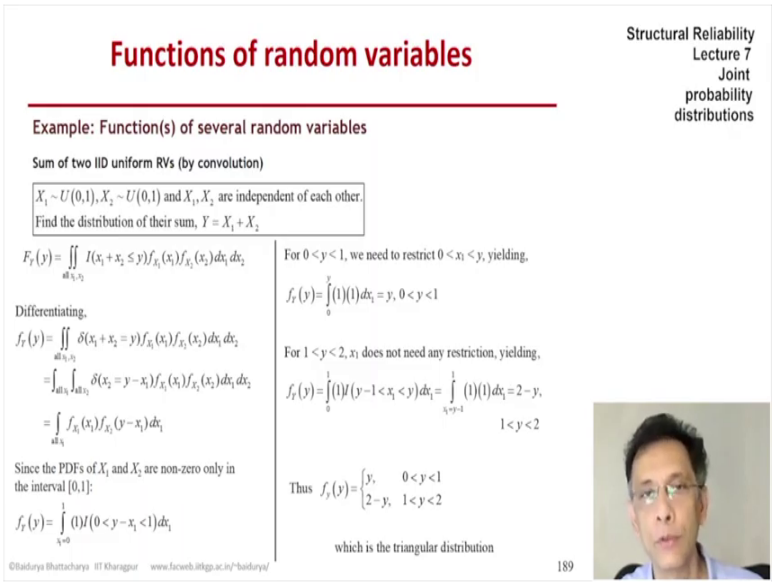
Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –62
Joint Probability Distributions (Part - 13)

(Refer Slide Time: 00:27)



In the second set of problems we look at the sum of two IID uniform random variables. We will solve this problem in two methods the first one is by convolution which is what you see on this slide and in the next one we are going to solve by transformational variables. So, X 1 and X 2 are two uniform random variables independent and identically distributed between 0 and 1. So, we want to find the distribution of their sum.

So, let us start with the basic definition of the CDF of Y which is the sum of X 1 and X 2 and we just write it as the double integration over X1 and X2 of the region in which the sum is less than little y for which we have used the indicator function as you see. So, now we can differentiate this to get the PDF of Y. Let us do that and when we differentiate the indicator function we get the delta function.

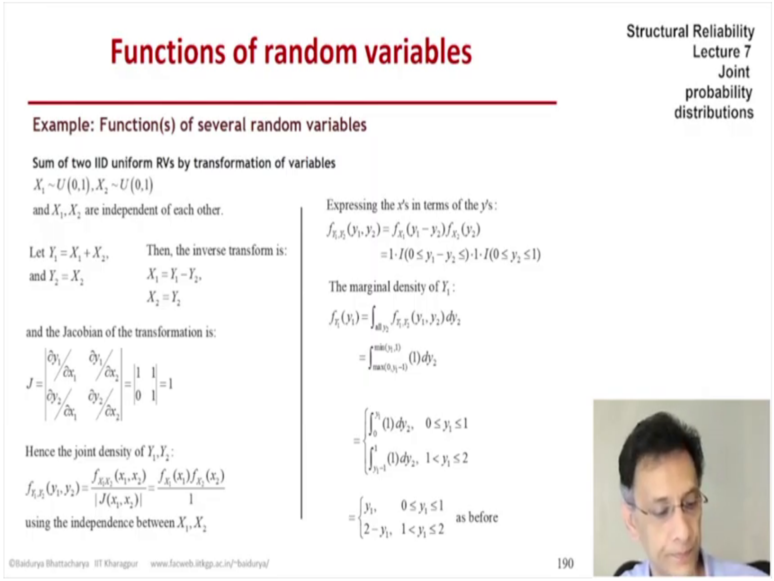
And if we write it out in terms of X 2 then we can get rid of the integration with respect to X 2 and end up with a one dimensional integration involving X 1 only y is a parameter that is not dependent on X 1 uh. So, now we make use of the fact that X 1 and X 2 are uniform and f x 1 and f x 2 are defined only in the interval 0 and 1. So, once we use that fact we can simplify the density of y in terms of just one indicator function.

So, we are now integrating over all values of x 1 but making sure that y - x 1 is positive on one end and less than one on the other ah. So, now it is inconvenient to split this integration for 2 sets of values of y. So, one in which y is between 0 and 1 and the other in which the y is between 1 and 2. So, in the first range the density of y simplifies to y itself that is between 0 and 1. And in the second range between 1 and 2 the density integrates to 2 - y.

So, that is between 1 and 2. So, putting these two things together it is clear that the sum of two independent standard uniforms is the triangular random variable. So, it is its density function is a triangle symmetric about the point one and in fact this is kind of a precursor to what we will see later when we add several such uniforms. So, if we added a few more uniforms it is going to look more and more like the normal density function and just by adding two of them you kind of start getting the shape.

Now we will solve the same problem by transformation of variables which we looked at a couple slides ago.

(Refer Slide Time: 04:42)



So, it is the same problem but now we have X 1 and X 2 on one side and just y on the other side. So, we need to introduce one sort of dummy variable as we discussed. So, to make the problem 2 by 2. So, let Y 1 be X 1 + X 2 and then we define a new variable Y 2 which is identical to X 2. And now we can we need to invert that that relation also. So, X 1 in terms of Y 1 and Y 2 and X 2 in terms of Y 1 and Y 2 in this case only X 2 is equal to Y 2.

So, this lets us write the Jacobean and it is a constant equal to 1 and that lets us describe the joint density of Y 1 and Y 2 in terms of the joint density of X 1 and X 2 and we conveniently make use of the fact that X 1 and X 2 are independent. So, the joint density is the product of the marginal densities. Once we do that we should express the X 1 and the X 2 in terms of Y 1 and Y 2. So, that gives us the joint density of Y 1 and Y 2 in terms of the product of two indicator functions one involving Y 1 – Y 2 and the other involving just Y 2.

So, the marginal density of Y 1 which is what we need we can get it simply by integrating out Y 2 from the joint density function. So, now that is that is what we do and now you know we just need to make sure that the indicator functions are properly accounted for. So, the limits of the integration can be found to be as I have written on the on the screen. So, the lower limit is maximum of 0 and y 1 - 1 and the upper limit is minimum of y 1 and 1.

And if you complete the integration then you get again two ranges of y 1 1 the first one is 0 to 1 and the second 1 is 1 to 2 and you get the same triangular density function as before.