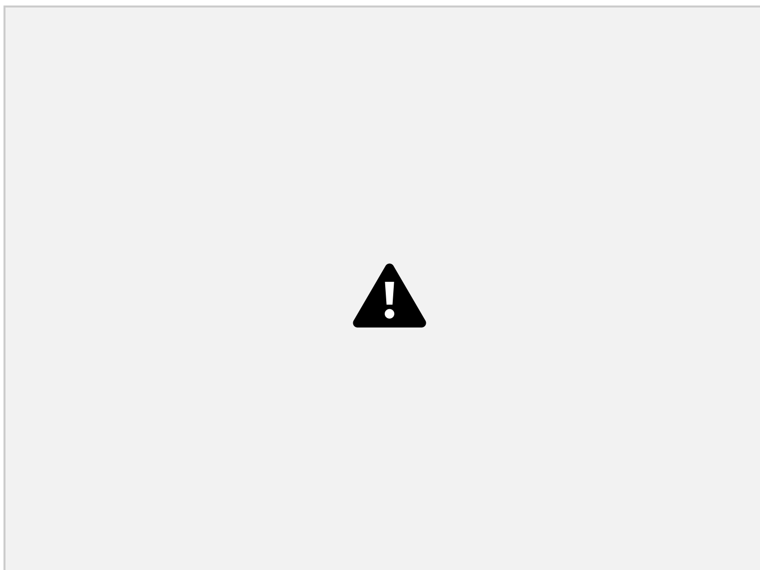

Continuing our discussion on simulating continuous random variables let us look at the most important one of them all the normal distribution. So, how do we generate independent normal deviates a sequence of standard normal's. So, the first thing to note is that the normal CDF cannot be inverted in closed form. So, it is not the most efficient to use the inversion that we did for the exponential in the previous slides.

But there are other ways we can obviously invert the distribution the normal distribution it is just going to take a little long but there are other more elegant methods. So, we can we can in fact the one that I am going to discuss at length is the box singular transform also known as the polar method there are efficient methods like acceptance rejection technique from the Laplace distribution and a few others.

The Matlab command is rand n. So, if you type random you get a standard normal deviate I am not sure what the algorithm behind rand n is but we are going to use it extensively in our simulation. And then once we have a standard normal deviate it is trivial to convert it to any arbitrary normal because we can always multiply that stronger that standard normal z with sigma and add mu to that and we get a normal with mean mu and standard deviation sigma.

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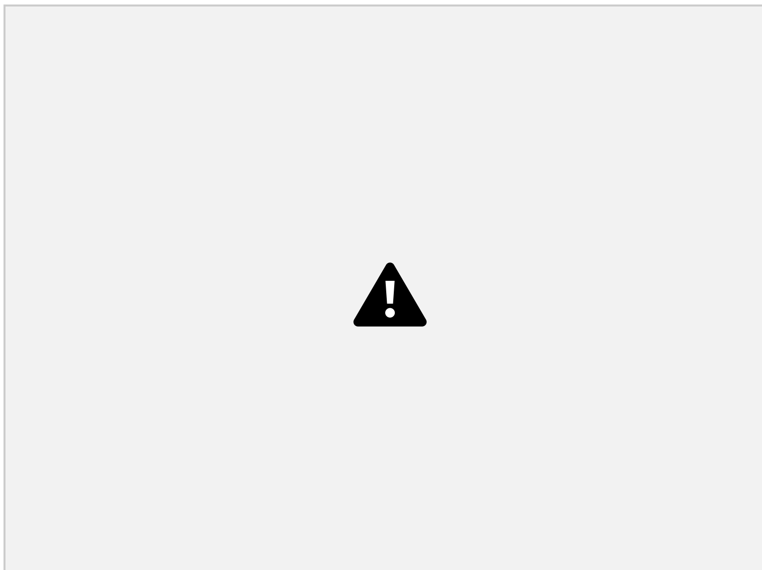

So, let us now look into a little detail at the Box Muller transformation. So, let us say we represent two independent standard normals z 1 and z 2 on the Cartesian plane. And so, we can represent that with in terms of polar coordinates the radius r and the angle theta. So, with this map and inverse map we can actually find out that the density are the joint density of r and theta ah the joint density of z 1 and z 2 is the bivariate normal with the bivariate standard normal with rho equals zero.

So, we can express f of r theta in terms of f of z1, z2 and the Jacobean the Jacobean is simple to compute it is equal to r. So, it is the radial distance once we plug that in the joint density of r and theta can be given in terms of that of z1 and z2 and if we do the calculus and we integrate out r from the joint density. So, we will be left with the marginal density of theta and likewise we can do it for the other variable and we will be left with the marginal density of r and clearly we end up with the uniform distribution for theta.

So, if our if z 1 and z 2 are standard independent standard normals then the angle thus formed is uniform between 0 and 2 pi and the radius thus formed is really distributed or r squared is chi squared with 2 degrees of freedom. So, this is quite powerful and more. So, because we clearly see that f theta times f r is actually f r theta. So, r and theta are independent. So, now we have a means of generating r and theta if we could do it efficiently then we have a clear way of getting z 1 and z 2 from them without the need for inverting the normal distribution function.

So that is exactly what we will do. So, here we have z 1 and z 2 in terms of U 1 and U 2 the first U the U 1 is used to generate the really random variable and the second U, U 2 is used to generate the angle.

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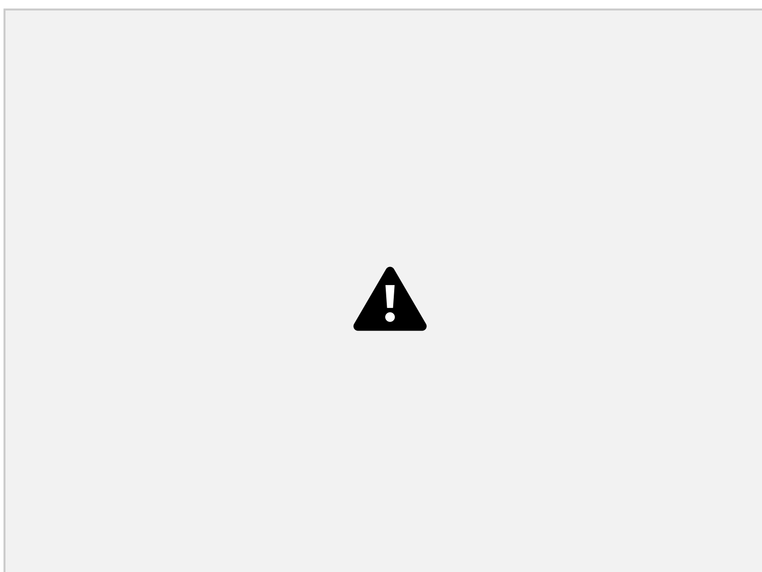

Let us look at uh just an implementation with with the Matlab code. So, r as i said is Rayleigh with the mean of root of pi by 2 and theta is uniform between 0 and 2 pi and we are going to use the polar representation for z 1 and z 2. So, this would be the code the little snippet of Matlab code we generate theta uniformly between 0 and 2 pi. So, the rand multiplied by 2 pi would give me that then we generate the radius which is the Rayleigh random variable and that is using the transformation of another rand.

Here i have used 1 minus rand as the same as rand they are identically distributed. So, that gives those theta and r pair that gives us the x and y coordinates which would be actually z 1 and z 2. So, this is what 10000 samples of this process would look like on the z 1, z 2 plane clearly you know we can get an idea get a sense that this is looking at the normal the standard normal dome that we have seen before from the top.

So, it is most dense near the origin and becomes less and less dense as we move away from the origin the mean the mode. And if we want to find the statistics of these 10000 samples clearly we are getting something close to the theoretical values the means of z 1 and z 2 are close to zero the variance are each close to 1 and the correlation coefficient is pretty small and close to the ideal value of 0 that we should expect.