

Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –63
Joint Probability Distributions (Part - 14)

(Refer Slide Time: 00:27)

Functions of random variables

Structural Reliability
Lecture 7
Joint
probability
distributions

Example: Function(s) of several random variables

Transformation of wave height and period

Given:

$$V = \frac{H}{T+1}$$

$$A = \frac{H}{(T+1)^2}$$

What is the joint density of V, A?

Inverting the relation:

$$H = \frac{V^2}{A}$$

$$T = \frac{V}{A} - 1$$

The Jacobian of the transformation is:

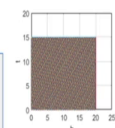
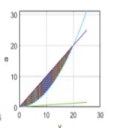
$$J = \begin{vmatrix} \frac{\partial h}{\partial v} & \frac{\partial h}{\partial a} \\ \frac{\partial t}{\partial v} & \frac{\partial t}{\partial a} \end{vmatrix} = \begin{vmatrix} 2v/a & -v^2/a^2 \\ 1/a & -v/a^2 \end{vmatrix} = v^2/a^3$$

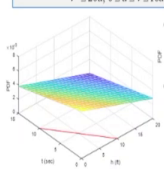
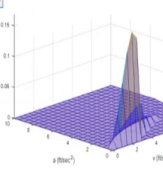
The joint density of V, A:

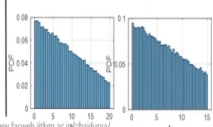
$$f_{v,a}(v,a) = J(v,a) f_{h,t}(h,t)$$

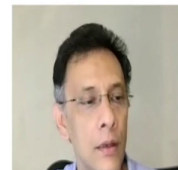
$$= \frac{v^2}{a^3} k \left(36 - \frac{v^2}{a} - \frac{v}{a} \right)$$

$$v^2 \leq 20a, 0 \leq a \leq v \leq 16a$$





©Baidurya Bhattacharya IIT Kharagpur www.facweb.iitkgp.ac.in/~baidurya/ 191

Let us look at one more problem involving functions of random variables we have seen this particular problem before when we were discussing joint distributions last week and we wanted to do two things one is find the value of k which you see here and we also wanted to find the probability that $h + t$ was less than a certain number. So now we are going to look at this problem and see if we can use this same joint density function to do a transformation of variables.

And let us just define two variables V and A as you see on the screen they have units of velocity and acceleration but they are by no means those quantities the question is that what is the joint density of V and A if the joint density of h and t is as you see on the screen. And the relation between h and t and V and A are as given. So, we proceed as before and we invert the relation first and then obtain the Jacobean of a transformation.

So, we can do the calculus and the Jacobean is V squared divided by A cubed. Now because the

way we have defined the Jacobean f of VA is equal to the Jacobean times the f of h t and if you do the algebra then we get the functional form as you see on the screen. We have to be careful about the limits unlike in the h and t space they are not fixed but they are functions of each other. So, if we can present the region of the ht space and the corresponding VA space where the density functions are non-zero.

You can see that the rectangle in the ht space transform to something a little more interesting looking in the VA space. Now the joint density function because of the transformation also looks very different. So, in ht space it was just a plane an inclined plane but when we transform and go to the VA space it actually looks substantially different and that is what you see on the on the screen and we could also from the joint density function just to see what things look like.

We can find out what the marginal densities of h and t are you can just integrate the joint density and get these and they are basically decreasing linear functions and that is what we get. These plots are obtained by simulation and we will look at that when we take our Monte Carlo simulations maybe we can come back to this problem. And we can also find the joint density function from the joint density function of V and A we can find the marginal's the marginal densities of V and A and that's what we get. So, h and t those linear densities are transform to something which look very different for V and A .