Structural Reliability
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Lecture –65
Joint Probability Distributions (Part - 16)

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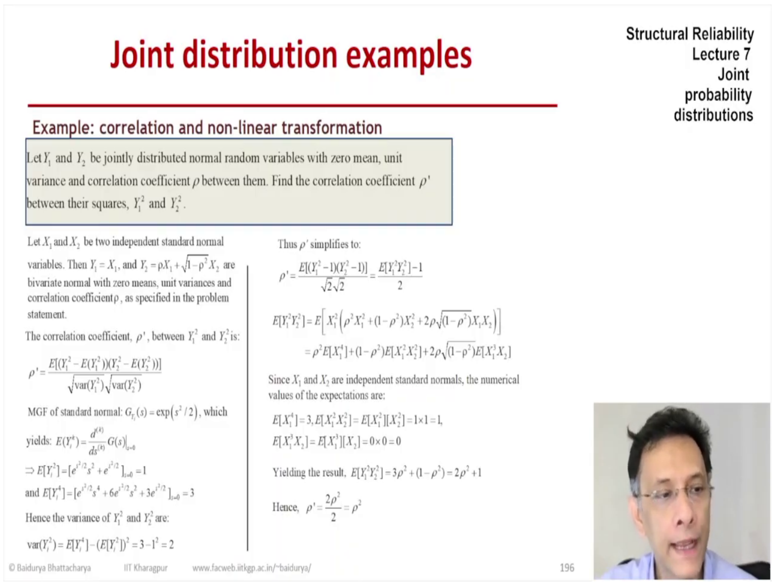

Let us solve some examples involving joint normal random variables. Let us take a minute to read the problem Y 1 and Y 2 are joint standard normal's they have unit variance they have zero mean and rho as their correlation coefficient. If we square them we get two new random variables as we have seen earlier in this lecture and what would be the correlation coefficient because of this non-linear transformation of Y1 and Y2.

So, we will call them call that rho prime. So, to start with let us define Y1 and Y2 in terms of two independent standard normal's. So, that the statistics of Y1 and Y2 are satisfied. So, let X1 and X 2 be these two individual independent standard normal's and we define Y1 as equal to X1 and Y2 in terms of a linear combination of X1 and X2 as you see on the screen and let us just take a minute to make sure that we get what we wanted.

So, obviously the statistics of Y1 is mean 0 variance one that is clear the statistics of Y2 is also mean 0 because the mean of Y2 would be rho times 0 plus root 1 - rho squared times 0 and the variance of Y 2 would again be the sum of the variances of X 1 and X 2 multiplied by rho and 1 minus multiplied by rho square and 1 - rho square respectively and that also checks out to be equal to 1.

And what would be the correlation coefficient between Y 1 and Y 2 we need to find the covariance. So, that would be e of Y 1 times Y 2 that turns out to be just rho from that we subtract the product of the means which is zero and we divide that by the individual standard deviations each of which is 1. So, it does check out. So, let us now define rho prime in terms of these new random variables Y 1 and Y 2.

So, rho prime as you see requires the mean of Y 1 squared the mean of Y 2 squared the variance of Y 1 squared and variance of Y 2 squared but it also requires the expectation of Y 1 squared times Y 2 squared. So, we will need to derive them one by one. Let us first get the expectations of 1 squared and also Y 1 to the power of 4 and likewise for Y 2 and Y 2 squared and Y 2 to the power of 4 for that we are going to use the moment generating function of the standard normal random variable which we did a few lectures back and which is exponential of s squared by 2 and the kth derivative of that evaluated at 0 gives me the kth mode or the kth raw moment of Y 1 and Y 2.

So, if we go through the steps the expectation of Y 1 squared is 1 which is also expectation of Y 2 squared and expectation of Y 1 to the power of 4 turns out to be 3 which is also the same for expectation of y 2 to the power of 4. This would let us write the variance of Y 1 and Y 2 squared each of them which turns out to be 2. So, we have of all the quantities that we needed in row prime we have derived most of them except the one that remains which is expectation of Y 1 squared times Y 2 squared.

And that we will now take up and to do that one of the straightforward steps would be to write Y 1 and Y 2 in terms of X 1 and X 2 uh. So, let us do that. So, V square Y 1 V square Y 2 and we multiply them when we express them in terms of X 1 and X 2 and this is what we get. So, the expectation of Y 1 squared and times Y 2 squared is given in terms of expectations of X 1 to the power of 4 and X 1 squared X 2 squared and so, on that you see on the screen.

We know most of these actually we know all of these because X 1 and X 2 are independent. So, the expectation of X 1 cubed being an odd moment is 0 because X is the standard normal the others we have already found out through the moment generating function. So, we can simplify this expression and it turns out that expectation of Y 1 squared times Y 2 squared is a simple function of of rho. So, it's twice rho squared plus 1.

So, now we plug this value back in the top expression for rho prime and the answer turns out to be rho squared. So, the non-linear transformation of Y 1 and Y 2 reduces the correlation coefficient between them because as we know the correlation coefficient measures the degree of linear dependence uh. So, rho prime becomes rho squared which in the range of 0 to one or -1 to plus one is always less than the absolute value of rho.

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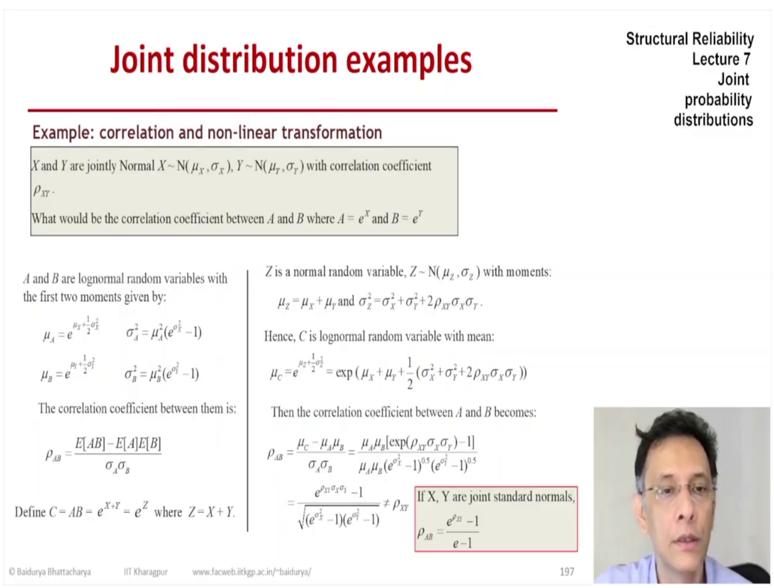

The next example we take up also involves bivariate normal’s and correlation coefficient we actually have seen this in the previous lecture. So, we would not spend a lot of time on this but if you want to rederive these especially in terms of x and y being standard normal then that would be a good exercise and let us go through the steps. We what we did was we found the the mean of a and b and the variances of A and B in terms of those of X and Y.

And then we started deriving the row between A and B and for that we had to define a new log normal random variable C and we went through the steps and ended up with an expression which was definitely different from rho xy but it would be interesting to see what the answer would be for the standard normal case when x and y are joined standard normal's and you would get the answer as exponential of rho - 1 divided by e - 1. So, that would be the result of this particular nonlinear transformation.