Structural Reliability
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Lecture –74
Monte Carlo Simulations (Part - 06)

The pseudorandom number generator gives us a sequence of independent uniform deviates. But in Monte Carlo simulations we need samples from all kinds of distributions discrete continuous including dependent samples, samples from joint distributions. So, in this lecture today in our second lecture in Monte Carlo simulations we are going to discuss how to generate continuous and uniform deviates from the sequence of IID uniforms.

And then discuss how to get dependent samples and finally a brief discussion on the need for variance reduction and how it might be achieved. So, let us start with generating continuous variance from a sequence of uniform random numbers.

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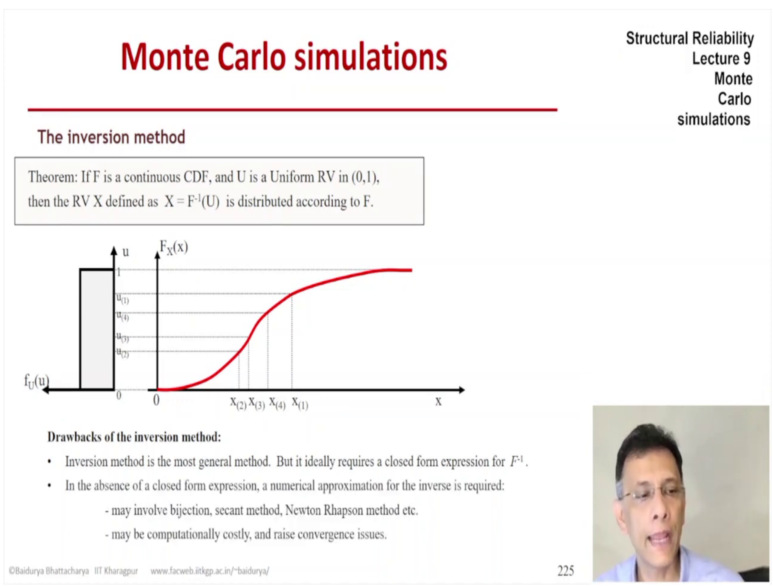

The inversion method it is the most general and very powerful method and the theorem states that if I have a uniform deviate and if I am able to invert that through a continuous CDF F then the value that I get after the inversion is distributed according to the distribution f itself. The proof is straightforward it can be found in any textbook but let us look at it pictorially. So, what you see on the left part of this figure is the uniform PDF.

So, FU is the uniform PDF and the values of U take between 0 and 1. So, it is in this range in that rectangular block on the left that the random number generator samples from. So, let us say the first sample is U subscript 1. So, that U subscript one is. Now used to invert through the distribution function F, so, on the right you have X and its CDF F the red line and so, U subscript 1 is inverted through the function F.

And what we get is X subscript 1 the next one suppose that it generates U 2 and again by inverting the function f we get X2 on the X axis likewise X3 X4 and so, on. So, this would give me a sample from the distribution F by starting with a sample of uniform deviate. Now as I said this is a very general method but the only shortcoming is that it is ideal for those distributions that can be inverted in closed form but there are many which are not invertible.

For example the most famous being the normal distribution we can invert it numerically but it is no closed from analytical expression for the normal inverse. We will discuss generating normal samples later in this lecture. So, once we are not able to invert the distribution function then obviously numerical approximations are required and it slows down the process somewhat and may raise convergence issues.

So, other methods are also used in place of inversion but where possible inversion is the ideal choice.

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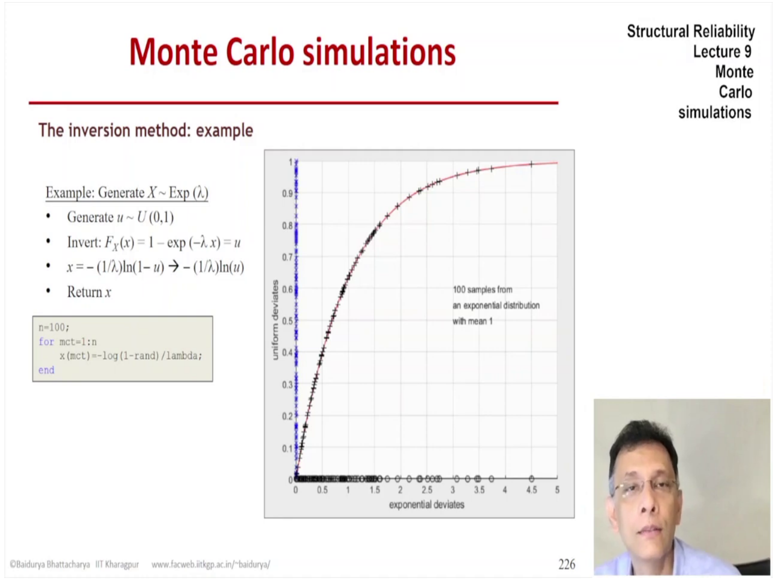

Let us now work through an example in which we simulate the exponential random variable. So, let us say the job is to generate X from the exponential distribution with parameter lambda. So, the steps would be that we first generate U from the uniform distribution and then invert the exponential CDF which is well known and the inversion would give me X as -1 over lambda times log of 1 - u.

Now in many texts and in many codes you will see that it's minus 1 over lambda times log of u because if u is uniform random then 1 - u is uniform random with the same density. So, we actually say one operation if we just start with u n instead of subtracting u from 1. So, then this X that we get would be the desired exponential deviate. Here I have this small Matlab code. So, let us say we are generating 100 exponentials and so, the counter goes from 1 to n and for each of those I generate one uniform by the command rand and subtract that from 1.

I did not use that shortcut that I mentioned and taking the log of that and multiplying with negative 1 over lambda gives me the desired deviate x and then I can use that x for whatever purpose I have in mind. let us look at it pictorially and here I have a small movie of 100 samples being generated. So, on the y-axis you see the uniform deviates being generated one by one and corresponding to that point we are reading off from the red line which is the CDF of x the exponential CDF.

And for each of those black plus points we are reading of the corresponding values of x in the x-axis. So, this is the application of the inversion method. Now as you can see clearly that the points the blue points are generated they look pretty much well dispersed on the y-axis as they should be they are coming from a uniform sample. But because of this particular inversion through that red line clearly you see more points near zero they are more dense there that is because exponential density is exactly that.

It starts it has its highest value at 0 and then falls so, exponentially with increasing x. So, it is clear that this is this is how we can reproduce or we can sample from the exponential. If the red line was some other CDF Gumbull or Y bull or anything else then obviously the points on the x axis would be different. So, I would then be generating points from that particular distribution.