Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –27
Review of Random Variables (Part - 10)

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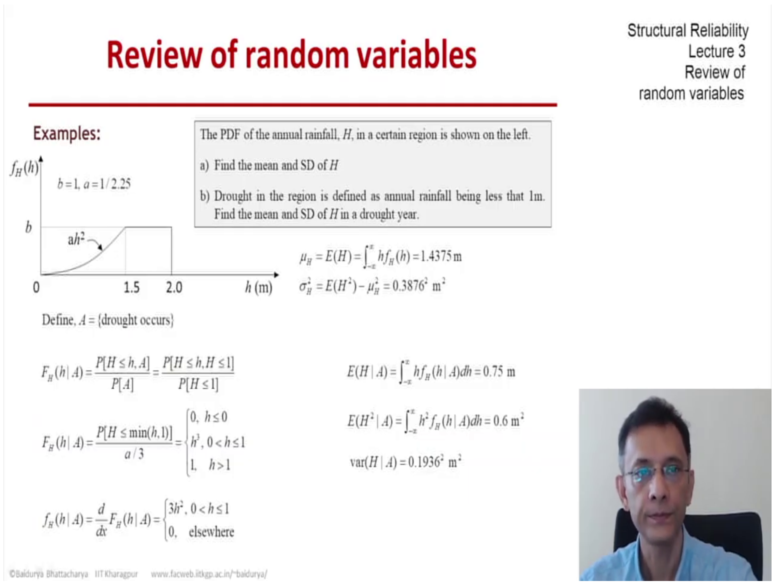

As our second example on conditional distributions and conditional expectations let us go back to an example we saw earlier in this lecture the rainfall distribution. So, h is the annual rainfall and it has a certain density function as you see on the screen we first would like to find out what its mean and standard deviation are and. Now let us put a conditional event. So, let us define drought and drought is defined in terms of h that the random variable h is less than 1 meter.

So, with this conditioning event I am going to find out what is the mean and standard deviation of h in a drought yeah. So, to do that let us first find the unconditional mean as as asked for and the unconditional standard deviation. So, if you just do the basic integration if my calculations are correct then the unconditional mean is about 1.4 meter and the standard deviation is 0.3876 meter. Now let us bring in the condition.

So, let a be the event that drought occurs and is defined as h is less than 1. So, the conditional distribution function is P of in the numerator we have P of capital H less than equal to h and A divided by P of A and now we bring in what is the event A itself. So, that is h is less than or equal to 1 because h is a constant variable the probability of h less than 1 and h is less than or equal to 1 are the same and so, if we now expand that we get the distribution function defined in 3 ranges as you see it cannot have negative values as before.

So, it is 0 there but greater than 1 it also cannot have any values. So, conditioned on the drought the CDF attains the value of 1 at h equal to 1. Now if we differentiate this CDF this conditional CDF we get the conditional density. So, the density of h given a is we have non-zero values only in the range 0 to 1 and 0 elsewhere whether it is greater than 1 or less than zero there is no density and we need to make sure that this density still integrates to 1.

So, we can see that it does. So, that is good. And now as requested in part b let us find out the conditional mean and the conditional standard deviation. So, the first would be just to integrate the conditional density with respect to h and the number comes to 0.75 meter which is much less than the unconditional value of 1.44. And if you now find the conditional expectation of h squared you get 0.6 and if you subtract the mean squared from that you get a standard deviation of 0.1936 which is also less than what it was at 0.38 in the unconditional case.

So, that would be another example looking at the effect of conditioning events on densities and expectations of random variables.