

Structural Reliability
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Lecture –24
Review of Random Variables (Part - 07)

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 Lecture 3
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Examples:

The manufacture of fire safety device tests every device before shipment. The non-destructive test has the following characteristics: (i) the probability that the test will identify the device as defective when it is actually defective is 99%, and (ii) the test will declare the device as OK when it is actually OK is 90%. From destructive tests, the company knows that 1% of the devices have some kind of defect in them. It costs \$500 to manufacture a device. The company ships out only those units that have passed the test. The device sells for \$1000. However, when a defective device is sold, the eventual loss including penalties amounts to \$1,000,000.
 What is the expected profit per device?

$D = \{\text{item is defective}\}, T = \{\text{test indicates defect}\}$
 Partition = $\{DT, \bar{D}T, D\bar{T}, \bar{D}\bar{T}\}$
 Given, $P(T|D) = 0.99, P(\bar{T}|\bar{D}) = 0.9, P(D) = 0.01$

Case	Probability	Sold?	Income, I
DT	$.99 \times .01 = .0099$	No	0
$\bar{D}T$	$(1-.9)(1-.01) = .099$	No	0
$D\bar{T}$	$(1-.99) \times .01 = .0001$	Yes	$1000 - 10^6 = -0.999 \times 10^6$
$\bar{D}\bar{T}$	$.9(1-.01) = .891$	Yes	1000

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$$\begin{aligned} \text{Profit} &= -c_0 + I \\ E[\text{Profit}] &= -c_0 + E[I] \\ &= -500 + 0 \times .0099 + 0 \times .099 \\ &\quad - 0.999 \times 10^6 \times .0001 + 1000 \times .891 \\ &= -500 + 791.10 \\ &= 291.10 \end{aligned}$$



We have looked at an example involving diagnostic tests in the previous lecture today we are going to build on that and bring in cost and profit. So, let us take a minute to read the problem. So, let us proceed step by step the first thing would be to define the events carefully as we always do. So, let D be the event that the item is defective and T is the test indicating the item is defective. So, we are not using $D+$ and $D-$ and $T+$ and $T-$ as we did last time.

We will use more standard notations D and \bar{D} and T and \bar{T} . So, that gives us the partition of the sample space in terms of $DT, \bar{D}T, D\bar{T}$ and $\bar{D}\bar{T}$. Now for these 4 cases we can find out using the information given as before the sensitivity and the specificity information we can find out the probabilities of these four events in the partition and if you do the algebra carefully the numbers are as you see here.

So, I am using basically some form of P of AB is P of A given B times P of B . So, if you do it

carefully you would arrive at the 4 probabilities and one good way to check that your answer is correct is add these 4 up and you should get the value of 1. Now this is not the entire problem the company takes decisions based on these test results. So, let us look at those now. So, if the test is positive. So, it does not matter whether the item is defective or not because the company only knows the test results.

So, if T is there then the item does not get sold it does not get shipped out. So, that is true whether it is DT or D bar t. So, the first 2 cases the item is not sold and the income is 0. In the last two cases when the test does not indicate the item is defective actually the test indicates the item is okay then it is shipped out. Now if it is defective then there is problem because the income is the sale price but eventually there will be penalties.

So, there is a lot of negative value there and the ideal case the one that the company always looks for is the D bar T bar and that is the item is sold and no problem one thousand dollars is earned. So, now let us look at the profit angle. So, the profit is $-C_0 + I$. So, $-C_0$, C_0 is the cost to manufacture and I is the income which we listed the 4 possible cases. So, let us interpret the problem as I is a random variable and it has 4 possible values and the 4r probabilities that are listed in the second column.

So, now we want to find the expected profit. So, the expected profit as we know that expectation is a linear operator the constant just remains as it is and the expected value of the income. The expected value of the income is the value times the corresponding probability. So, that would be -500 for the manufacturing cost that is always there and then the 4 possibilities the first 2 being zero income and the third income being negative and the fourth possibility has a positive income. So, if you now do the algebra you get an expected profit of 291 dollars and 10 cents.