Structural Reliability
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Lecture –67
Joint Probability Distributions (Part - 18)

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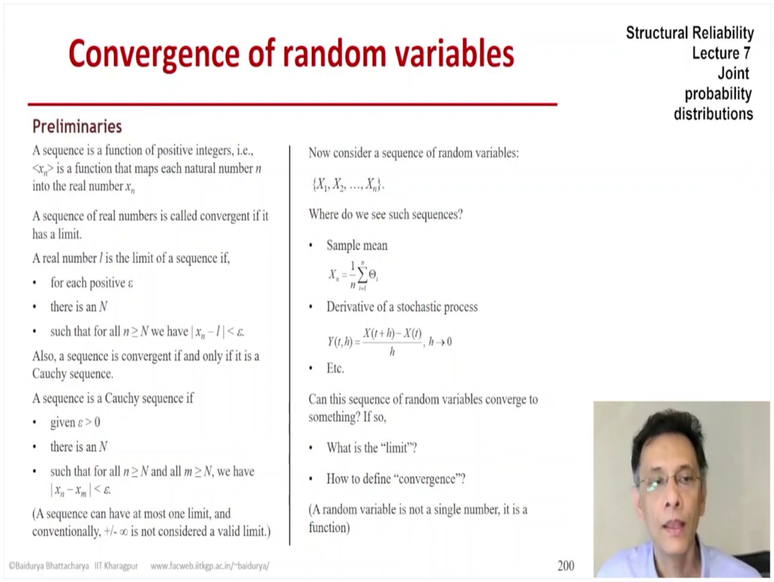

The last topic in this lecture is the convergence of a sequence of random variables we will discuss four such modes of convergence and end the discussion with the law of large numbers. So, when we talk about the convergence of a sequence of real numbers we have in mind a limit. And as we progress with the sequence as we go down the sequence the distance from the limit becomes smaller and smaller and this is formalized in terms of very precise definitions.

Now when we have a sequence of random variables what does it mean to talk about convergence of such a sequence. So, let us first think of cases or situations where such a sequence even occurs. So I have put two examples on the screen one is the estimation of a population mean by taking samples. So, as the sample size becomes larger and larger the sample mean would constitute such a sequence of random variables.

The other example would be the definition of the derivative of a stochastic process we have not talked about stochastic process yet but it's basically a random function in time. So, if we take the ratio as you see on the screen for different values of h for smaller and smaller values of h every time we get a new random variable which we call y t , h and. So, does this sequence of random variables converge to something so, that we can define that as the derivative of the process.

So, in such situations uh what would be the limit and how to even define convergence because we need to keep in mind that a random variable is not a single number it is it is a function defined on the sample space.

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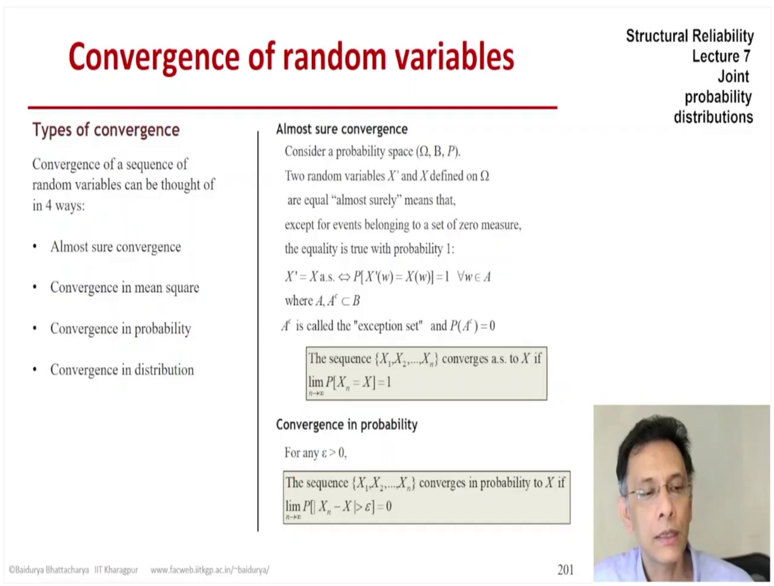

So, there are 4 ways that we can think of a sequence of random variables can converge and those four ways are almost sure convergence, convergence in mean square, convergence and probability and convergence in distribution and in the end we will discuss the hierarchical relation between pairs of these. So, in talking about almost shear convergence let us say that we define two random variables X prime and X on the same sample space.

So, if it happens that for every sample point omega the random variable omega X of omega and X prime of omega are equal with probability 1 except an exception set with zero probability measure then we can say that X prime is equal to X almost sheerly. So, we if we extend that idea to the sequence of X1, X2 up to X n then that sequence converges to the random variable X almost surely if the in the limit the probability that X n equals X is exactly equal to 1.

Now that is a very strong statement if we weaken that statement we get a convergence in probability. So, if now we have any positive epsilon and we define the convergence in terms of a probability in the limit in terms of X n - X the absolute value greater than epsilon, epsilon being any positive number then if that probability is zero then we get convergence and probability. If you compare the two the almost shield convergence and convergence probability you can see how the requirement has been diluted in the convergence and probability case.

Now if we replace the probability in the limit in convergence probability with the expectation the expectation of the difference or the expectation of the Pth norm between X and X n then what we get is the convergence in LP naught.

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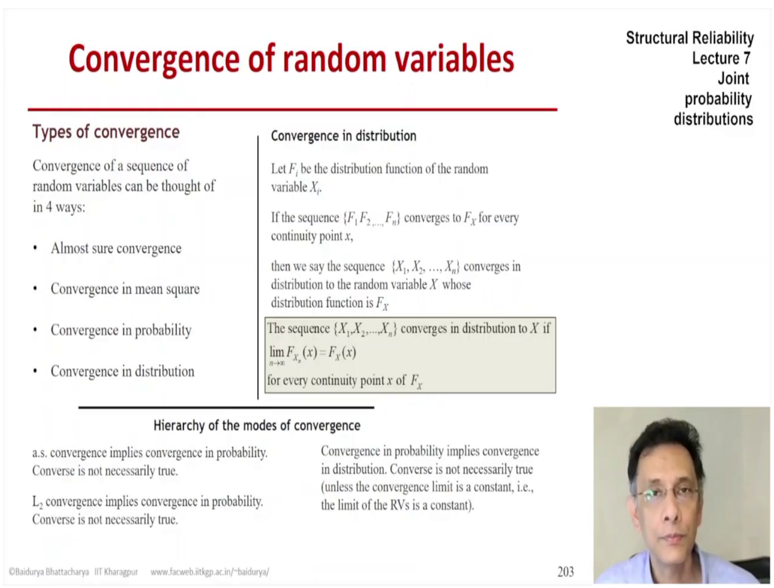

So, and if P is 2 that gives mean square convergence which we also call limit in mean and obviously it makes sense only for those cases where the second order the expectation of X squared actually exists but this would be the definition of the mean square convergence. The fourth mode of convergence is convergence in distribution. So, if now we have a sequence of distributions. So, each random variable Xi has the distribution F i and if the sequence converges to F X for every continuity point of the function.

Then we say that the sequence of random variables X1 up to Xn convergence and distribution to the random variable X whose distribution is F X. Now how do they stack up against each other. So, what is the hierarchy the almost shear convergence implies as we discussed converges to probability but converge is not necessarily true. So, convergence probability is a much weaker statement.

Likewise mean square convergence also implies convergence and probability but the converse is not necessarily true and convergence and probability in turn implies convergence and distribution and the conversion is not necessarily true. Some lectures back we discussed the central limit theorem and the convergence we talked about in that context was actually convergence in distribution. For further reading I would refer to the two volumes by Feller and the book by Sydney Resnick.