

Structural Reliability
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Lecture –22
Review of Random Variables (Part - 05)

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Review of random variables

Structural Reliability
 Lecture 3
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Examples:

A university computer network is subject to X hacking attacks every day with probability: $P(X=x) = c(n+1-x)$, $x = 1, 2, \dots, n$. The probability of outage increases with the number of attacks: $P[F | X=x] = x/n$. Find the probability that the network will work uninterrupted during the day.

$$p_x(x) = c(n+1-x), x = 1, 2, \dots, n$$

Find c :

$$c \sum_{x=1}^n (n+1-x) = 1$$

$$\Rightarrow \frac{1}{c} = n(n+1) - \sum_{x=1}^n x = n(n+1) - n(n+1)/2$$

$$= n(n+1)/2$$

$$\Rightarrow c = \frac{2}{n(n+1)}$$

$F = \{\text{outage occurs}\}$

$$P[F | X=x] = x/n$$

$$P[F] = \sum_{x=1}^n P[F | X=x] P[X=x]$$

$$= c \sum_{x=1}^n \frac{x}{n} (n+1-x)$$

$$= c \frac{n+1}{n} \sum_{x=1}^n x - c \frac{1}{n} \sum_{x=1}^n x^2$$

$$= c \frac{n+1}{n} \frac{n(n+1)}{2} - c \frac{1}{n} \frac{n(n+1)(2n+1)}{6}$$

$$= c \frac{n+1}{2} \left[n+1 - \frac{2n+1}{3} \right] = \frac{1}{3} + \frac{2}{3n}$$

$$P[\bar{F}] = \frac{2}{3} \left(1 - \frac{1}{n} \right)$$

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This example involves attacks on a university computer network. So, let us take a minute to read the problem. So, up to n attacks can occur in a day and the probability decreases with increasing x and there is the constant c which we first need to find out and then we can find the probability that the network will not go down on a given day. So, we are going to use the basic property of PMF's that they all add up to one if you include all the possible values.

So, that is what we are going to do and this gives us the equation for c and to solve this we need to remember the formula of the sum of the first n natural numbers. And if you do the algebra you get the value of c as 2 divided by n times $n + 1$. So, now we need to find the probability of no outage. So, if there are X attacks the probability of failure is proportional to x . So, it is x by n where n as you remember is the maximum number of possible attacks.

So, we are going to use the theorem of total probability once more and that gives me a

probability of F which involves the constant c and a sum involving x and n. So, again if you remember the sum of n the first n natural numbers and the sum of the squares of the first and natural numbers we can solve this. So, the algebra leads us to the P of F as $\frac{1}{3n^3 + 2n^2}$. So, what has been asked for is $P(F^c)$ there will be no outage.

So, if you subtract this from 1 you get the answer it is $\frac{2}{3n^3 + 2n^2}$ times constant which depends on n. So, if you have possibly a large number of attacks then it is going to get close to the value of $\frac{2}{3n^3}$ that there will be no outage on the given day.