Structural Reliability
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Lecture –12
Review of Probability Theory (Part -04)

Now let us solve some examples involving the counting principle which will invoke the classical definition of probability.

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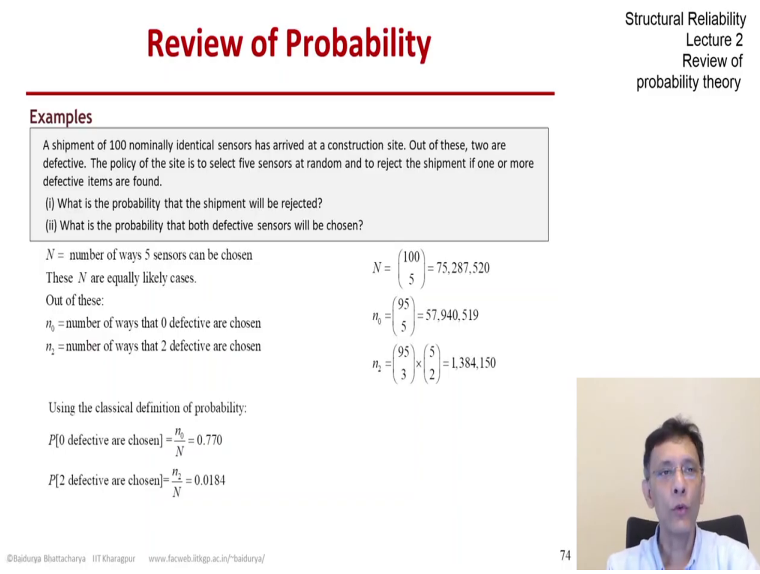

As usual if you would like to solve this problem please pause the video otherwise let me proceed with the solution. So, what we have is a shipment of 100 identical sensors and out of which two are known to be defective. The site policy is to select 5 sensors at random that at random is a key word there. And to reject the shipment if one or more of those 5 selected turn out to be defective. So, the first question is what is the probability that the shipment will be rejected? And the second one is that that both defective sensors will be chosen.

So, the way this problem is set up we see that it lends itself to the classical definition of probability. So, we would use probability as the ratio of the favorable outcomes to the event A divided by the total number of possible outcomes these must be equally likely. So, we define the variables carefully. So, capital N is the number of ways that 5 sensors can be chosen from the

100 and n 0 lowercase n 0 is the number of ways that we choose none of the defective items and n 2 is the number of ways that we choose two defective items.

So, let us count those three numbers as we k. Now that capital N this case would be 100 choose 5 and that is a very large number 75 million plus n 0 is we do not choose any defective so we choose 5 from the non-defective one. So, 95 choose 5 that is about 57 million and then n 2 where we choose 3 non-defective and two defective that can be done. You first choose the three non-defective ones from 95 and for each of them you can select two defective from the five. So, the answer is about 1.3 millions using.

Now the definition of probability in equally likely cases the probability that zero defective census are chosen is about 77% and 2 defective items are chosen is about 1.8%. In lecture 4 we will see that this basically is an application of the hyper-geometric distribution.

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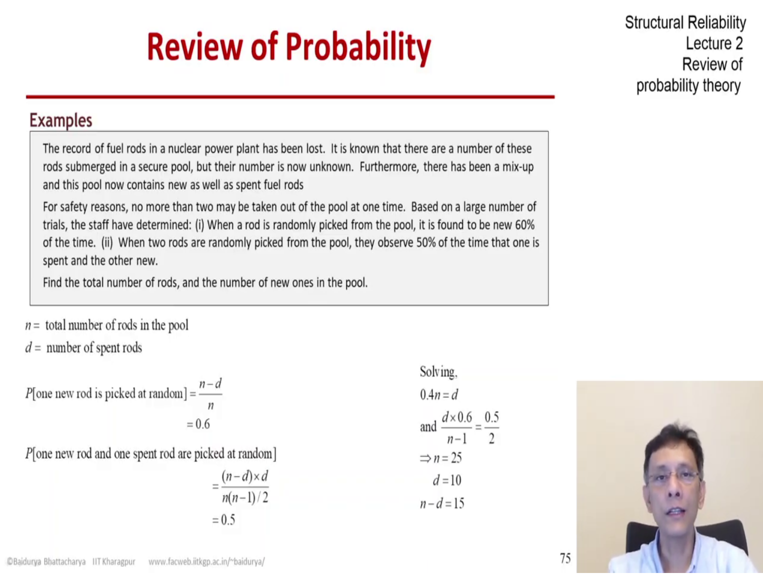

Let us move on let us have another application of the counting principle and the case of equally likely outcomes. So, here we have again a set of marked and unmarked or defective and non-defective or new and spent types of otherwise nominally identical specimens and if you want to pause otherwise let me go ahead with the solution. So, let n be the total number of rods in the pool and d out of them is the number of spent rods.

So the way that we can pick one new rod out of these when we pick n is n minus d over n that is simple and it is given that that is equal to 0.6. Now we can pick one new and one spent rod when they are picked at random is that the denominator is n choose 2. So, that is n times n - 1 divided by 2 and the numerator is n - d choose 1 and d choose 1. So, that ratio is equal to 0.5 and it is a very simple 2 equations in two unknowns.

So, d is equal to 0.4n and the other equation is 0.6d is about 0.25 times n - 1. So, solving them I find that the total number of rods in the pool is 25 and out of them d10 are spent and the new are 15 in number. Let us move on to another problem.

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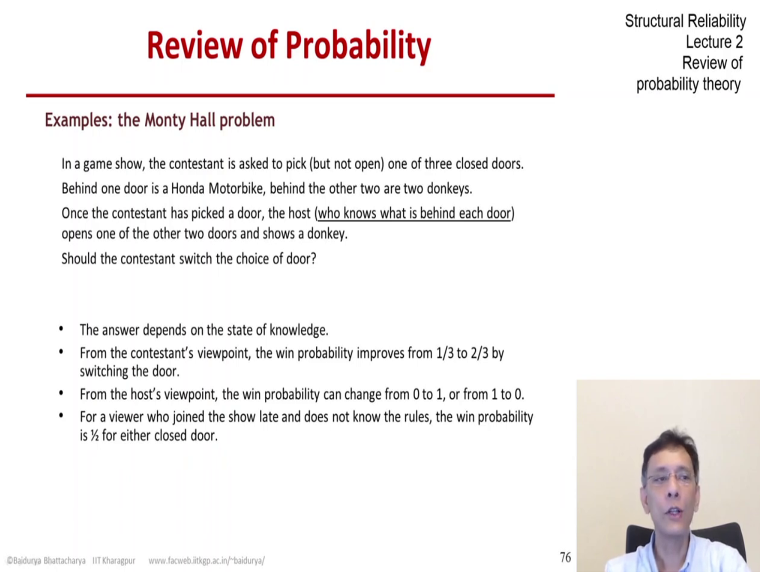

This is a classic the Monty Hall problem and I am sure many of you know this in a game show the contestant is asked to pick a door and get a prize . Now the key here is the host knows the host knows everything. So, once the contestant picks a door the host would open another one but only the undesirable outcome the donkey in this case will be revealed. So, the question is that should the contestant switch his or her choice.

Now again I am sure you know this answer many of you at least the answer is that it is to the contestant's advantage to switch the probability of winning increases from 1 over 3 to 2 over 3.

But what I would like to point out is this answer depends on the state of knowledge. So, we are now invoking the third definition of probability here. So, the contestant who knows the rules and knows that the host will only open a gate a door with the donkey behind it would if you do the math with that piece of knowledge the probability was 1 over 3.

And now it increases to 2 over 3 but from the host's viewpoint the host knows everything. So, the host probability can only change from 0 to 1 or 1 to 0 because the host is operating from his or her vantage point of knowledge. For another person a viewer who joined the show late and does not know the rules for that person the probability does not change from the information that this late viewer has to that person the probability was half and will remain half by changing the door. So, the answer would depend on the state of knowledge.

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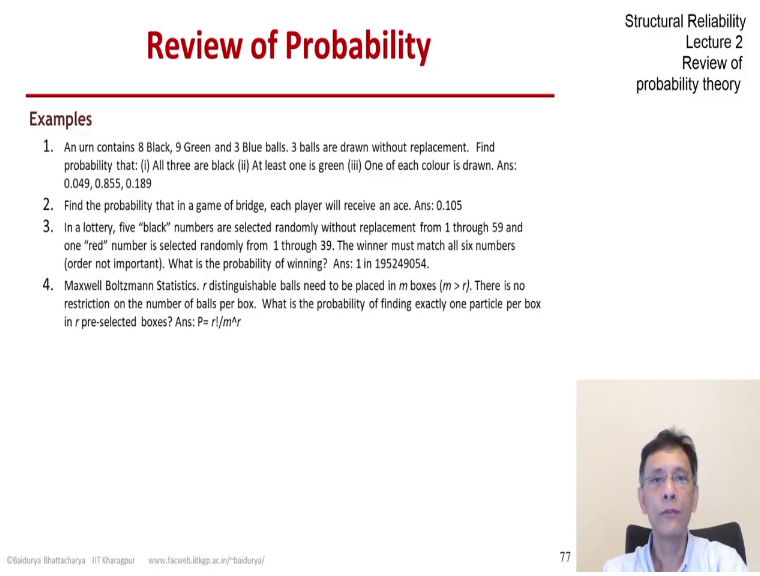

There are many problems that we can solve using this classical definition and I have some examples here I have also listed the answers. So, if you have time please work through them some of these are well-known classical problems but I am sure you will enjoy solving them.