Advanced Foundation Engineering
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Lecture - 59
Pile Foundation: Under Lateral Load and Uplift - IX

So, in the last class I have discussed that how we can determine the deformation of laterally loaded pile for single pile as well as given the concept that how we can determine it for the group by using elastic analysis. And the bearing capacity or the load carrying capacity of the pile I have already discussed for single as well as the group is the same as we did it for the single.

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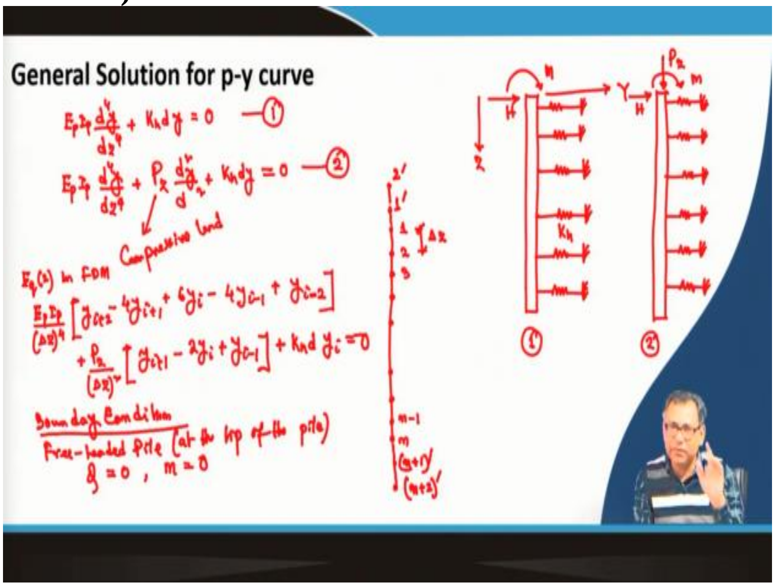

So, now, in this class I will discuss how we can or I will give a solution or the general solution for *p*-*y* curve. Now, for *p* is basically stress and *y* is the deformation. So, it is stress versus deformation curves for the laterally loaded pile. So, now, in this case we are assuming that pile is modelled as beam and which is resting or attached with the springs and we are applying say horizontal load and moment at the head of the pile or top of the pile suppose this is your *y* axis and this is *z* axis and this is the value of k_h.

Now, if under this condition it is subjected to only horizontal load and the moment and then under this condition the general solution of the beam equation will be so, that means, we are applying this solution will be $E_P I_P \frac{d^4y}{dz^4} + k_h dy = 0$. So, this is your generalised solution for the beam subjected to *H* and the moment but here why it is 0? Because no UDL is acting.

So, it is only a horizontal load and the moment is acting. So, that will be taken care during the application of the boundary condition. So, this is the common beam equation that I have already discussed all I have derived even. So, how this equation is coming during my shallow foundation classes, or beams on elastic foundation. So, you can see those lectures and then you will find that how this equation is coming?

And this equation only difference is that in that case it was only E_i and here it is $E_P I_P$ that means elastic modulus of the pile and moment of inertia of the pile and that shallow foundation case we used $k_h \times b$ or sorry $k \times b$, b is the width of the foundation and k is the subgrade modulus, but here we are using $k_h \times d$, d is the diameter of the pile or if it is a square pile it will be width of the pile.

So, this is the condition that the beam with the spring. Now, if we can go for this is our case 1, we can go for another condition. Suppose this is your beam which is again attached with spring but here it is subjected to H, moment plus a compressive load P_z, why this compressive load? Because when the pile will be subjected to lateral load definitely at the same time pile will be subjected to compressive load.

Because of the load which is coming from the superstructure or so, that means, pile will be not only subjected to horizontal load at the same time pile will be subjected to compressive load also. So, in the first case, we have not considered the effect of that compressive load the second case we have considered the effect of that compressive load, so, the basic equation will slightly change. So, this will be the basic equation $E_P I_P \frac{d^4 y}{dz^4} + P_z \frac{d^2 y}{dz^2} + k_h d y = 0$.

So, this P_z is the compressive load which is acting on the pile along with the lateral load or the horizontal load H. So, the second one is more generalised equation because in that case the effect of compressive load is also included and if you put $-P_z$ it will be tension or uplift but here it is under compression you were talking about. So, now, how we will solve this equation how I solve this equation, so, for the solution purpose, so, we have to again solve these equations by finite difference technique.

Now, when you are talking about these finite difference techniques, so, we have to divide this pile into number of node points. So, suppose this is the pile and we have to apply the number of node points. So, this is the number of node points so, for this is node number 1, 2, 3 similarly this is n $n-1$. So, we have divided it into number of node points with this distance between the node is Δz this we did it for the beams on elastic foundation concept case also but here how many boundary conditions are required so, we need four boundary conditions.

So, if I express these equations in finite difference form. So, these equations will be I can express this equation 2. So, equation 2 in FDM in finite difference method if I want to express that will be $\frac{E_P I_P}{(\Delta z)^4}[y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}]$. So, I have already discussed how these equations are coming in finite difference form then $+\frac{P_z}{(\Delta z)^2}[y_{i+1} - 2y_i + 4y_{i-1}]$ then $+k_h dy$ that is equal to 0.

So, this is the finite difference form of equation number 2 because that is the more generalised equation because where we have considered the effect of compressive load also. So, now, these equations we have to apply on all the nodes. So, that means, here you can see there is a term $i+2$ there is a term $i-2$ also. So, that means it is $i-2$ then if I will apply this equation as node number 1, so, then there will be $i-2$.

So, that means 2 imaginary nodes will be required if I want to use these equations at node number 1 if I want to use these equations at node number 2, then one imaginary node will be required. Similarly, for *n* if I want to apply these equations at node number *n* then there will be 2 imaginary nodes will be required. So, $n + 1$ and $n + 2$ and for $n - 1$ node there will be one imaginary node will be required.

So, that means, 2 + 2, 4 imaginary nodes will be required 2 in each side. So, that means, we have to apply 2 imaginary nodes. So, this is 1 and this is 1 so, this is $2'$ and this is $1'$. So, now for point node number 3 to $n - 2$ you can apply these equations no issue no imaginary nodes will be required. But, so, that means here also there will be $(n+1)'$, then another equation $(n+2)'$.

So, this $1'$ $2'$ and $(n+1)'$ $(n+2)'$ these are the imaginary nodes. So, and you know that these nodes because these are not the real nodes these are the imaginary nodes the deflection of these nodes we have to convert it to the deflection of the real nodes and that you have to do by using the boundary conditions. Now, what are those boundary conditions? So, that means here how many boundary conditions will be required?

This is 1, 2, 3, 4, 4 boundary conditions will be required. So, for the boundary conditions first we will go for the free-headed pile then we will go for the fixed-headed pile. So, for the free-headed pile, at the tip of the pile that means at the base of the pile shear force will be equal to 0 and moment will be equal to 0 because the tip is considered as a free-end because it is within the soil and it will be considered as a free-end.

So, the shear force will be 0 and moment will be 0 that is the condition for free end so, that means for free-headed pile both the ends will be treated as a free-end and for the fix-headed pile, the top end will be treated as a fixed and the bottom in will be treated as a free end. So, if it is shear force and bending moment is 0.

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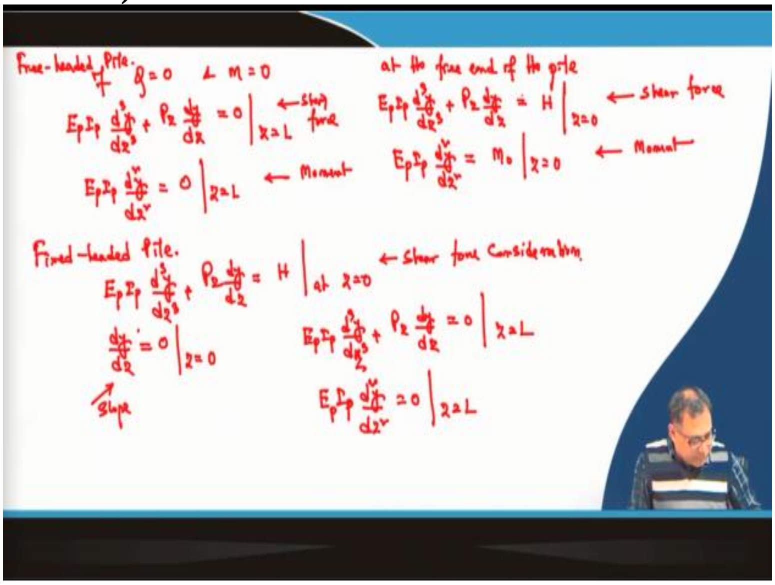

So, I can write that if $Q = 0$ and $m = 0$. So, for the moment part or the shear force I can write that this is $E_P I_P \frac{d^3y}{dz^3} + P_z \frac{dy}{dz} = 0$ at $z = L$, z is the length of the pile. Similarly, for moment is equal to 0 I can write that $E_P I_P \frac{d^2y}{dz^2} = 0$ at $z = L$. Now at the free end of the pile so, here the shear force will not be equal to 0 because the shear force there is a horizontal force acting because as I mentioned we have incorporated the compressive load effect in the equation.

But we have not incorporated the H and m effect in the equation anywhere. So that H and m will be incorporated through the boundary condition. So, for the free end of the pile, this shear force $E_P I_P \frac{d^3y}{dz^3} + P_z \frac{dy}{dz} = H$ that will not be equal to 0 because now a horizontal force is acting, so, that is at $z = 0$. Similarly, for the bending moment $E_P I_P \frac{d^2y}{dz^2}$ that will not be equal to 0 that will be equal to m_0 at $z = 0$. So, this is the free end of the free-headed pile.

Now, for the fixed-headed pile that shear force equation will be same as this equation $E_P I_P \frac{d^3y}{dz^3} + P_z \frac{dy}{dz} = H$ at $z = 0$. But $\frac{dy}{dz} = 0$ at $z = 0$ because in fixed-headed pile moment will not be 0 shear force that is developed, but moment will not be 0 because for the free-headed pile we put also applied some moment, but here the condition is that for the fixed-headed pile one is the shear force equation that I have written another is due to the slope and that at $z = 0$ will be 0.

So, this is from the slope consideration this is a shear force consideration. So, this is shear force consideration this is the bending moment consideration this is shear force this is moment this is at $z = 0$ and for at the tip of the pile, the boundary condition will be same as the free-headed pile. So, that means at the tip of the pile these will be $E_P I_P \frac{d^3y}{dz^3} + P_z \frac{dy}{dz} = 0$ at $z = L$ that is shear force consideration and $E_P I_P \frac{d^2y}{dz^2}$ will be 0 at $z = L$ that is bending moments consideration.

So, you can use these 4 boundary conditions for free-headed pile and the fixed-headed pile and then you can solve these equations and you will get the deformation or once you get the deformation then you will get the bending moment, slope, shear force whatever you want, but the one thing I want to mention that suppose if you want to use the $k_h$variation. So, the one variation will be that you can simple write that I can consider that k_h variation.

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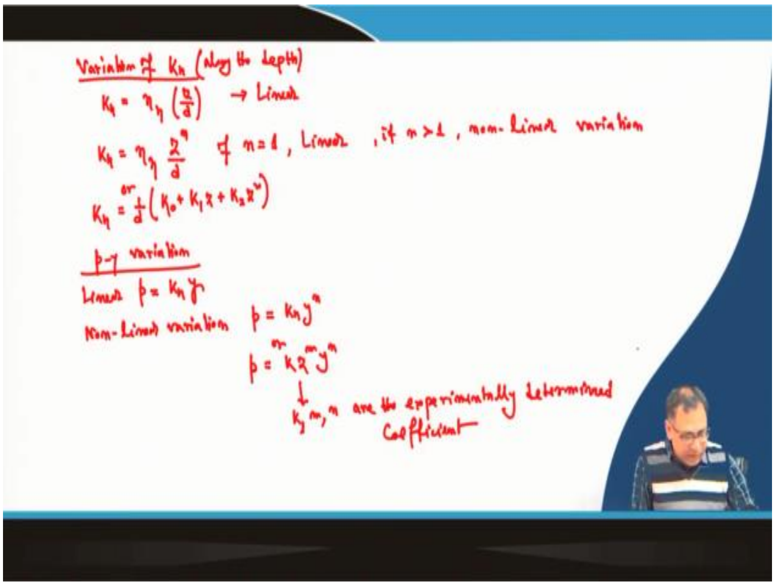

So, variation of k_h here we have not talked about. Now, what is the k_h variation? Now k_h variation will be linear then in that case that $P = k_h \times y$. So that means at any point that this stress is linearly varying with the deflection. So, that is the linear variation or I should write so, this linear variation I will write later on first I am writing the variation of k_h.

So, variation of k_h can be of different types so, this is the $k_h = \eta_h \left(\frac{z}{d}\right)$, so, that is one variation we have already used. So, this variation is linear so, k_h is varying linearly with depth. So, this is the variation, so, another variation can be that the $k_h = \eta_h \frac{z^n}{d}$. So, now, if $n = 1$ then this will be the linear variation would be same as this one but if $n > 1$ then it will be a nonlinear variation.

You can write k_h in different form that is $k_h = \frac{1}{d}(k_0 + k_1 z + k_2 z^2)$. So, these also you can write in different forms. So, this is the different variation of the k_h it can be linear it can be nonlinear depending upon your requirement you can put them in the equation because now it is so flexible that you can put that different variation in the equation and you can apply these variations at different nodes and you will get the result by using the different variation or the variation you want.

So, then the *p*-*y* relationship that is the stress and the deflection relationship or the *p*-*y* variation I should say when p is the stress y is the deflection. So, the linear variation is that $p = k_h y$ is a linear variation then the nonlinear variation also that means $p = k_h y^n$ or $p = kz^m y^n$. So, here these k, m, n are the determined coefficient. So, these k, m, n are the coefficient those can be determined experimentally.

So, that means, here not only you can give different variation of k_h, but you can give the different variation of the *p*-*y* that means, whether you will put a linear variation that means, in previous expressions that k that means, here the stress strain behaviour or the load settlement behaviour is linear, but you can put nonlinear variation also because the first one is the linear variation and second one is the nonlinear variation.

So, that means here the beams are not only resting on a linear spring that means, it can be resting on a nonlinear spring also that you can incorporate the non-linearity in the load deformation behaviour you put the non-linearity in the variation of the k_h along the depth. So, these variations of the k_h is along the depth that we can incorporate in this equation and then we can solve it. And another thing is that as you know this k_h value is not constant with the depth it is varying for the granular type of soil.

So, you can vary this k_h and if you want to make it uniform, because for the cohesive soil it is uniform then you can make these things uniform also even this stress strain relationship or the load versus deformation or the stress versus deformation relationship also changes along the depth. So, deformation versus your load or the load versus the deformation behaviour that also changes along the depth.

So, that also you can incorporate here because, as I mentioned for different node you can put different variation and if you decide node there will be different variation and different node that also you can incorporate here. So, that means, all these types of flexibility are there in these types of solution, where you can put a different variation of k_h along depth you can define variation of p and y you can put it linear nonlinear or you can have a *p*-*y* variation along the depth also. So, those things you can incorporate here.

And another thing I want to mention that if you put $P_z = 0$ then everything will be converted to equation number 1 or your case number 1, here it is case number 2. So, that case 2 is the more generalised solution, where we have taken the consideration of the compressive load along with the lateral load. So, this way you can get a solution and then you will get that deformation in the other quantities. So, this is the end of this laterally loaded pile part. So, now, I will start the next topic that is the pile under uplift load.

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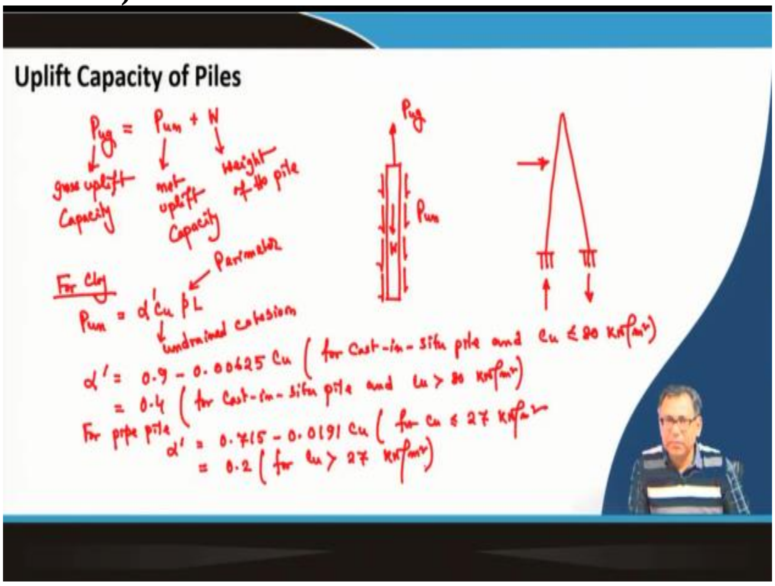

So, now, as I mentioned the pile can be under uplift load also and that uplift load means when suppose pile is under a transmission tower, where pile can be subjected to uplift load suppose, if this is a transmission tower and this is a pile and when the wind force is acting in this direction, so, it will try to rotate this structure. So, that means this portion of the pile in such case will be under uplift load and this portion of the pile will be under compressive load and this can be reversed also.

If the direction of the wind will change then the direction of this uplift load and compressive load that will also change. So, that means when you check a pile under uplift load at the same time you have to check it under compressive load also. So, now, first I will discuss that how I can develop the equation for the pile under uplift load. So, that means pile under uplift load that P_{ug} that means pile under uplift.

So, that is your or this is called gross uplift load. So, that will be equal to P_{un} this is net uplift $+W$ this is gross uplift capacity I should say gross uplift capacity net uplift capacity and this is weight of the pile. So, this is the equation that means when pile will be subjected to uplift load. So, in that time this tip resistance will not be active so, only the skin friction will be active. So, it is acting in this direction, so, skin friction will act in the downward direction, but the tip resistance will not be there.

So, this is your P_{un} so, that means $P_{un} + W$ will be equal to P_{ug}. So, now for the clay how we can calculate P_{un}? P_{un} will be equal to $\alpha' c_u p L$. So, that means, here p is the perimeter of the pile and c_u is the undrained cohesion L is the length of the pile and α' is the adhesion factor now how α' can be calculated?

$\alpha' = 0.9 - 0.00625 c_u$ that is for cast-in-situ pile and if c_u is less than equal to 80 kN/m^2 and $\alpha' = 0.4$ for cast-in-situ pile and c_u greater than 80 kN/m^2 now for pipe pile this alpha dash = $\alpha' = 0.715 - 0.0191 c_u$ for c_u is less than equal to 27 kN/m^2 and this is equal to 0.2 for c_u greater than 27 kN/m^2.

So, this way we will get the adhesion factor and if you know the undrained cohesion, if you know the length and perimeter, we can get the P_{un} and at the same time we can calculate the weight of the pile and you will get the gross uplift capacity of the pile.

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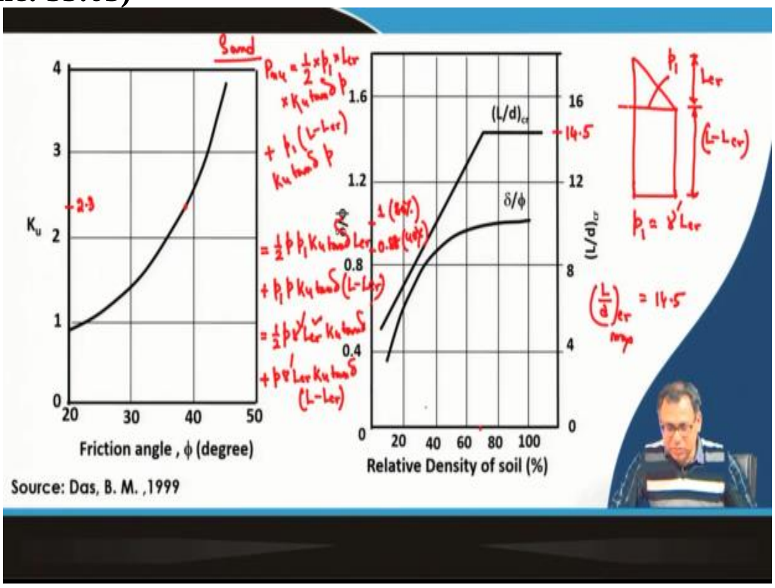

Now, this is for the cohesive soil now, for the granular soil, how we can calculate this value and also that means, for the granular soil there will be again the critical depth we have to calculate. So, this is the critical depth where this is the critical length, L_{cr}, this is $L - L_{\text{cr}}$. So, this is for the granular soil or sand the initial was for the clay and this is for the sand. Now, how I will calculate the critical length?

So, these charts will help me to determine the critical length suppose, if I know the relative density of the soil then this particular chart corresponding to relative density will give me the critical length divided by diameter that means $\left(\frac{L}{d}\right)_{\text{cr}}$. So, that means say this is the critical length axis this is the δ/ϕ axis. So, in this axis we will find for say 80% relative density it is around this is 12, 13, 14 this is 12, 14, 14.4, 14.5.

So, that means the maximum value is around 14.5 for the critical length. So, that means $\left(\frac{L}{d}\right)_{\text{cr}}$ maximum is equal to 14.5 for uplift case because for compressible case we get that means for loose sand we took a critical length by d ratio is 15 and dense sand we took it for 20 but here it is dense sand it is 14.5 and for the loose sand around these value you can say this is around 10 or something. So, that means it varies from 10 to 15 so, slightly less compared to the pile under compressive loads.

So, first we have to determine the critical length and then here we can calculate the δ/ϕ for different relative density also by using these charts suppose if it is 80% relative density then this chart we use so, δ/ϕ is around 1. So, this value is 1 for 80% relative density. So, that means for 80% relative density it is 1 for 40% relative density will be around 0.88.

So, this is the 0.88 for 40% relative density. So, this chart is for δ/ϕ this chart is for critical length and this axis is for the δ/ϕ axis this is for critical length axis and x is the axis for relative density. So, that means finally, for granular soil that P_{nu} for this type of case we can determine suppose at the critical length this is p_1 and that will continue here. So, for this particular case it will be $\frac{1}{2}p_1 \times L_{\text{cr}}$ and then there will be K_u and $\tan\delta$ then + there will be perimeter × length.

So, that means and then for the rest of the portion there will be $p_1 \times (L - L_{\text{cr}}) \times K_u \times \tan\delta \times p$ so, this is will not be there. So, that means finally I can put here the equation is that $\frac{1}{2} \times$ perimeter $\times\, p_1$ is the stress at this level $\times\, K_u \times \tan\delta \times L_{\text{cr}} + p_1 \times$ perimeter $\times\, K_u \times \tan\delta \times (L - L_{\text{cr}})$. So, now, if $p_1 = \gamma' \times L_{\text{cr}}$ so, finally, if I put this value here this will be $\frac{1}{2}$ I should write γ' this will be perimeter $\gamma' L_{\text{cr}}^2$.

Then $K_u \tan\delta + p\gamma' L_{\text{cr}} K_u \tan\delta \times (L - L_{\text{cr}})$. So, this is the equation of the P nu so, once you get the P_{nu} or the net ultimate resistance for the granular soil then you can calculate what will be the gross ultimate load carrying capacity or uplift capacity of the pile this is the equation.

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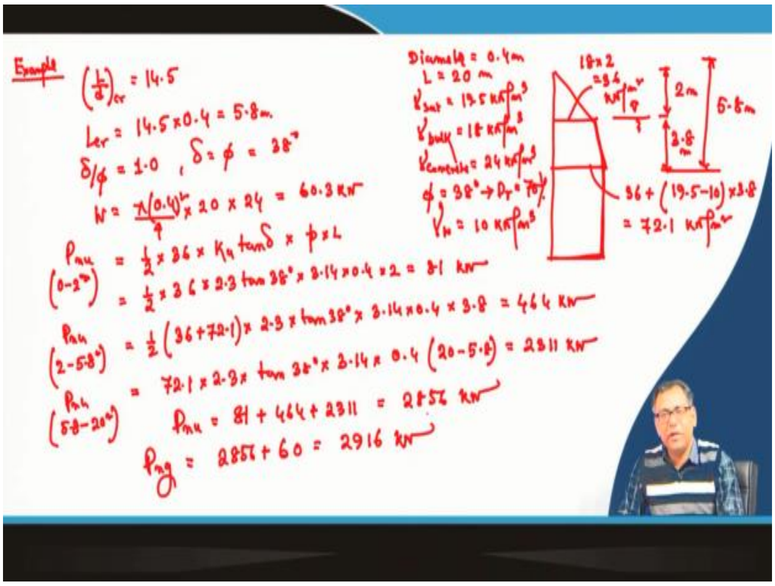

Now, let me solve one particular example problem. So, this example problem will help you to determine or to know the uplift capacity of the pile. So, for the diameter of the pile was given 0.4 m and variation is this is the water table, this is the critical and this is the uniform. So, these are the three variations here the water table is present here which is at a depth of 2 m diameter is 0.4 m then the length of the pile is 20 m.

Then a γ_{sat} is given 19.5 kN/m^3 γ_{bulk} is given 18 kN/m^3 $\gamma_{concrete}$ is given 24 kN/m^3 ϕ value is given 38° corresponding relative density D_r is given 70%. So, what would be the critical length? So, which is again 14.5 the same 14.5. So, corresponding $\left(\frac{L}{d}\right)_{cr}$ that will be 14.5 so, L_{cr} will be 14.5 × 0.4 which will be 5.8 m.

So, critical length will be up to this which is 5.8 metre. So, that means that distribution will be something like that and then it will be uniform. So, this is how much? So, this will be 3.8 m and then rest will be the uniform then δ/ϕ will be how much? This is for 70% relative density δ/ϕ value is close to 1. So, that value is close to 1 so, $\delta = \phi$ which is equal to 38° for 70% relative density.

Now, weight of the pile will be $\frac{\pi d^2}{4} \times L$, L is 20 m × the density which is 24 so, this is 60.3 kN. So, P_{nu} if I calculate from 0 to 2 m so, 0 to 2 m this γ_{bulk} is 18 and it is × 2. So, that will be 36 kN/m^2 and at this point this value will be $36 + (19.5 - 10) \times 3.8$. So, that will be equal to 72.1 kN/m^2.

So, we have considered unit weight of water is 10 kN/m^3. So, for this particular part this will be $\frac{1}{2} \times 36 \times K_u \times \tan\delta$ ×perimeter so, $\frac{1}{2}$ because we have taken the average that is into perimeter or we can write that is equal to $\frac{1}{2} \times 36$ and K_u value is how much? Here, ϕ is 38° so, K_u will be 2.3. So, I will take $2.3 \times \tan\delta$ is $\tan 38°$, then perimeter is π is 3.14 × diameter is 0.4 × length is 2.

So, this is perimeter × length because or you can put length here also $\frac{1}{2} \times 3L6 \times$ then $K_u \times \tan\delta$ ×perimeter that also you can write, but remember that these equations that I have developed in this slide is applicable if your unit weight is uniform, if the water table is at the ground surface, then you can use these equations directly, but, if it is not uniform.

Then first you draw the pressure distribution then calculate segment wise like these what I am doing, if the unit weight is uniform, then you can use those equation directly otherwise, you have to do it segment wise. So, that is $3.14 \times 0.4 \times 2 = 81$ kN then P_{nu} for 2 to 5.8 m. So, that is $\frac{1}{2}$ now, it is $\frac{1}{2}$ taking the average the same thing that we did for the pile under compressive load to determine the frictional resistance.

So, this is $\frac{1}{2}$ then $(36 + 72.1) \times 2.3 \times \tan 38°$ ×perimeter, perimeter is $3.14 \times 0.4 \times 3.8$ this will be 464 kN then P_{nu} for 5.82 m to rest of the pile 20 metre that is equal to again that is uniform. So, this will be $72.1 \times 2.3 \times \tan 38° \times 3.14 \times 0.4 \times (20 - 5.8)$ so, this value is 2311 kN. So, total P_{nu} will be 81 + 464 + 2311 so, this will be 2856 kN.

So, the gross will be 2856 + 60 this will be 2916 kN. So, this is the uplift capacity of the pile for granular soil similarly, you can calculate it for the cohesive soil also. So, that means, remember that the equation that I have given directly you can use for uniform unit weight, if there is water or it is not uniform then you have to do it segment wise, but you have to consider the critical length that I have discussed by using these two charts.

So, this is the end of this pile part. So, next class I will start the new topic which is the well foundation and then there will be one more topic that is foundation on difficult soil. Thank you.