

Structural Reliability
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Lecture –19
Review of Random Variables (Part - 02)

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Review of random variables

Structural Reliability
Lecture 3
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Expectation:

The expectation of any function $g(X)$ of the random variable X is defined as:

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx \text{ if } X \text{ continuous}$$
$$= \sum_{x_i} g(x_i) P_X(x_i) \text{ if } X \text{ discrete}$$

The expectation of a constant is the identity operator:

$$E(c) = c \text{ where } c \text{ is a constant}$$

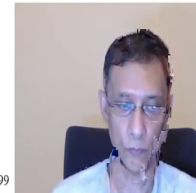
Expectation is a linear operator:

$$E(ag(X) + b) = aE(g(X)) + b$$

and if $Y = g_1(X) + g_2(X) + \dots$, then

$$E(Y) = E(g_1(X)) + E(g_2(X)) + \dots \quad (0.4)$$

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Moments and Expectations: The expectation of a function g of a random variable X is defined as the integration or the sum as you see depending on if X is continuous or discrete. Some properties of expectation the expectation of a constant are the constant itself. The expectation is a linear operator so if a and b are constants the expectation of $a gX + b$ is a of $E gX + b$ and if Y is a sum of several functions of X then the expectation of the sum is the sum of the expectations.

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Review of random variables

Moments:

Thus the mean of X is its expectation:

$$\mu = E(X) = \begin{cases} \int_{-\infty}^{\infty} x f_X(x) dx, & \text{continuous RV} \\ \sum_{all\ x_i} x_i p_X(x_i), & \text{discrete RV} \end{cases}$$

and its variance is the expectation of its squared deviation from the mean:

$$\sigma^2 = E[(X - \mu)^2] = \begin{cases} \int_{-\infty}^{\infty} (x - \mu)^2 f_X(x) dx, & \text{continuous RV} \\ \sum_{all\ x_i} (x_i - \mu)^2 p_X(x_i), & \text{discrete RV} \end{cases}$$



If the function g is the random variable X itself then we have the expected value of X or the average of x or the mean of x . So, this is the simplest moment that we can have for a random variable and we can generalize this we can generalize this by raising it to powers more than one or we take the difference from the mean and then raise it to an integer power. So, the first moment about zero the first raw moment of the random variable X is the mean itself.

The second moment about the mean so, when g is X minus μ whole squared we have the well-known variance of the random variable.

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Review of random variables

Moments (contd.):

The k^{th} central moment of X :

$$\mu_{r,k} = E[(X - \mu)^k]$$

Hence, by definition, $\mu_{r,0} = 1$ and $\mu_{r,1} = 0$. The variance of X is the second central moment:

$$\text{Variance, } \sigma^2 = \mu_{r,2} = \begin{cases} \sum_{i=1}^n p_i (x_i - \mu)^2 & \text{for discrete RV} \\ \int_{-\infty}^{\infty} (x - \mu)^2 f_X(x) dx & \text{for continuous RV} \end{cases}$$

The k^{th} raw moment of X : $\mu_{0,k} = E[X^k]$. Hence, the mean of X is simply the first raw moment:

$$\mu = \mu_{0,1} = \begin{cases} \sum_{i=1}^n p_i x_i & \text{for discrete RV} \\ \int_{-\infty}^{\infty} x f_X(x) dx & \text{for continuous RV} \end{cases}$$



So, this way we as I said we can define the k th central moment it is $X - \mu$ whole to the power of k and the expectation of that is the k th central moment the first central moment is 0 the second central moment as I said is the variance likewise you can define the k th raw moment. So, E of X to the power of k and the first as I said the first raw moment is the mean itself.

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Moments (contd.):

General formula for central moments (by simply expanding the binomial series):

$$\mu_{r,n} = \sum_{k=0}^n \binom{n}{k} \mu_{0,k} (-\mu)^{n-k}$$

Similarly, the general formula for raw moments:

$$\mu_{0,n} = \sum_{k=0}^n \binom{n}{k} \mu_{r,k} (\mu_r)^{n-k}$$

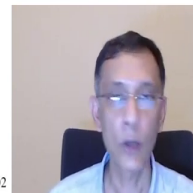
In particular,

$$\mu_{r,3} = \mu_{0,3} - 3\mu_{0,2}\mu_r + 2\mu_r^3$$

$$\mu_{0,3} = \mu_r^3 + 3\mu_r\sigma^2 + \mu_{0,3}, \text{ etc.}$$

If n is an integer, the binomial series is given by:

$$(x + y)^n = \sum_{i=0}^n \binom{n}{i} x^{n-i} y^i = \sum_{i=0}^n \binom{n}{i} x^i y^{n-i}$$



You can express the central moments in terms of the raw moments and vice versa using the simple binomial series. So, I have the expressions here and particular cases like the third central moment in terms of the raw moments or the third raw moments in terms of central moments and

at the bottom of the screen you see the binomial series of X and Y.