

**Structural Reliability**  
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**Lecture –69**  
**Monte Carlo Simulations (Part - 01)**

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### Recap: course contents

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simulations

Weekly Course Plan	
<b>PART A: BASICS</b>	<b>Week 1</b> Pre-requisites Introduction and overview. Review of basic probability. Random variables, probability laws, common probability distributions – origins and interrelations. Simple one variable example problems.
	<b>Week 2</b> Random variables Functions of random variables. Joint probability distributions, conditional distributions. Joint Normal distribution. Concepts of point process. Simple one- and multi- variable example problems.
	<b>Week 3</b> Monte Carlo simulations Introduction to Monte Carlo simulations Generation of samples from various discrete and continuous distributions, generation of dependent samples Variance reduction techniques Examples: simple coding problems



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Monte Carlo simulations with this we come to the last topic of the part A basics of our course syllabus. Let us just review it for a second we have looked at the basics of probability, random variables, and distributions. We have looked at several univariate distributions and then moved on to joint probability distributions, conditional distributions, derived distributions, joint normal's and also convergence of a sequence of random variables.

We did borrow one lecture from this week the third week to complete the discussion on random variables and joint distributions.

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# Monte Carlo simulations

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- Why simulate random events
- Random number and tests of randomness
- Generation of random samples from various univariate distributions
  - Discrete
  - Continuous
- Generation of samples of dependent random variables
- Variance reduction techniques
- Examples and applications



So, we have two lectures on Monte Carlo simulations and in these two lectures I would like to cover the basics of why we research simulations and Monte Carlo simulations in particular. How we get random numbers and how to test for randomness, how to generate random samples from various universe distributions both discrete and continuous and then generation of samples of dependent random variables and then a few words on variance reduction techniques in Monte Carlo simulation.

And as we have done all through we will solve examples and see applications during these lectures.

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# Monte Carlo simulations

## Why simulation?

### Consider the scenarios:

**New design** or strategy has been produced. We want to know its performance and implications. Is the design safe? Is it economical? Is it feasible? Which design is the best? It may be too costly, too risky or too time-consuming to build the prototype and then test it under real conditions.

Modification to an **existing design** has been proposed. How well will it work? Will it satisfy the requirements? It may be too costly to fix first and then test.

An existing system needs to be **verified for adequacy**, but *test risk* is too high.

**A Real system** needs to be mastered. But consequence of failure is too high (e.g., flying aircraft, surgery, operating power plants etc.), so operator must practice on a simulated system.

The **future outcome** of a process needs to be predicted. The initial/ current condition and evolution rules are known.



Now the first question is why **why** simulation let us consider a few scenarios. Let us say a new design has been proposed a new strategy has been proposed and we would like to know what the performance the implications of this new design is. What about its safety its economy its feasibility if there are more than one designs which one would be the best. And instead of building the prototype we probably we would like to we would like to restart the simulation first because of the cost and the time involved.

The next **the next** scenario is let us say some modifications to an existing system or structure has been proposed. We would like to know how well that modification would work would serve the purpose but it may be too costly or too risky to fix first and then test. The next situation could be that an existing system like an existing structure a bridge structure needs to be verified for adequacy for continued service.

But the test risk is too high which means the load at which we would know something useful could be. So, high that there is an appreciable failure probability associated with that that stress. So, that the test risk could be too high and we would like to do some simulation. The next scenario could be that a real system needs to be learned or mastered and there are quite a few examples you see and but to learn on the job it might be too risky. So, some sort of simulation would be preferred.

The last scenario is that there is a process whose evolution law we know whose initial conditions we know and then we need we would like to know what happens how it evolves in the future what the end point is. So, we would need to simulate the process uh.

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## Monte Carlo simulations

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**Why simulation?**

In each case, we know, roughly or in detail, the analytical modeling of the system, i.e., the rules governing the system.


But closed form solution relating input to output is not available.

So we resort to simulation.

In today's world, computers have a central role in simulation.

For our purpose: **Simulation is the procedure in which a mathematical/physical/hybrid model of a real system is subjected to experiments by supplying a set of external stimuli (input) and the response (output) of the model system is studied.**

***A note of caution:** simulation can be only as good as the model and its assumptions.  
Do not put misplaced faith on simulation results.*



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So, the common theme running through all of this is that we know the **the** rules governing the system but there is no closed term solution which relates the input with the output. And in today's world when we undertake simulation the computers have a central role in fact some simulations are performed entirely in the computer. For **for** our purpose therefore is it is the process of having creating a mathematical or a physical or a mathematical physical hybrid model of a real system and then subjecting it to experiments by supplying a set of inputs and then studying the output of the response of the model.

So, that is simulation. Now once more let us have a bouncing animation just to underline the fact that any simulation can only be as good as **as** the model and its assumption. So, one should be careful about putting misplaced faith on the on the simulation results.

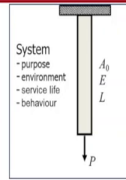
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# Monte Carlo simulations

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## Simulation of probabilistic events:

- When a system is such that there are non-negligible uncertainties or randomness in:
  - inputs/demand
  - properties
  - capacity
  - modeling
- of the system,
- then the probabilistic nature of the parameters must be incorporated into the simulation.
- **That is, one needs to simulate random events**
- **This is achieved using Monte Carlo simulations**



## Outcomes of MCS:

- Evaluate expectations of functions of random variables
  - estimate probabilities of (rare) events
  - Establish statistics of system response
- Obtain spatial and/or time histories of evolution of stochastic systems



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Now simulation of probabilistic events that is what this lecture is about. So, we have seen **this** this description of a concept system it might have uncertainties in one or more aspects namely inputs properties the capacity and the model of the system and if that is. So, then we need to when we simulate the system we need to incorporate the random aspects of the problem. So, we need to be able to simulate random events.

And that is what is achieved through Monte Carlo simulations and Monte Carlo **is** a famous district in the principality of Monaco in the south of France and it is famous for its gambling casino. So, that is that is where the **the** name comes from now the outcomes of Monte Carlo simulation is basically it lets us evaluate the expectations of functions of random variables and in our case this often takes a form of estimation of rare probabilities like probabilities of failure.

Or the expectation could be used to establish statistics of a system response mean variance etc or just the evolution of **of** a random system which could be random both in space and or time and obviously this is relevant very relevant when closed some solutions are not available. So, Monte Carlo simulations is one of the best ways to tackle such problems.