

Structural Reliability
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Lecture –25
Review of Random Variables (Part - 08)

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Review of random variables

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Examples:

Consider a jackup platform used for oil production in Bombay High. Its initial cost (construction, installation etc.), C_i (in Rs crore) is proportional to the amount of steel used (x in tons) besides having a fixed component: $C_i = 10 + 0.05x$

The life-time failure probability of the platform depends on the amount of steel: $P_f = \exp[-(x/10000)^3]$.

If the platform fails (i.e. topples or collapses), massive pollution will result in the Arabian Sea. The cost of cleanup and penalty in case of failure will be Rs 10000 crore.

Find the optimal solution by minimizing the expected total cost.

$$C_T = C_i + C_f$$

$$E[C_T] = C_i + E[C_f] \quad (C_i \text{ is non-random})$$

$$E[C_T] = 10 + 0.05x + 10^4 P_f + 0(1 - P_f)$$

$$= 10 + 0.05x + 10^4 \exp[-(x/10000)^3]$$

For optimum: $\frac{d}{dx} E[C_T] = 0$

$$\Rightarrow 0.05 - 3 \exp[-(x/10000)^3] (x/10000)^2 = 0$$

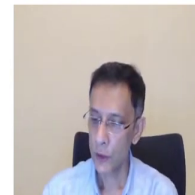
$$\Rightarrow x^* = 17316 \text{ (tons)} \rightarrow \frac{d^2}{dx^2} E[C_T] > 0 \rightarrow \text{minimum}$$

$$\Rightarrow C_i^* = 876 \text{ (Rs cr.)}$$

$$\Rightarrow P_f^* = 0.0056$$

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Suppose the Coast Guard requires the life-time failure probability to be no more than 0.0001. Does this change your answer? If so, what is the new optimum?



The last problem we will look at in this series has to do with an optimization problem in one variable where we optimize the total cost. So, let us read the problem. So, as you see there are two costs the initial cost the construction cost which is constant it is not random and a failure cost. This failure cost is actually a random variable it is a binary random variable it can have two possible values if everything goes all right there is no failure cost.

But if there is failure then it is quite a huge cost 10 to the power 4 crore rupees. So, when we want to find the expected value of the total cost C I is a constant. So, it remains as such but the expected value of the failure cost. Now looks at the 2 possibilities so, it is 10 to the power 4 times P_f and 0 times $1 - P_f$. So, if you put in the values of C I and P_f in terms of the resource x then we have an objective function which we can minimize using any standard method.

And if you do it right then you should get the answer of about 17300 tons would be the optimal

resource to spend and the corresponding value of the initial cost would be about 875 crore rupees and the optimal failure probability for using that amount of resource would be 0.0056. So, that is a nice optimization problem that can be solved but here is an interesting question is that what if this cost-based optimization answer a solution is not acceptable to the regulators.

The regulators might have other considerations for setting a value of in this example say 0.0001 that comes from considerations other than minimizing the total cost on behalf of the owner and. So, your original answer of 0.0056 is too high that amount of failure probability is not acceptable. So, in that case we have to revise the answer. So, that would give us an x star of almost 21000 tons. So, going up from 17300 is 21000 tons and obviously it will cost somewhat more instead of 876 it would cost almost 200 crore rupees more.

So, about 1058 crore rupees so, that would be the revised answer. Now I want to also bring out one interesting feature which is present in such problems and for that we need to look at the look at the graph of the costs.

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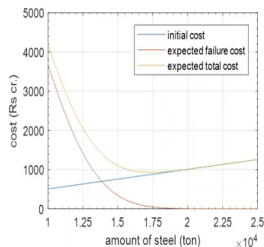
Examples:

Consider a jackup platform used for oil production in Bombay High. Its initial cost (construction, installation etc.), C_i (in Rs crore) is proportional to the amount of steel used (x in tons) besides having a fixed component: $C_i = 10 + 0.01x$.

The life-time failure probability of the platform depends on the amount of steel: $P_f = \exp[-(x/10000)^2]$.

If the platform fails (i.e. topples or collapses), massive pollution will result in the Arabian Sea. The cost of cleanup and penalty in case of failure will be Rs 10000 crore.

Find the optimal solution by minimizing the expected total cost.



Suppose the Coast Guard requires the life-time failure probability to be no more than 0.0001. Does this change your answer? If so, what is the new optimum?

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So what you see on the x-axis is the resource that we have x the steel in ton and then we have three graphs the blue graph is the linear construction cost the orange curve is the expected failure cost. So, that is $P_f \times C_f$ and P_f decreases with increasing x and the yellow curve which is

the sum of the two the sum of the blue and the orange. So, that is the total expected cost, now the optimization that we did in the first which gave us about 17300 tons that would be the lowest value of the yellow curve.

Now in such problems you will see that the cost is not symmetric. So, if you move to the left of the optimal point of that optimal point of 17300 there is a much steeper rise than if you moved to the right of the optimal point it is a much more gentle rise. So, there is a case for here to hear on the side of caution. So, if you have to you know if there are uncertainties if some error has been made inadvertently then it is probably much safer to spend a little more to employ a little more resource because the consequence is very unsymmetric.