Structural Reliability
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Lecture –14
Review of Probability Theory (Part -06)

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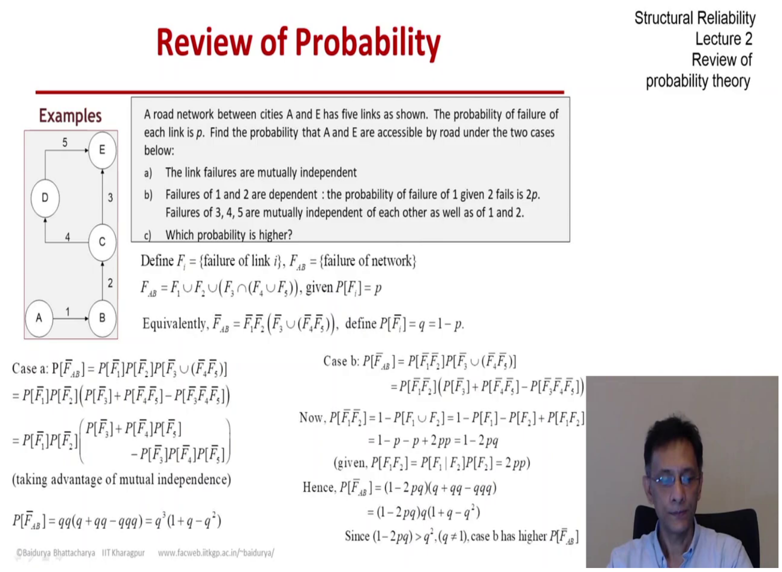

Let us solve some problems involving joint probabilities. The first one we look at is a transportation ne2rk there are 5 links or 5 roads between cities A and E and we want to know the traveler will be able to successfully travel from A to E. There are 2 cases one in which the 5 links are mutually independent and the second case in which there is a degree of dependence between failure of links 1 and 2.

So, for such problems it is a good idea to start by defining the relevant events. So, here we have Fi which is failure of link i going from 1 to 5 and the system failure which is which we call F AB. Now given the system logic as you can see in the diagram we can define F AB in terms of the element failures we can also define the complement the success event F AB bar in terms of the link successes.

So, the roads are open the roads are open to travel. So, let us look at F AB bar for that to happen link 1 must be good the road must be open road 2 must be open and between C and E either 3 must be good or 4 and 5 together must be good or both. So, that gives me the F AB bar expression that you see on your screen. In fact we are going to start we are going to use this F AB bar in solving the problem in both cases because that is a little more convenient.

Expressing the probability of F AB bar we take advantage of the mutual independence. So, the probabilities are multiplied with each other. So, P of F 1 bar is multiplied with P of F 2 bar and P of the bracketed term F 3 bar union F 4 bar intersection F 5 bar. So, let us expand the last term in terms of P of A union B. So, that's PA + PB - P AB and. Now we can use the independence of all the events F 1 bar F 2 bar up to F 5 bar.

Now because we discussed this if link failures are mutually independent link successes are also mutually independent. So, that would let us write the system success in terms of all the individual element success probabilities and the answer for case a is given in terms of q, q is the link success probability which is q cubed times 1 + q - q squared. Now when there is dependence between failures of 1 and 2 we need to be a little careful.

So, we can't write P of F 1 bar F 2 bar as the product of the 2. For the second term on the right hand side we can still do what we did before but for the first term a P of F 1 bar F 2 bar we need to consider the dependence that has been given. So, how do we do that P F 1 bar F 2 bar. So, these 2 events are complementary F 1 bar F 2 bar and F 1 union F 2. So, we can write P F 1 bar F 2 bar in terms of P of F 1 P of F 2 and P of F 1 F 2.

Now F and F 2 they are not independent anymore. So, we cannot multiply the probabilities. So, we have to use the information given. So, P of F 1 F 2 is P of F 1 given F 2 times y of F 2. So, it is 2 P times p. So, that gives me the probability of F 1 bar F 2 bar that both are working as 1 - 2pq and. Now I can go back to what I did before and the success probability is. Now 1 - 2 pq times q + q 2 - q 3.

Now which one is better which one will use the word reliability a little later but which situation is more reliable or which success probability is a little higher. So, we can just compare the 2 answers and because 1 - 2pq is always greater than q 2 unless q is equal to 1 then case B has a higher success probability.