Advanced Foundation Engineering
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Lecture - 65
Well Foundation - VI

So, last class I have discussed how we can determine the total soil reaction force which is acting on the side wall of the well and that expression was determined as P.

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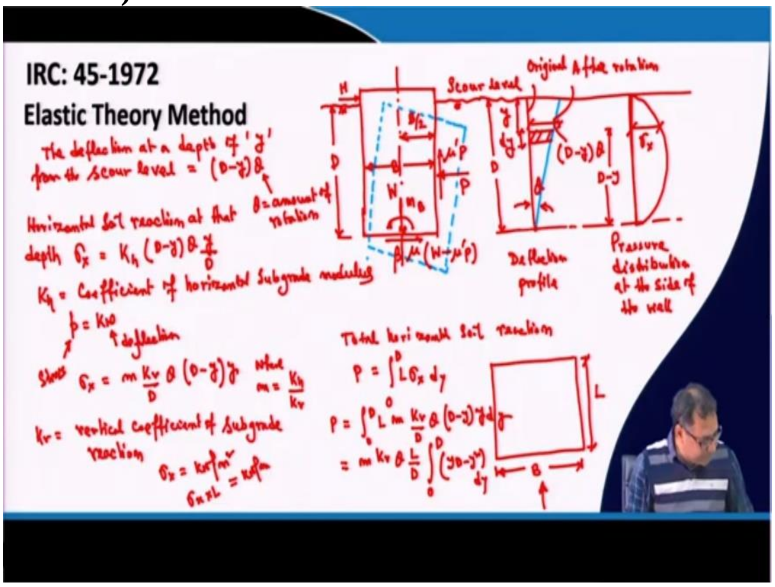

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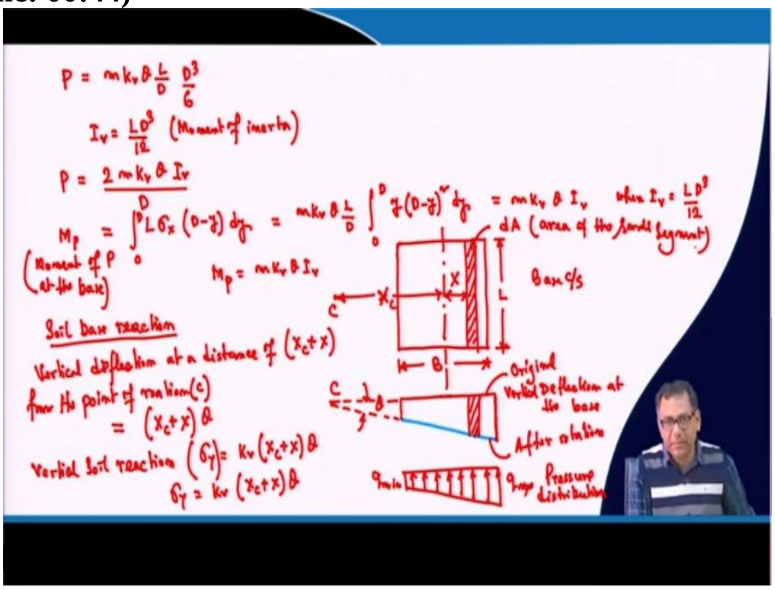

So, this is the P expression. And this is the P which is acting on the side wall of the well and there will be a friction between the sidewall of the well and the soil and that is $\mu' \times P$. So, now, there will be a base reaction also and before that, so what I will calculate. So, because of this

P which is acting at the side of the well, a moment will develop at the base of the well because this P is acting. So, there will be a moment.

So that moment is if I tell that moment is M_P, so that means, if I say this is the M_P where M_P is the moment due to P at the base. So that M_P will be equal to again we have a stress that is $L \times \sigma_x$. Now, we have to apply the lever arm. So, what is the lever arm? Because we are taking moment suppose in small segment that force is $L \times \sigma_x$ and then the lever arm from the base to that point is $D - y$.

So that means, the lever arm will be at that point in this point this is $D - y$. So, because of this small segment the moment that will develop for that moment the lever arm is $D - y$. So, this is the force into the lever arm, $D - y$ and that you have to integrate from 0 to D this is dy. So, you will get the total moment because of this P at the base and so that means we can write that σ_x is this value $m\frac{K_v}{D}\theta(D - y)y$.

So that we can write this is $mK_v\theta$ then L/D because we have multiplied with L/D then this is 0 to D along the depth then, there is a y term also, so this is y, and then $(D - y)^2$ and this is dy. So, now after integrating this term and then putting the limit and simplify this then we will get this $M_P = mK_v\theta I_r$. So, this is your I_r or I can write this is I_v also.

So, because later on I have to use one I_r, so you can put I_r or you can put I_v, so I am putting this I_v, so you can put this I_v. So, I_v is this one. So, here also it will be I_v because later on I will use one I_r part. So, this is I_v, so where $I_v = \frac{LD^3}{12}$. So, this is the expression of M_P. So that means the $M_P = mK_v\theta I_v$ and I_v expression I have given.

So, next one is that soil reaction or this was the soil reaction. So, to do that, so if I draw this figure this is the base of the well or the cross section of the well, so this is the base where this is the B and this is the L. Now, this well is rotating. So that means there will be a point of rotation of the base also. So that point of rotation say, this is the point of rotation C, this is the point of rotation.

And that C is at a distance of say X_C from the center of the well and we have considered one small segment in the well base, whose distance is X from the center of the well. So that means, the amount of rotation of the well base will be, this is your initial position, now after the rotation this is the rotated position of the base. So that means, if I use the different color, so then I can say. So, this is the rotated position.

So that means, here the rotation again will be with this point. So, the amount of rotation again this is the θ and this is the point C again. So, this is the amount of deflection of the base. So, if I take again a small segment here, so this is original and this is after rotation. So, because of that, so there will be a stress difference at the base of the well because it is rotating in this direction, towards the right direction.

So that means this will be the stress distribution at the base of the well because it will not be uniform because it is rotating because you are applying the horizontal load. So, this is $q_{\max}$, this will be $q_{\min}$. So, this is the pressure distribution at the base and this is the deflection at the base. This is the base cross section. So, now that vertical deformation, so because this is the vertical deformation at this edge and this is the vertical deformation of the other edge.

So, these are the vertical deformations, this is the deflection or I should say vertical deflection at the base. So that means the vertical deflection at a distance of, this is from the center is $X_C + X$, at a distance of $X_C + X$ from the point of rotation will be how much? Because the amount of rotation is θ and the distance is $X_C + X$, so that small one, this curvature will be $r\theta$, *r* value is how much, *r* value is $X_C + X$, this is θ and the deformation is small.

So that is why we can consider it as a straight portion. So that is why we can take this total deflection as $X_C + X$ from the point C × θ, C is the point of rotation. So, because of the rotation the vertical soil reaction σ_y will be how much? σ_y will be, the deformation is this amount and we know the K_v that means the coefficient of vertical sub grade modulus.

So that means that stress will be $K_v \times (\text{deflection} + X) \times \theta$. So, this is the σ_y at any distance from the point of rotation or I should write σ_y is $K_v \times (X_C + X) \times \theta$. So, because of this rotation we can write what will be the base moment that will develop?

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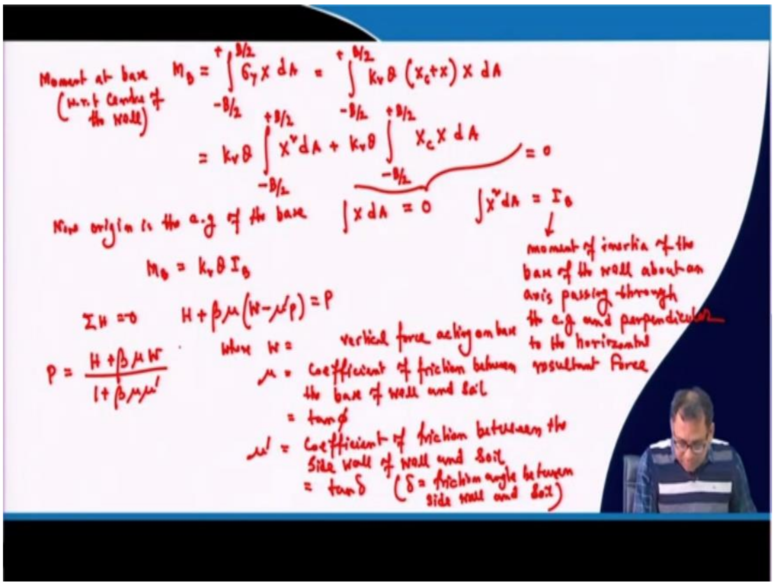

So, the moment at base, with respect to center of the well and that is M_B that will be equal to that means, we have a vertical soil reaction is σ_y and so now, if I want to take the moment that is generated at the base of the well and that we are taking from the center and this is at the small segment, the stress is σ_y. So, σ_y is stress and we are taking with respect to center, center mean this one, so distance will be X, center of the well, not the center of rotation or a point of rotation, it is center of the well.

So that is center of the well, remember that is in respect to center of the well. So that will be $\sigma_y \times X$ and the X is the lever arm. So, if the stress is acting in this small segment and the area of this small segment is dA, this is the area of the small segment then the total force will be $\sigma_y \times dA$, so that means the total force will be $\sigma_y \times dA$ and then the lever arm is X because that is from the center of the well.

So that means then if I integrate that will give me the moment at the base, I mean the soil reaction for that small segment is σ_y and then the area of that small segment is dA, σ_y is the vertical soil reaction for that small segment and then the area of the small segment is dA. So, total force for the small segment is $\sigma_y \times dA$ and the lever arm is X. So that is the moment. So, now this moment, what will be the limit? So, limit will be because it is taking from the center to this right side is positive and left side is the negative.

So that means we can write these to the limit will be $-\frac{B}{2}$ to $+\frac{B}{2}$ because this is $\frac{B}{2}$, this half portion is $\frac{B}{2}$ and this half portion is $\frac{B}{2}$, so for $-\frac{B}{2}$ to $+\frac{B}{2}$ because center is considered as the origin. So, now we can write that this is $-\frac{B}{2}$ to $+\frac{B}{2}$ and σ_y is $K_v\theta(X_C + X)$ then we can write this as $K_v\theta(X_C + X)XdA$ or we can take $K_v\theta$ outside then this will be $-\frac{B}{2}$ to $+\frac{B}{2}$.

Then I can take this is $K_v\theta \int_{-\frac{B}{2}}^{+\frac{B}{2}} X^2 dA + K_v\theta \int_{-\frac{B}{2}}^{+\frac{B}{2}} X_C X dA$. Now, this origin that means, this origin where we are taking this moment is basically the centre of gravity of the well or the base. Now, origin is at the centre of gravity of the base that means, this is the origin and that is itself the centre of gravity of the base. So that means, if I take a moment with the origin, so in that case this X value is equal to 0 because we are taking because your centre of gravity of the base on that centre of gravity this reaction will act.

So that means, X will be 0. So, I can write in that case that $\int XdA$ because your origin is at the centre of gravity of the base. So that means, if I take the dA the total area and then $\times X$, so that case this will be 0 because your origin is the centre of gravity of the base, so XdA will be equal to 0. And we can write that X^2dA that will be equal to I_B. So, this I_B is moment of inertia of the base of the well about an axis passing through the center of gravity and perpendicular to the horizontal force.

That means this is the moment of inertia of the base of the well about an axis which is passing through the center of gravity. So that means here you can see that the origin that we have taken is the center of gravity of the base. So that means the total area $\times X$ or the area $\times X$, if I consider then that area, center is the origin of the point from which we are taking the moment. So that is why this term will be 0 and this is the I_B and then finally, I can write here that M_B will be equal to this is, this term will be 0, first term will be 0 and second term there is a $K_v \times \theta$.

And there will be X_C and so this is $X_C \times$ this second term is 0. So that mean this is $X \times dA$. So, this term is equal to 0, second term is equal to 0 and this first term X^2dA, so this is I_B. So, this will be I_B. So, this is your expression of M_B. So, this total term is 0 and the second term is 0 because this $XdA = 0$. So, this will be the X^2dA, so that is equal to I_B.

So, now the total summation of the horizontal force or I should say the summation of the horizontal force, so what are the horizontal forces acting in this well? So, you can see that there will be P is one horizontal force. So, now, there will be an M_B also, base moment is acting or there will be a W and there will be H is acting and P is acting. So, there will be a friction at the base of the well also, so that friction we have W, this is $\mu \times W$.

So, we have a friction, so that direction of the friction force to indicate that direction multiplies a term β. So that friction force is $\beta\mu W$. W is the weight of well that means the vertical force which is acting at the base of the well. So, W is the vertical force which is acting at the base of the well and then there will be $\mu \times W$ that friction force and to indicate the direction that we have multiplied a term which is β. Because P is also acting, H is also acting, so direction of the β will be defined based on that.

So that means our value of the β will be defined based on that. So that means here we are putting now, these are the forces. Now, if I take these are the forces then what is the horizontal force, total horizontal force? The total horizontal force there will be H which is acting at the top, so this is H then P and then this $\beta\mu W$. So, this is I should say this is W, so now, this $\beta\mu$ instead of writing W, I should write this is equal to W because there is a W component and that is upward component also.

So, this is $\mu' P$. So that means the friction force at the base that is the total vertical force $\times\, \mu$ and to note that direction we have multiply one β term. So that value will be discussed later on. And so that means that this is $\beta\mu$ then the W is the vertical one and the vertical downward direction and the side wall friction that is acting in the upward direction. So, we are taking the net downward force, so that is $(W - \mu' P) \times \beta\mu$.

So, now that if I write here, so this will be H, so H is acting this way, now, we have considered this friction, base friction is also acting towards the H direction and P is the opposite direction. So, this is $H + \beta\mu(W - \mu' P) = P$. So, now where W is the net or vertical force or W is the vertical force I should write, downward vertical force acting on base, μ is the coefficient of friction between the base of well and soil which is generally taken as $\tan\phi$.

And μ' is the coefficient of friction between the side wall of well and soil, which is taken as $\tan\delta$. So, δ is the friction angle between the soil and the side wall. So, now, if we simplify this equation then we can write that there is a term P also this side, so after simplifying we will get

$P = \frac{H+\beta\mu W}{1+\beta\mu\mu'}$. So, here δ is the friction angle between side wall and soil. So, P also I will get by this equation. So, now we have taken the summation of the horizontal force.

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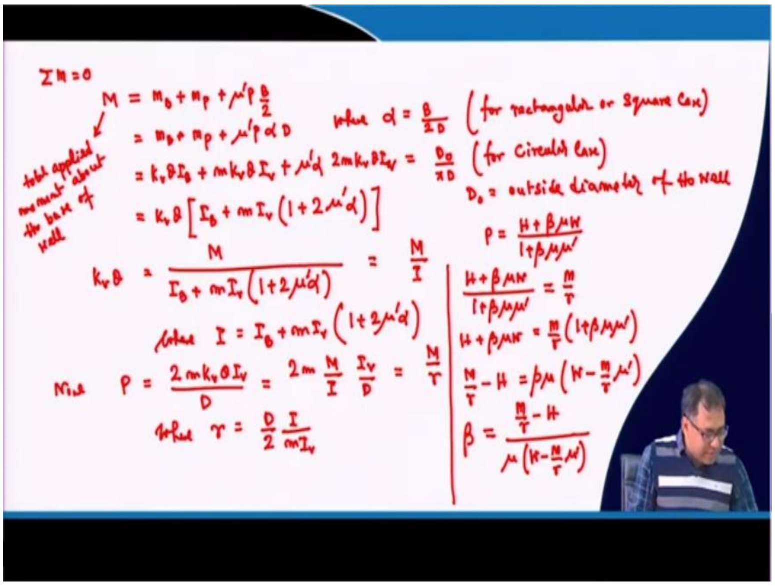

Now, if I take the summation of the moment that is equal to 0 or the summation of the moment, so that means, if I take the M which is the total applied moment about the base of well that means, if M is the applied moment, so suppose in this figure you can see the M is the applied moment then what are the moment that are counter balancing this applied moment M?

Suppose M is the applied moment with respect to base. So that total moment is applied with respect to base then what is the moment which is acting opposite to the M. So that moment is M_B which is due to the base soil reaction that is counter balancing the applied moment. Then moment due to the P which is the force due to the side soil reaction then because of the $\mu' P$ which is due to the friction between the side wall and the soil.

So, summation of these three moments will be equal to the applied moment. Because applied moment direction is from left to right and the direction of the other three moments is from right to left or I should say the direction of the applied moment in clockwise direction, the other moments are in the anti-clockwise direction, we are taking the moment from the base.

So, this moment due to W and then this W is acting at the center of gravity, so we are also taking moment from the origin and origin itself is the center of gravity of the well and the moment will be the acting in the base of the well. So, these things we will not consider because this we are taking moment from the base. So, now, only three moments that we have to consider, the moment due to P, moment due to $\mu' P$ and moment due to this M_B that we have already considered.

Then M_B is due to the soil the reaction at the base, P is the soil reaction at the side and mu dash P is the friction resistance from the side and the soil. So, if I take this moment and applied moment, so this you will be equal to the M_B, base moment then moment due to P, M_P then the $\mu' P$ and that will act at a distance of $B/2$. So, this will act at a distance of $B/2$ because this is B. So, this is equal to $B/2$ where it is acting. So, now, if I put these values further $M_B + M_P + \mu' P \alpha D$.

So, where αD will be $B/2$. So, $\alpha = \frac{B}{2D}$ for rectangular or square case and this will be equal to $\frac{D_o}{\pi D}$ for circular case where D_o is the outside diameter of the well. And D is the depth of well below the scour level. Now, we know what are the value of M_B and M_P? So that we have derived, so M_B is $K_v \theta I_B$ and the M_P is $m K_v \theta I_v$.

So, these values I am putting here. So, M_B is $K_v \theta I_B$ and M_P is $m K_v \theta I_v$ then $+\mu' \alpha$ and what is P value? So, P value is, this is the P value and there is a term D, so this is $2 m K_v \theta I_v$. So, this P we are putting in that equation this is the P value. So, this P we are putting here $2 m K_v \theta I_v$ and there is a term D and, in the P, also there is a term D, so these 2 will cancel. So, this is the equation.

So, now if I take $K_v \theta$ common then this will be $I_B + m I_v (1 + 2\mu' \alpha)$. Because this is also I_v, so we have taken $K_v \theta$, so $2\mu' \alpha$. So, we can take the $K_v \theta = \frac{M}{I_B + m I_v (1 + 2\mu' \alpha)}$. That we are taking as $\frac{M}{I}$ where $I = I_B + m I_v (1 + 2\mu' \alpha)$. So, now we have two expressions of P, this is the one expression of P and this is the one expression of P.

So, now we are using that expression that we are taking that $P = \frac{2 m K_v \theta I_v}{D}$. Now, $K_v \theta$ is $\frac{M}{I}$. So, if I replace that $K_v \theta$ by $\frac{M}{I}$ and this is $\frac{I_v}{D}$, remember that M is the applied moment. So, now we can write this is equal to $\frac{M}{r}$, where $r = \frac{D}{2} \frac{I}{m I_v}$. Because this is replaced as $\frac{M}{r}$, so r will be $\frac{D}{2} \frac{I}{m I_v}$.

So, now, we have another expression of H. So that expression is what? This is the expression. So, now, if I write that expression that $P = \frac{H+\beta\mu W}{1+\beta\mu\mu'}$. So that we can equate with this equation, so we can write that $\frac{H+\beta\mu W}{1+\beta\mu\mu'} = \frac{M}{r}$. So, you can write that $H + \beta\mu W = \frac{M}{r}(1 + \beta\mu\mu')$.

If I take H this side, this is H then this will be equal to if I take $\beta\mu$ common, so this will be W then $-\frac{M}{r}$ and $\beta\mu$ I have taken, so μ'. And I have already mentioned what is μ and μ'. So, μ' is between the sidewall and the soil and μ is between the base and the soil. So, this is the equation. So, finally, $\beta = \frac{\frac{M}{r}-H}{\mu\left(W-\frac{M}{r}\mu'\right)}$, this is the expression of β.

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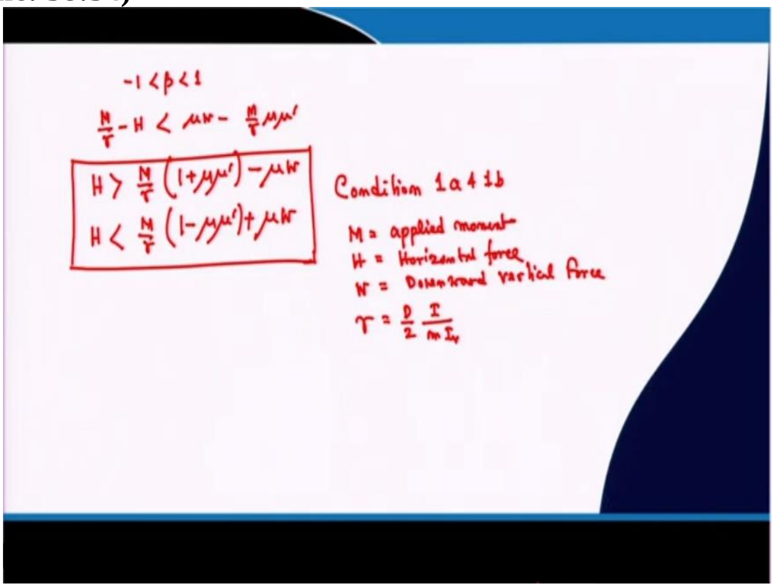

But now, this expression is valid if this is β value is in between -1 and $+1$, so that is why as I have discussed that we have to give the value of the β. So, β is in between $+1$ to -1 if it is $+1$ then the direction is the same as I have shown, if it is -1 the direction will be the opposite because depending upon the P value and the H value, this force direction may change.

So that mean the limit of β is -1 to $+1$. So that we have to now apply here, so that means this is the condition and β expression is given, so if I consider that one then I can write that $\frac{M}{r} - H$ because β value is now given, $+1$ to -1, so that should be less than $\mu\left(W - \frac{M}{r}\mu'\right)$. So, if I consider the β should be less than 1 then this is the equation. If you consider beta should be or beta should be greater than -1, so then also another condition will appear.

So, now these two conditions, so if I take this one then I have to write the horizontal force should be greater than $\frac{M}{r}(1 + \mu\mu') - \mu W$ and if I consider the another condition because we have two conditions then H should be less than $\frac{M}{r}(1 - \mu\mu') + \mu W$. So, these are the two conditions we have to satisfy for the lateral stability of the well. So, this is our condition number 1 or there are two conditions. So, I should write 1 A and 1 B.

So, this is condition 1 A and 1 B. So that means, we have to satisfy this condition that $H > \frac{M}{r}(1 + \mu\mu') - \mu W$ and $H < \frac{M}{r}(1 - \mu\mu') + \mu W$. So, what is *M*? M is the applied moment about the base and H is the horizontal force and μ and μ' are the friction coefficients, *W* is the downward vertical force and *r* value is given. What is *r*? Expression of the *r* is given here, so this is the expression of the *r*. So, *r* is $\frac{D}{2}\frac{I}{mI_v}$.

Then what is *M*? What is *I*? What is I_v? All the expressions are given, so we have to satisfy these conditions. So, these are the conditions, so that we have to satisfy. So, there is one more condition also. And there are not only one more condition there are few other conditions also. So, we have discussed or I have discussed only one condition or there are two conditions.

But there are other conditions also. So, we have to satisfy all the conditions when you check the stability of a well under lateral force, this is one of them. So, in the next class, I will discuss the other conditions as for the elastic method then I will discuss the other methods also by which we can design or we can check the stability of well under lateral load. Thank you.