

Structural Reliability
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Lecture –43
Common Probability Distributions (Part - 14)


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Structural Reliability
Lecture 5
Common
probability
distributions

Common Continuous Distributions

Return period revisited

- Sequence of independent and identical exponential random variables ("trials"):
 - Occurrence rate = λ (per unit time)
 - Mean occurrence time = $1/\lambda$
 - "Mean Return Period" = mean of these IID exponential RVs = $1/\lambda$
- If a random variable X is associated with each occurrence
 - CDF F_X is known
 - Occurrence (or success) = $\{X_i > x_p\}$ in i^{th} trial
 - $p = P(\text{success}) = P\{X_i > x_p\} = 1 - F_X(x_p)$
 - "Mean Return Period" = mean of these IID exponential RVs = $1/(p\lambda)$



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Our next example involving the exponential takes us back to the return period which we saw in the case of the geometric distribution. So, instead of a sequence of independent and identical geometric random variables we have a sequence of IID exponentials. And the trials are not occurring at fixed instance of time equally separated but random instance of time on the time axis with an occurrence rate of lambda and the mean occurrence time is 1 over lambda.

So, the mean return period in this case would be instead of the mean of the IID geometric random variables the mean of IID exponential random variables which is as we saw one over lambda. So, that is the mean return period. Now as we had in the case of the geometric distribution those Bernoulli trials we could associate with each point uh a random variable x . So, it would be a marked process and if that random variable exceeded certain threshold x_p then only I would have the event that I am worrying about.

So, p small p is the probability of success with that occurrence and that would be 1 minus the CDF of x evaluated at $x = p$ and the mean return period because of this marked process I would have as 1 over p lambda not 1 over lambda let us. Now look at one example involving the person process and the return period.

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Common Continuous Distributions

Return period revisited (example)

What is the mean return period of the earthquake that has a 10% probability of exceedance in 50 years?

Solution: Earthquakes occur according to a Poisson process with rate λ . We need to find λ such that:

$$P[N(50 \text{ yr}) \geq 1] = 0.10$$

Hence,

$$1 - P[N(50 \text{ yr}) = 0] = 0.10$$

$$\text{or, } 1 - \exp(-50 \text{ yr } \lambda) = 0.10$$

$$\text{or, } \lambda = 2.1072 \times 10^{-3} / \text{yr}$$

Thus,

$$\bar{T} = 1 / \lambda = 1 / 2.1072 \times 10^{-3} / \text{yr} = 474.6 \text{ yr}$$

Ans: approximately 475 years.


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Compare with Bernoulli trial based approach:

$$(1 - p)^{50} = 0.10$$

$$\Rightarrow p = .00210 / \text{yr}$$

$$\Rightarrow \bar{T} = 1 / p = 475.1 \text{ yr}$$



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So, uh the mean return period of an earthquake that has a 10 percent probability of exceedance in 50 years. So, it is a standard way of describing rare earthquakes and these earthquakes occur according to a Poisson process. So, what we need is to find that lambda because if you can find lambda we would need 1 over lambda is the mean return period. So, that statement is interpreted as P of N 50 greater than or equal to 1 is only 10%.

So, uh that when expressed in terms of the exponential CDF gives me a lambda of about 0.0021 that you see on the screen that is per year and then I uh take the reciprocal of that and my mean occurrence time is roughly 475 years. Now you might be wondering what if we did uh what we have done before. So, what we've we took the geometric distribution the Bernoulli trial approach. And so, let us do that and it would be something similar.

So, $1 - P$ whole to the power of 50 now so, we this is the probability of non-occurrence every year for 50 years. So, that is 0.1. So, if you solve for P that comes to a 0.0021 and the reciprocal

of that would be also 475 years. So, with a slight difference instead of 474.6 you have 475.1. So, these two approaches would basically give you almost uh identical answers.