Advanced Foundation Engineering
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Lecture - 58
Pile Foundation: Under Lateral Load and Uplift - VIII

So, during my last 2 classes I have discussed how we can determine the deflection of a laterally loaded pile by considering pile as a finite beam and pile as a semi-infinite beam. But in both the cases it was assumed that k_h value or the horizontal subgrade modulus value is uniform along the depth of the pile. So, that is suitable if the soil is cohesive soil, but if the soil is granular soil, then this k_h value changes linearly with depth. So, in today's class I will discuss how we can determine the deflection of a laterally loaded pile if this k_h value changes linearly with the depth.

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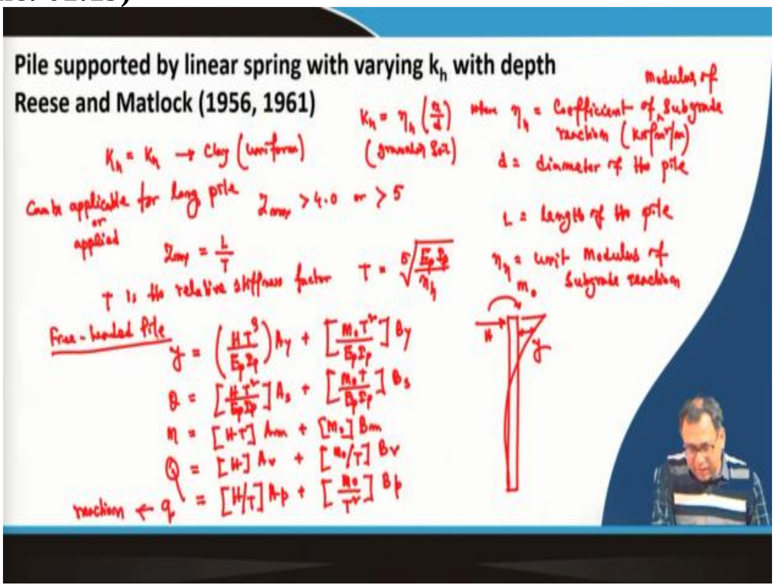

So, now, here that is the concept which is proposed by Reese and Matlock in 1956 and then in 1961 it is modified. So, the piles are supported by linear springs with varying k_h with depth. So, now here the k_h is taken as $\eta_h \left(\frac{z}{d}\right)$ whereas, *z* is any depth and *d* is the diameter of the pile. And where this η_h is equal to coefficient of subgrade reaction and whose unit is also kN/m^2/m or sometimes it is called unit subgrade modulus or unit modulus subgrade reaction.

Coefficient of subgrade reaction was coefficient of modulus of subgrade reaction sometimes it is called unit subgrade reaction or subgrade modulus. So, now, that means here we can say that $k_h = k_h$ for clay soil that means uniform and for the granular soil we can use this concept we can use this one. So, now, this approach is can be applied for long pile.

So, how we can say the pile is a long pile if $Z_{\max}$ is greater than 0, what is $Z_{\max}$? $Z_{\max} = \frac{L}{T}$. So, sometime it is greater than 4 or greater than 5 also. So, that means if $Z_{\max}$ so, I will recommend if $Z_{\max} > 4$ you can use this approach and $Z_{\max}$ is $\frac{L}{T}$ where L is the length of the pile and T which is called the relative stiffness factor can be determined as $\sqrt[5]{\frac{E_P I_P}{\eta_h}}$.

So, as I mentioned η_h is coefficient of modulus of subgrade reaction or it is sometimes called unit modulus of subgrade reaction. Now, for the free-headed pile the expressions are given, for the deflection can be calculated as $y = \left[\frac{HT^3}{E_P I_P}\right] A_y + \left[\frac{m_0 T^2}{E_P I_P}\right] B_y$. So A_y, B_y are the coefficients which can be obtained by solving the basic differential equation for the beam on elastic foundation.

So, that means here again this pile is modelled as a beam resting on the spring. So, this beam which is resting on the spring that means beams on elastic foundation. So, that equation can be solved by finite difference techniques. So, I have discussed that how you can solve the beams on elastic foundation those equation why finite difference technique and after solving that you will get these coefficients.

So, that means these H is the load which is acting and here it is the moment. Because here there is H and m_0 both are acting at the same direction that means this moment is not due to the fixity. So, this is the external moment which is acting. So, that means here we have to add the deflection due to this H and due to this moment. So, that is added here and remember that in the previous case when we solved the pile as a finite beam, we applied H and moment in opposite direction.

So, that means here H and moment both are applied because the moment is applied due to the fixity of the fixed head. But if the moment and H both are applied in the same direction that can be also solved by using that equation in that case you have to add the deformation due to the moment and due to the horizontal force. But here that is applied due to the moment and due to the horizontal force.

So, that means here that total deformation will be something like that and this deformation is y which is the summation of the contribution due to H and the contribution due to m. So, now, the expression for the slope is $\theta = \left[\frac{HT^2}{E_P I_P}\right] A_s + \left[\frac{m_0 T}{E_P I_P}\right] B_s$ then the shear force or sorry bending moment is given or sorry these equations this is the moment one is given with theta the expression is $\left[\frac{HT^2}{E_P I_P}\right] A_\theta + \left[\frac{m_0 T}{E_P I_P}\right] B_\theta$ or here the table is given for B_s.

So that is why I am giving B_s as slope. Because these values will be given in tabular form, that is why I am writing the same, I am giving the same notation. So, this moment, $M = [HT]A_m + [m_0]B_m$ and shear force $Q = [H]A_v + \left[\frac{m_0}{T}\right] B_v$ then the reaction $q = \left[\frac{H}{T}\right] A_p + \left[\frac{m_0}{T^2}\right] B_p$. So, these equations are valid for free-headed piles.

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$Z = \frac{z}{T}$	A_y	A_s	A_m	A_v	A_p	B_y	B_s	B_m	B_v	B_p
0	2.435	-1.623	0.000	1.000	0.000	1.023	-1.750	1.000	0.000	0.000
0.1	2.273	-1.618	0.100	0.989	-0.227	1.453	-1.650	1.000	-0.007	-0.145
0.2	2.112	-1.603	0.198	0.956	-0.422	1.293	-1.550	0.999	-0.028	-0.259
0.3	1.952	-1.578	0.291	0.906	-0.586	1.143	-1.450	0.994	-0.058	-0.343
0.4	1.796	-1.545	0.379	0.840	-0.718	1.003	-1.351	0.987	-0.095	-0.401
0.5	1.644	-1.503	0.459	0.764	-0.822	0.873	-1.253	0.976	-0.137	-0.436
0.6	1.496	-1.454	0.532	0.677	-0.897	0.752	-1.156	0.960	-0.181	-0.451
0.7	1.353	-1.397	0.595	0.585	-0.947	0.642	-1.061	0.929	-0.226	-0.449
0.8	1.216	-1.335	0.649	0.489	-0.973	0.540	-0.968	0.914	-0.270	-0.432
0.9	1.086	-1.268	0.693	0.392	-0.977	0.448	-0.878	0.885	-0.312	-0.432
1.0	0.962	-1.197	0.727	0.295	-0.692	0.364	-0.792	0.852	-0.350	-0.364
1.2	0.738	-1.047	0.767	0.109	-0.885	0.223	-0.629	0.775	-0.414	-0.268
1.4	0.544	-0.983	0.772	-0.056	-0.761	0.112	-0.482	0.688	-0.456	-0.157
1.6	0.381	-0.741	0.746	-0.193	-0.609	0.029	-0.354	0.594	-0.477	-0.047
1.8	0.247	-0.596	0.696	-0.298	-0.445	-0.030	-0.245	0.498	-0.476	0.054
2.0	0.142	-0.464	0.628	-0.371	-0.283	-0.070	-0.155	0.404	-0.456	0.140
3.0	-0.075	-0.040	0.225	-0.349	0.226	-0.089	0.057	0.059	-0.213	0.268
4.0	-0.050	0.052	0.000	-0.106	0.201	-0.028	0.049	-0.042	0.017	0.112
5.0	-0.009	0.025	-0.033	0.013	0.046	0.000	0.011	-0.026	0.029	-0.002

Source:
Ranjan and Rao, 1991

And coefficient will be determined in this table so, this table is given for different *z* value this is given so, here $Z = \frac{z}{T}$. So, small $\frac{z}{T}$ so, these equations are given or these coefficients we can determine for different z, so here z is any depth. So, at any depth we can determine these coefficients. So, this is A_y, A_s, A_m, A_v, A_p then B_y, B_s, B_m, B_v, B_p.

So, all these coefficients can be determined for any Z value. So, and then these equations are valid for a free-headed pile. So, what will happen if it is a fixed-headed pile, so, for the fixed-headed pile. So, I can write here for fixed-headed pile again a moment will be developed due to the fixity and that moment is equal to m_0 and it is given as $-0.93HT$.

So, this moment will be developed due to the fixity and that means this is moment due to fixity. So, that means for fixed-headed pile the moment generated due to the fixity is $-0.93HT$, H is the horizontal load which is acting. So, now let us solve one problem and you will see that how we can determine the deformation for a pile where k_h is varying with the depth.

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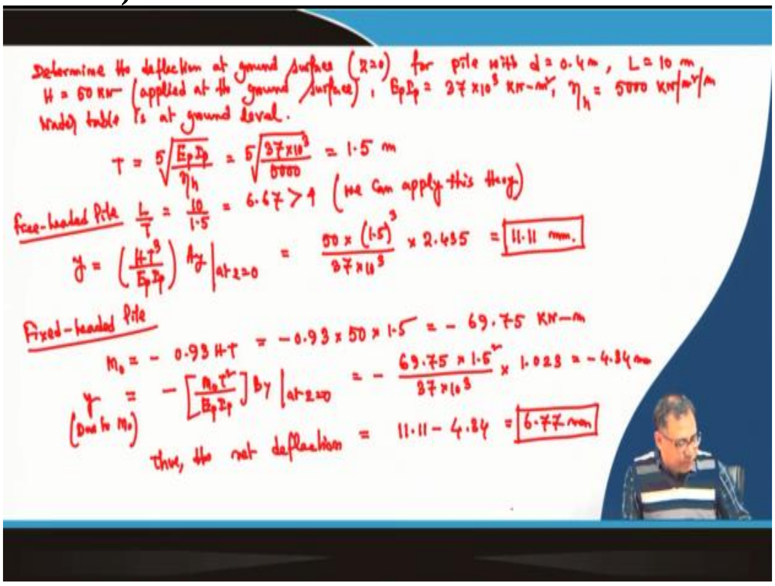

So, again the same problem I am taking that determine the deflection at ground surface that means it will be equal to 0 for pile with $d = 0.4$ m and $L = 10$ m. Because it is applicable for the long pile and H is 50 kN which is applied at the ground surface. So, $E_P I_P$ is 37×10^3 kN/m^2 then η_h is 5000 kN/m^2/m because this variation can be observed for soft clay and the granular soil.

Now, water table is at ground level so that value is not required for this particular approach but anyhow, that information is given that water table is at the ground surface. So, first we will check whether these approaches can be used or not. So, first we will take T value which is $\sqrt[5]{\frac{E_P I_P}{\eta_h}} = \sqrt[5]{\frac{37\times10^3}{5000}}$ so, this will be 1.5 m.

So, this is 1.5 m now $\frac{L}{T} = \frac{10}{1.5}$. So, which is 6.67 which is greater than 4 so, we can apply this theory. So, now a deflection for free-headed pile is $\left[\frac{HT^3}{E_P I_P}\right] A_y$ at $z = 0$ surface we have to determine and we are not taking the moment contribution because moment is not applied here.

So, that will be equal to H is 50, T^3 is 1.5^3, $E_P I_P$ is 37×10^3 then A_y at $z = 0$ so, what is A_y at $z = 0$? So, A_y at $z = 0$ is 2.435 so, we will write 2.435 because we are calculating the deflection if you want to calculate the moment then you have to take the A_m at $z = 0$ which is equal to 0 because it is free-head condition.

Now this is equal to 11.11 mm. Now for fixed-headed pile, we have to determine what would be the moment that will develop due to the fixity that is $0.93HT$ similar to the previous concept, so, this is 0.93 × H is 50 × 1.5. So, this is equal to -69.75 kN-m. So, deflection due to this moment is equal to now I will take the contribution due to the moment these contributions and that we are putting as a minus sign because now moment is minus.

So, this will be $\left[\frac{m_0 T^2}{E_P I_P}\right] B_y$ at $z = 0$. So, what is B_y at $z = 0$ because this is the moment contribution because now, in second case when pile is a fixed-headed pile you are considering deflection due to this moment and that time again the free-headed deflection you will consider, but you have to subtract this deflection due to this moment m_0, which is developed due to the fixity of the pile.

So, that means here B_y is how much? B_y is 1.023 at $z = 0$. So, you will write $\left[\frac{-69.75\times1.5^2}{37\times10^3}\right] \times 1.023$. So that is equal to -4.34 mm. So, the net deflection will be 11.11 - 4.34 so, that will be 6.77 mm. So, for the fixed-headed pile deformation at the head is 6.77 mm, but for the free headed pile that deformation is 11.11 mm.

So, that means for the fixed head there will be deformation but the slope will be 0 and free headed there will be definitely a deformation and the slope will be also there. So, this is the settlement for both the cases that I have discussed when your k_h is uniform and with varying k_h, but all the cases I have discussed that for the single pile. So then for the group pile how we can determine this deformation or the deflection for pile under compressive load?

I have discussed that how you can determine the deflection for a pile group in clay specifically. So, here also I am giving one idea that by using the elastic approach you can determine the deflection of a pile group under lateral load also.

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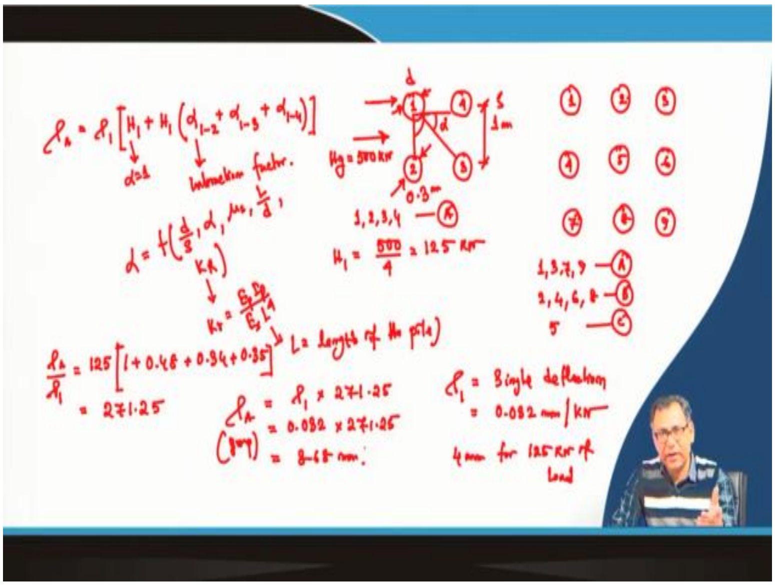

So, how you can do that suppose you have a pile group of say 4 piles or you can say one case is 4 pile. So, another case is a 9 pile this is a pile group and in first case this is the pile group whose diameter is 0.3 m and spacing between the pile is say 1 m. So, now, I want to determine the deflection of this pile group where the lateral load applied in this group is 500 kN. A 500 kN lateral load is applied in this pile group.

So, in this pile group we have to identify the similar type of piles. So, here all the piles will be similar type of pile, but for this group all the piles will not be similar type of pile. For example, these will be similar type pile these will be similar type of pile suppose if I name these piles, this is 1, 2, 3, 4, 5, 6, 7, 8, 9. So, this 1, 3, 7, 8, are in 1 group say A group because these are similar type of pile 1, 3, 7, 9.

Similarly, 2, 4, 6, 8 is in B group will be a similar type of pile and 5 will be one particular group single pile because that is a single. So, no other pile will be similar to the 5th pile. So, for these type of group, 9 pile group so, you have to make a group of similar types of pile and but here all 1, 2, 3, 4 are in 1 group these all are same type of piles.

So, suppose I am solving these types of problems. So, first case, this is the problem where every pile is taking the similar amount of horizontal load and that is H_1 say H_1 is the 500 divided by 4. So, this will be 125 kN say single pile is taking a load of 125 kN and then here the equation is that we are taking the deformation of a pile group.

Suppose, if we know the deformation of a single pile and if I take that deformation for a single pile that is fine for that deformation will be there for a single pile, but there will be the effect of other piles on a particular pile in the deformation that means, there will be interaction between the piles and because of that interaction the deformation of a particular pile will change or will increase.

That means, here the reason is the deformation of one pile where that will be the deformation of the one pile itself plus the deformation due to the other points that means, the deformation of the one pile will be the deformation of that pile plus the deformation due to the effect of pile 4 due to the effect of pile 3 due to the effect of pile 2 so, that will be there.

So, that is why I can write the deformation of that particular group will be the deformation of that pile into the load which is acting on that pile plus same load is acting on the other piles also this is H_1 plus the interaction factor. So, this is interaction between 1 to 2 plus the interaction between 1 to 3 plus the interaction between 1 to 4 why 1 is taken because all the piles will behave in similar way. Because these are in same group in case of 3×3 pile there will be 3, 3 such piles or group.

But here there is one pile group that means, we have taken the deformation due to that pile itself. So, here the interaction factor value is equal to 1 because it is that pile itself then the interaction factor for other piles. So, these are called interaction factor and these interaction factors are the function of diameter and spacing between the piles. So, this is spacing say this is the diameter.

So, this is function of diameter, spacing, the angle between the directions of horizontal load suppose this is the direction of the horizontal load. So, here this angle is how much? Here this angle is changing here this angle is changing. So, here it is 0 degree then it is 90 degree then it is 45 degree, so, this angle of the direction of the pile with respect to the direction of the load so, that angle it is function of that angle.

So, now, it is a function of this Poisson's ratio of the soil, length by diameter and the stiffness K_r which is $\frac{E_P I_P}{E_s L^4}$, where L is equal to length of the pile. So, that means this is function of diameter, spacing, angle with respect to the direction of the load and Poisson's ratio of the soil.

It is function of length and diameter of the soil, then it is function of K_r, K_r is basically the stiffness of the pile. So, that means the E_P, I_P, E_S so, all these things are involved for interaction factor and these interaction factor values are given in tabular form for different cases in the book by Carlos and Davis. So, you can get those values from that book. For different cases I am just explaining what is the concept of using that interaction factor where you will get the chart and the different conditions.

You should know the angle you should know the Poisson's ratio, then length by diameter, then diameter and spacing and K_r value. So, if you know all these values based on that, you will get the interaction factor. So, you will put those interaction factors then you will get this deformation. For example, in this particular case this is $\frac{\rho_A}{\rho_1}$. So, H_1 is 125.

So, this will be α_{1-2} so, this is given is 0.48, then this is α_{1-3} is 0.34 + α_{1-4} is 0.35. So, these are the values and then finally, I will get this is 271.25. So, ρ_A will be for the group if it is a rigid pile cap then we will assume that all the pile will deform by same amount if the piles are with rigid cap. So, this will be $\rho_1 \times 271.25$ and ρ_1 is basically the single pile deflection and that is given as a 0.032 mm/kN that means, it is for per kN.

And generally, it is given as such as that 4 mm for 125 kN of load. So, you can get it for per KN and then now, you can determine that ρ_1 is given this is also in terms of kN or sorry this is in a non-dimensional parameter. So, if I multiply with 0.032 × 271.25 it is per kN then the deformation will be equal to 8.68 mm for this group.

So, that means, for the single pile deformation was say 4 mm for 125 kN, so, that you have to convert it for mm/kN, then we multiply that. So, these particular group these 4 piles with this spacing this diameter the settlement of the group will be 8.68 mm, whereas, single pile settlement was 4 mm for 125 kN of load. So, this interaction factor values you can get from those books those charts are available.

So, I am not giving here all those charts here it is only the concept I have discussed. So, charts those value you can get from that book and then you can put these values here and you will get the deformation for particular group of the pile. So, that means here so, how you will get the single pile deformation you can get it by elastic analysis also the single pile deformation you can get it and I have already discussed that how you can get the single pile deformation for this using these two concepts.

One is with varying k_h and other is uniform k_h or by pile load test also under lateral load you can get the deformation of the single pile by using this approach you can get the deformation of a group pile and for the pile under compressive load deformation of a group pile I have already discussed. So, this way I am finishing today's class so, next class I will discuss that how you can develop a generalised *P-y* curve for a pile under lateral load. Thank you.