Structural Reliability
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Lecture –17
Review of Probability Theory (Part -09)

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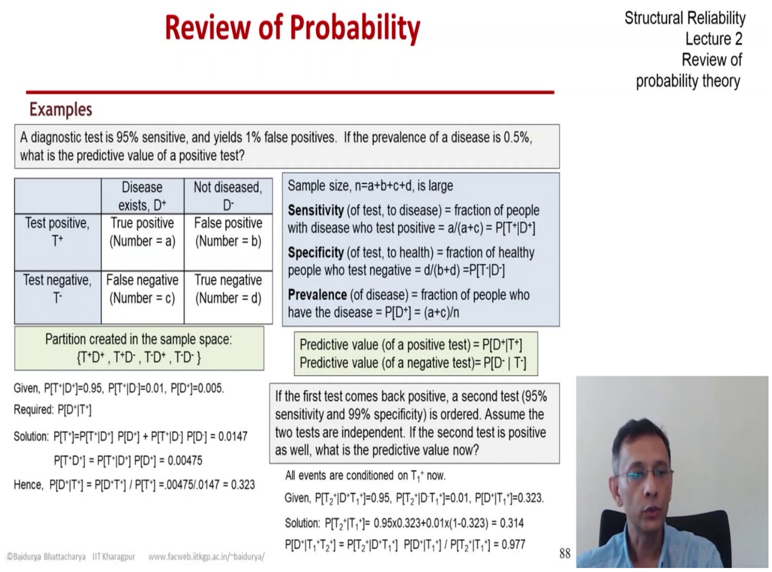

The last example in this series on joint probabilities concerns non-destructive tests although the problem is presented in terms of medical tests the logic would hold for any non-destructive test with binary outcomes. So, let us understand the terms systematically. What we do is we split the sample space into a partition of four sets. So, those are T+ D+ T+ D- T- D+ and T- T-. T+ is the test is positive that T- is test is negative.

Now we could use T and T bar but just to emphasize where I have put the plus and minus notations and likewise disease exists is D+ and this does not exist is D-. So this gives us a 2 by 2 truth table of true positive false positive false negative and true negative So, in terms of these numbers a, b, c and d which sum up to n we can define the sensitivity, the specificity and the prevalence.

So, the sensitivity of the test to disease is the fraction of people with disease who test positive.

So, that's a over a + c and that is the conditional probability of T+ given D+ and likewise the specificity to health of the test is P of T- given D- and the prior knowledge about the situation is the prevalence of the disease So, that is P of D+ and what we aim to do or what these tests aim to do is to update this prior knowledge of P of t

plus with the test outcome So, the predictive value of a positive test is P of D+ given T+ and that of a negative test is P of T- given T-. So, now let us just put numbers from the problem statement. So, P of T+ given D+ is 95% and P of T+ given D- is 1% and the prevalence of the disease is half a percent. So, from this point on we could apply our old friend the theorem of total probability So, P of T+ ah is given in terms of D+ and D- and if you put the numbers in it comes to 0.0147.

Now what we have been asked is P of D+ given T+. So, for that we need the intersection probability and for that we use the definition of the conditional probability multiplied by the marginal. So, that comes to 0.0147 times 0.005 so, that is 0.00475. So, Now we can again use the definition of the conditional probability or in effect we are using Bayes theorem and P of T+ given T+ so, the predictive value is 0.323.

Now this might look small that even if a person or a system comes out as tested positive it is only 32.3% but remember that our prior knowledge had a probability of only 0.005. So, that goes up to something like you know by 65 times to about 32.3%. So, now suppose we go one step ahead and let us say there is a decision that if the first test comes back positive the sample is sent for a second test.

Now this second test has the same properties in terms of sensitivity and specificity but it is independent of the first test. If not then you know we will not gain as much as we could in terms of increased knowledge. So, if the second test is also positive what is the predictive value now. So, now what happens all events are conditioned on T1+. So, the first test came out as positive. The same sensitivity and the same specificity but the prevalence is now 32.3% not half a percent.

And now if you do the math once again the solution comes out to be 97.7%. So, now my predictive values through 2 tests which are independent of each other goes up from half a percent to 32.3% and tend to 97.7%. Now I will just leave you with this thought is that what if the 2 tests instead of being independent of each other were fully dependent. So, I just want you to think about it a few of these numbers would become actually one and then you could find out is what would be the predictive value instead of 97.7% t what we would be left with?