Structural Reliability
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Lecture –66
Joint Probability Distributions (Part - 17)

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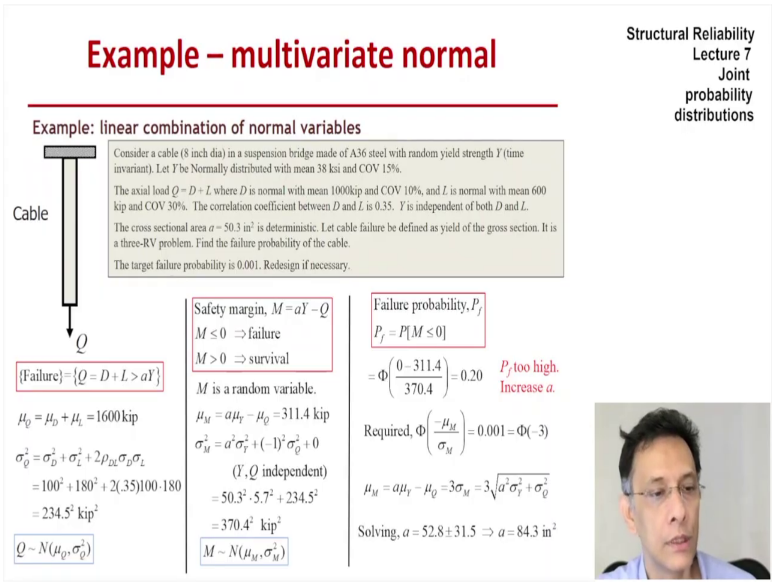

This example involves a linear combination of dependent normal random variables let us take a minute to read the problem. And as you see this you have seen this problem before uh as a simple one variable problem uh in which uh Y the yield strength was the only random variable and it was viable distributed. Here we consider it to be a three random variable problem uh in which we have introduced two more random variables uh the dead load and the live load D and L and because we want to look at combinations of normal random variables linear combinations.

So, we have made all of them normal. So, yield strength is normal the dead load is normal and the live load is normal uh and not only that there is a little bit of dependence between D and l. So, uh as before we define the failure event uh as Q the load uh exceeding uh the strength a times Y, a is a constant deterministic and actually we need to find uh an appropriate value of a if the failure probability is not small enough. uh

The first step would be to actually find out the statistics of Q uh the combined load the mean of Q is simply the sum of the means as we have seen before several times and the variance of Q. Now not only depends on the variance of D and L but also on their covariance because they are correlated. So, if you put in all the numbers uh the variance of Q turns out to be 234.5 squared the units being kept.

Now the important realization is that Q because it is a linear combination of normal's uh Q is also a normal random variable. Next we define a new random variable which we call the safety margin. So, the safety margin is aY. So, strength minus the load and if M is 0 or negative we have failure if M is positive we have survival. So, these are complementary events M is also random variable as i said. So, it has its own mean which turns out to be 311.4 kip and it has its own variance which here Q and Y are independent.

So, there is no correlation term between them. So, the variance is simply 370.4 squared and just as Q was a normal random variable m also is normal because it is the linear combination of two normal's uh Y and Q. So, uh we have m as normal with mean mu M and variance sigma squared. Now we are in a position to find the failure probability uh simply defined as P of M less than or equal to 0 and we use the normal table to find that.

So, P of Mm less than equal to 0 is the standard normal distribution function evaluated at uh minus 311 divided by 370. So approximately comes to 20%. Now this is too high because our limit is 0.001 uh we discussed several options uh when we looked at this problem before but here our uh our preference is to increase the cross-sectional area a. And so, that our failure probability is 0.001 or the argument of the normal CDF would be minus 3.

And if we go through the algebra we have one quadratic equation in terms of a and if we solve this the feasible solution is 84.3 square inch. So, it goes up from 50.3 to roughly 84.3. So, that would be the answer.