Structural Reliability
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Lecture –48
Common Probability Distributions (Part - 19)

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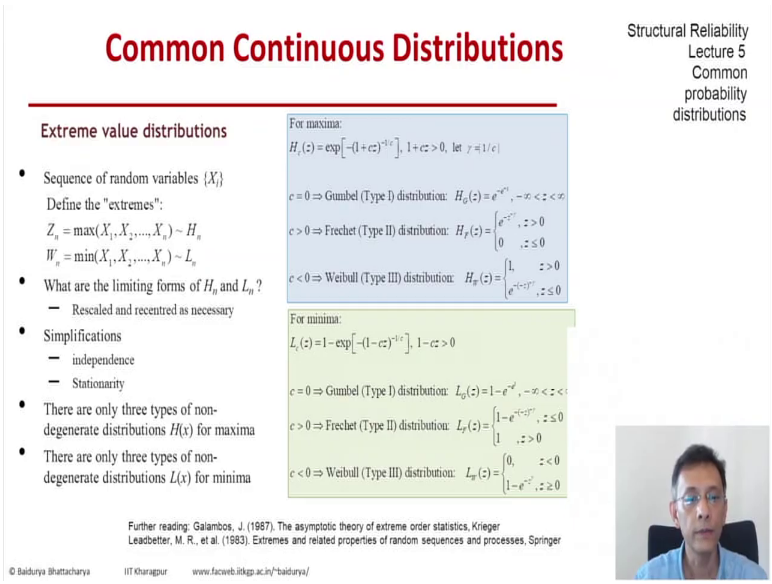

We end this discussion on common continuous random variables in structural reliability with extreme value distributions. So, this is the problem statement we have a sequence of random variables X1 up to Xn and we are interested in how the maximum or the minimum of the sequence are distributed. So, Zn is the maximum of the axis and it is distributed as H which is a function of n the number of elements and likewise W n the minimum is distributed according to L, Ln.

Now we do understand that as n goes to infinity these distributions H and L will become degenerate basically step functions. So, the question is that if we rescale and re-center them properly what are the limiting forms of H and L and it turns out that under some simplifications namely that the X's are IID we have the classical result. That there are only 3 types of possible distributions of maxima and likewise there are only 3 types of possible distributions for minima.

And in many texts you will find that the treatment is only with either maxima or minima that is simply because the minimum of a set of X's is nothing but the negative of the maximum of the negative X's. So, the books often just focus on one uh. So, here is the final result for maxima the limiting distributions are here which are parameterized by c. So, if c is 0 we have the gumball type which is type 1 for c positive we have the fresh a or type 2 and for c negative we have viable or type 3 and a similar set of expressions for minima.

Now just one point to note is that when I say two distributions are of the same type. So, if distributions f and g are the same type it means that f of x is equal to g of a x + b where a and b are constants. And these two texts are excellent for further reading the book by Galambus and the book by Leadbetter and others.

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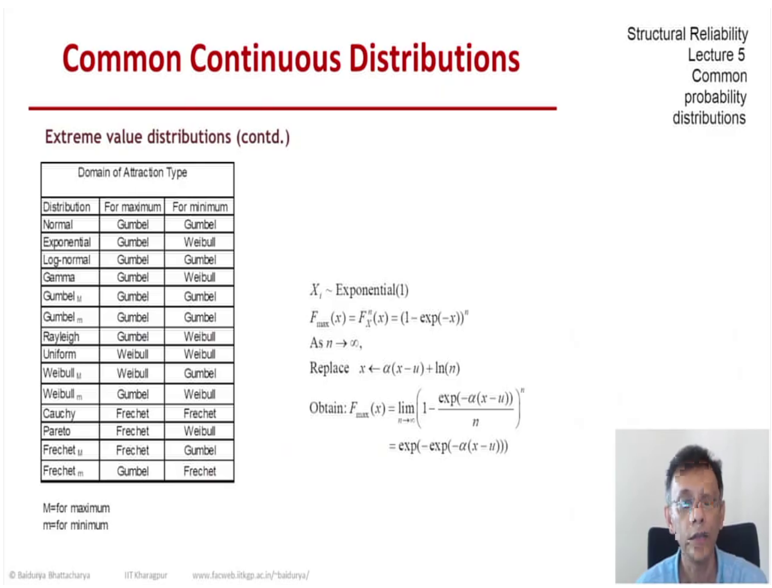

Now one thing to remember is that parent distributions which are basically distributions of the X’s the IID of X’s would give rise to only 1 of the 3 possible types for maximum and minimum for example if the parent distribution is normal under the IID assumption the asymptotic distribution for the maximum will invariably be Gumbel if and likewise for exponential and so, on.

So this table gives the parent distribution on the left column and the asymptotic distribution for maxima and minima in the second and third column for common distribution that we see. So, just let us go through the exponential case for maxima and let us see what this means. So, if I have say exponential with mean one as the parent distribution the distribution of the maximum is given in terms of what you see on the screen in terms of n and as n goes to infinity we locate this.

So, we replace x by alpha times x - u two constants and log of n so, that is how we rescale and if we now plug it in the maximum the asymptotic distribution of the maximum becomes the double exponential form which we identify to be the Gumbel distribution. So, this is one application of what we said in the previous slide.