Structural Reliability
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Lecture –40
Common Probability Distributions (Part - 11)

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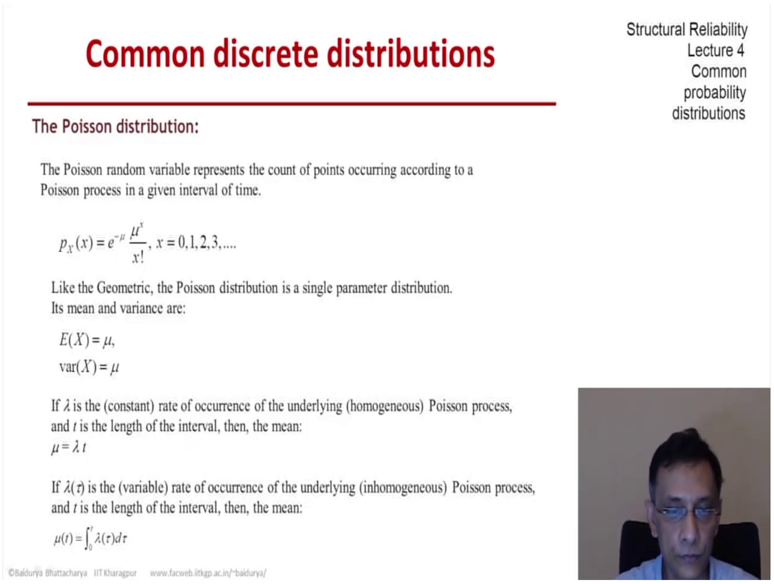

Our last topic in the discussion on common discrete distributions is the Poisson distribution the partial random variable represents the number of points the count of points that occur according to a Poisson process in a given interval of time. So, its PMF is defined as you see in terms of the integers x going from 0 to infinity there is just one parameter there just like the geometric distribution that parameter is mu.

You can expand the sum for the expectation and the variance and you can derive that the PMF that you see on your screen yields the mean of mu and the variance also mu. Now there is an intimate relation between the Poisson random variable and the person process which we are going to see later. But I just wanted to bring this fact out that if lambda is the constant occurrence rate in a partisan process which would be a homogeneous Poisson process and we are counting over an interval 0 to t.

So, the mean mu which we already discussed that would be lambda times t it is often more natural to express mean from this point of view that it is rate times the length of the interval. Now it does not have to be time it can be any index set in fact we are going to look at an example in which the index is length it is along some linear dimension it is not necessarily time. Now if the process if the underlying partisan process is not homogeneous.

If it is inhomogeneous we can still use the Poisson random variable representing the total number of counts in that interval provided we know what the variable rate is lambda being now a function of tau and integrate it and that would give me the mean of the Poisson random variable and I can proceed as before by plugging in the value of mu in the Poisson PMF which you see the first equation on your screen.

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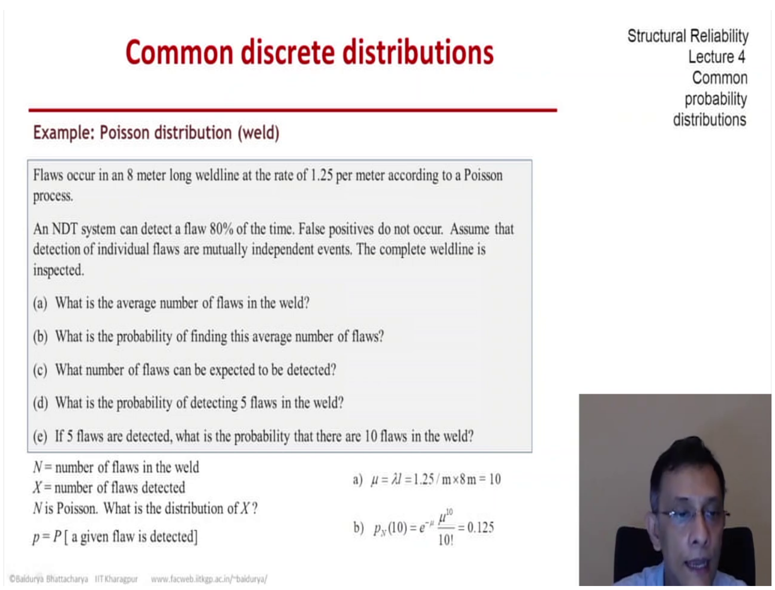

Now let us solve a couple of sets of examples the first one is a direct application of the person PMF of an earthquake process occurring on time with a constant rate that rate is 0.025 per year. So, we need to answer the probability of no earthquake occurring in certain duration and two or more articles occurring in duration. So, this is my person PMF we need to find mu, mu depends on time on the length of the interval.

So, in this case it is 1.25 and then plugging in the value of 0 for that particular PMF we get a

28.65% chance of no earthquake occurring in 50 years of that intensity. In part b the mu is different because my rate is the same but the interval is now longer. So, mu is 2.5 and we are required to find two or more earthquakes. So, we look at the complementary event. So, it is one minus p of no earthquake and p of one earthquake.

So, if you do the math the answer is about 71% in our next example we as I said it does not have to be on the time axis it can be on some length axis and here we look at a problem of weld defects along a line of wet. So, let us take a minute to read the problem and then we will start solving one by one. So, let us define capital N as the number of flaws in the world and it is given that capital N is Poisson distributed but we also have another issue here that is there these laws may or may not be detected.

So, not only do we need to define the number of flaws actually present in the world it would be good if we define another random variable x which is the number of flaws that are detected and it will be very helpful if we proceed in this manner. So, N is Poisson as it is given. So, what is the distribution of capital x for that we would need to have this little p it is defined as 0.8 is the probability that a given flaw is detected.

So, we will need to come back to this but for parts a and b we just need to apply the Poisson PMF without any regard to x is just to do with N. So, mu is lambda times l which gives me 10 and the average number with that average number I can find out part b which is the probability of finding 10. So, that would be just applying the same PMF formula and get 12.5 percent probability there.

So, now let us look at the more involved questions is what number of flaws can I expect to be detected and then detecting five exactly five flaws and if five flaws are defect detected. So, it is kind of a Bayesian turnaround is that there are actually ten flaws in the world.

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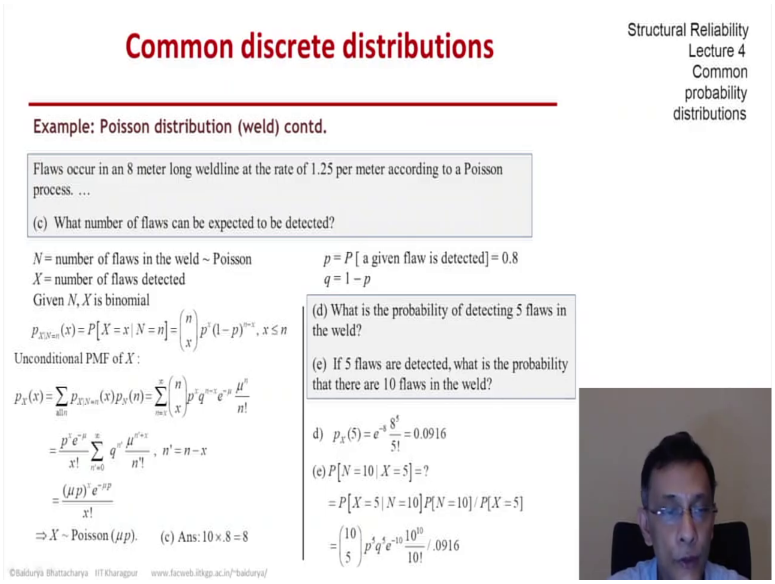

So, let us tackle them one by one in the next slide we know that N is Poisson we know that x is number of flaws detected we have defined the parameter little p whose value is 0.8. Now we need to find what the distribution of x is. Now it is important to recognize that if I fix the value of N if the number of flaws are known then the number of detected flaws would be binomial random variable because I have a fixed number of trials and each of them has a certain probability of success and they are mutually dependent.

So, that is clearly a binomial situation and hence i can write the conditional PMF of x conditional because I have fixed the value of the random N and I can write it out as the the usual binomial PMF. Now I need the unconditional PMF of x. So, for that I employ the theorem of total probability just making sure as we did in one other example earlier today is that the the sum starts not from 0 but starts from x.

And once I do appropriate substitutions and everything I actually come up with an expression that is very familiar that is nothing but a Poisson PMF but with a reduced mean. So, capital N was a personal random variable with mean mu but because I am effectively filtering that set with the probability of p for each of them. So, my mean effectively becomes mu times p and I get a new person random variable x.

So, that is extremely useful to know and with that knowledge we can now solve parts d and e for c the answer is mu p. So, that is 8. So, there are on an average 10 flaws in the world but the average number of detected flaws would be 8. Now let us solve d and e. So, the probability of detecting five flaws in the well would be simply the Poisson PMF for x and that is about 9.16% just make sure that you know use the right mean. So, we use the mean of eight for this particular question.

Question e is clearly a conditional probability. So, we are going to use the Bayesian formula. So, we need p of n equal to 10 given that x is equal to 5. So, let us just plug in the formulas we need p of x equal to 5 given n is 10 and need the prior PMF of n at 10 as well we have already derived p of x equals 5 which is in part d. So, plugging in all those numbers the first conditional PMF of x given n is as you remember a binomial PMF that multiplied by a person PMF for N and divided by p of x x equal 5 which also happens to be a Poisson planer the final answer is 0.0361.