

Structural Reliability
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Lecture –11
Review of Probability Theory (Part -03)

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Review of Probability

Structural Reliability
Lecture 2
Review of
probability theory

- Three approaches to probability
- Axioms of probability
- Counting principles
- Probability of joint events
- Conditional probability
- Independence
- Theorem of total probability
- Bayes' theorem
- Examples

Further reading:

A First Course in Probability by S Ross, Pearson 2019
Probability - Random Variables and Stochastic Processes,
by Papoulis and Pillai, McGraw Hill 2017
A Probability Path by SI Resnick, Birkhauser, 2005



In this review of the basic concepts of probability we would like to cover the 3 approaches to defining the concept the 3 axioms of probability some applications of the counting principles probability of joint events the concepts of conditional probability and statistical independence the theorem of total probability and base theorem and as we will usually do solve examples during the lecture. These three texts will serve as further reading the book by Sheldon Ross, the book by Papoulis later revised by Pillai and once again the book by Sydney Resnik .

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Review of Probability

- We need to:
 - to infer properties/state of a system
 - to predict outcome of a future event
 - to judge the truth of a hypothesis
- Lack of complete certainty
 - in state/properties of the system
 - in outcome of the future event
 - in truth of the hypothesis
- Knowledge of
 - thought experiments
 - repeatability of given experiment, observed data
 - context of problem and similar situations



Now why do we need to define probability? We might find ourselves in a situation as we saw in the first lecture where we need to infer the properties or the state of a system or to predict the outcome of a future event or to judge the truth of a hypothesis or the accuracy of a model. The problem is we lack complete certainty we do not have all information about the quantities we are trying to define.

But what we do have is access to thought experiments or access to actual experiments under nominally identical conditions repeated many times or some understanding of the context of the problem and experience with similar situations.

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Review of Probability

Three approaches to probability

- **Throw of a fair die.** How likely is a six?
 - Other examples: energy states in Maxwell Boltzmann distribution; fair coin toss; cryptology; lottery design;
- **Throw of a loaded die.** How likely is a six?
 - Other examples: pre-disposition to genetic disease; effectiveness of new drug; age-specific mortality; annual maximum wave height; sensitivity & specificity of diagnostic test; psephology
- **This is a fair die.** How likely is it to be true?
 - Other examples: accuracy of financial model/ weather model/ global warming/ finite element model/ seismological model etc.; correct location of oil well; sports betting;



Now let me pose three questions and these would actually give rise to the 3 definitions of probability that we use. The first one is there is a fair die I want to throw it how likely is a 6. Now this represents a vast number of problems and I have listed just a few. The second one is the die is loaded how likely is a six and again this represents a large class of problems and I have listed a few examples. The third question sounds similar but each is subtly different from the others.

So, this one is there is an assertion that this die is fair. Now how likely that assertion to be true is this also represents a wide class of problems I have listed just a few examples.

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Review of Probability

Three approaches to probability

- **Classical:** equally likely outcomes – thought experiment
- **Frequentist:** large number of identical trials – actual experiments
- **Judgmental/Bayesian:** degree of belief – use of experience, association, intuition etc.
- **Issues:**
 - Are these three approaches compatible with each other?
 - Can I mix them to get useful results?



Now in the first example we can take advantage of the fact that all the outcomes are equally likely because the die is fair there are 6 faces. So the probability of getting a 6 is 1 over 6 and this constitutes the classical definition which involves thought experiments but requires as I said the **the** outcomes to be equally likely. In the second one I do not have the advantage of equally likely outcomes.

So, I have to resort to experiments I need a large number of nominally identical trials and then I can estimate the probability this is the frequency's definition of probability. Now in the third type of problem I may not even have access to experiments I may have access to some data but I have understanding of the problem its context and I might have experience also. So, in that particular case I can set up the hypothesis I can perform some tests and I can say that I am 50% or 90% or even 100% certain that the die is fair.

But I will have to bring in my judgment and my value system and how much I trust the data the experimenter or the model. So, the question is that whichever way I define probability are these approaches compatible with each other and if I have a problem that involves two or more such definitions can I mix them and get results.

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Review of Probability

Probability as a Measure - Axioms of probability

A probability space (Ω, \mathcal{F}, P)

- 1) $0 \leq P(A) \leq 1$ for every measurable set $A \in \mathcal{F}$.
- 2) $P(\Omega) = 1$
- 3) If A_1, A_2, \dots are disjoint sets in \mathcal{F} then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Whatever the definition or interpretation of "probability," it must conform to these three axioms



The answer is yes provided your definition satisfies the three axioms of probability. So, we have a probability space consisting of the sample space an appropriately defined algebra defined on the sample space a collection of subsets of omega and an appropriate measure defined on that collections which we call probability. So, it is non-negative that is the first axiom. The second axiom uses normalization.

So, the probability of the sample space the shear event is one and the third axiom says that probability is additive in nature. So, if I have a finite or a countably infinite collection of sets in \mathcal{F} the probability of the union is the sum of the individual probabilities.