Structural Reliability
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Lecture –52
Joint Probability Distributions (Part - 03)

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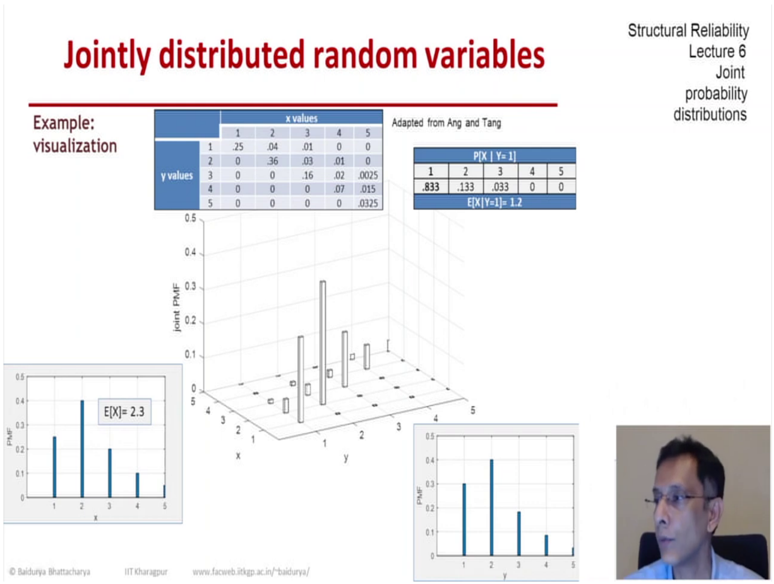


Let us look at a few bivariate cases in the next three slides which would help us visualize what is going on. So, this example is taken from Ang and Tang's book we have the joint PMF of two discrete random variables x and y. So, as you can see there are 5 possible values of x and 5 possible values of y and if you plot these on the plane with x and y being that the 2 axis and the third dimension being the probability mass then this is what it looks like.

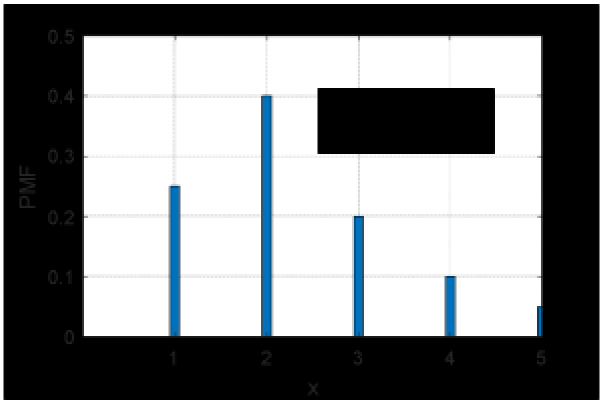
		x values				
		1	2	3	4	5
y values	1	.25	.04	.01	0	0
	2	0	.36	.03	.01	0
	3	0	0	.16	.02	.0025
	4	0	0	0	.07	.015
	5	0	0	0	0	.0325

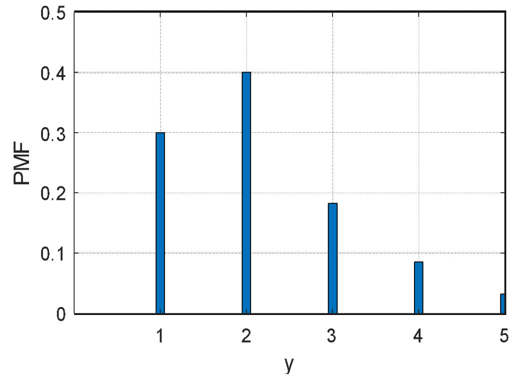
So, the height is proportional to the probability and we can clearly see that a few of these points in the grid have no probability mass they are 0. For example x 1 and y 4 has no probability mass and the highest mass is at 2, 2 which is 0.36. Now we could obtain the marginal mass functions for x and y. So, in this table if you add all the columns you would get the marginal mass functions for x and if you add all the rows.

P[X | Y= 1]				
1	2	3	4	5
.833	.133	.033	0	0
E[X|Y=1]= 1.2				

So, if you add all the elements in a given row and write on the margin that's how the name margin comes actually you get the marginal mass function for y. So, let us do that this on the left we obtain the probability mass function which is basically just the discrete probabilities of x. So, and just make sure that they add up to one if you like you can also find the mean value of x the expected value which in this case is 2.3 we could do the same thing for y and obtain the marginal mass functions for the random variable y we could go one step further and try to just implement the conditional mass functions that we just looked at.

So, let us say we fix the value of y at one. So, if you would like to look at the grid on that three-dimensional figure we are looking at the line where y is one and we are looking at all the possibilities of x only along that line. So, now we need to normalize that is how the conditional mass function of x given y is 1 would become a legitimate mass function. So, what you see in the table on the top right corner is the conditional mass function of x given that particular value of y and if you look at the table which we can easily guess from the grid also is that there is no mass for x equals 4 or 5 because that is what the problem states.





So, given y equals 1 the entire mass is constrained for 1, 2 and 3 and after normalization those three numbers must add up to one as they do. We could also find out the conditional mean the conditional expectation which we are going to define formally in a few slides from now but it is easy to do that just multiply the value of x with its conditional probability. So, one times 0.833 + 2 times 0.133 and so, on. So, that conditional mean of x given y is 1 is 1.2 we can compare that with the unconditional mean of 2.3.