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**Lecture - 57**  
**Pile Foundation: Under Lateral Load and Uplift - VII**

So, last class we have discussed how we can model the pile as a finite beam and then we can determine the deflections, bending moment and shear force at different depth along the pile and we assumed that the horizontal modulus of subgrade reaction is uniform along the depth of the pile. Now, today's class I will discuss how we can model the pile as a semi-infinite beam because if the pile is short pile then you have to go for finite beam but if the pile is a long pile then we can go for semi-infinite beam because the solution first I mean finite beam is easier compared to the infinite beam.

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**Free End (Both)**

Force (H)

$$\delta = \frac{2H\lambda}{K_{hd}} K_{\phi} H$$

(Deflection)

$$\theta = -\frac{2H\lambda^2}{K_{hd}} K_{\phi} H$$

(Slope)

$$M = -\frac{H}{\lambda} K_{\phi} H$$

(M)

$$Q = -H K_{\phi} H$$

(Shear Force)

where  $\lambda = \sqrt[4]{\frac{K_{hd}}{4E_p I_p}}$

**Free headed Pile.**

Finite beam.

$$\delta = \frac{2m_0 \lambda^2}{K_{hd}} K_{\phi} m$$

$$\theta = \frac{4m_0 \lambda^3}{K_{hd}} K_{\phi} m$$

$$M = m_0 K_{\phi} m$$

$$Q = -2m_0 \lambda K_{\phi} m$$

And before we go to the semi-infinite beam part so, there is a small correction and this that correction in expression for slope due to moment will be  $\lambda^3$ . So, this is  $\lambda^3$ . So, that this is the correction.

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For Fixed-headed Pile.

Slope due to  $H = \frac{-2H\lambda^2}{k_{qd}} K_{\theta H}(at=2\lambda)$


Slope due to  $M_0 = \frac{4M_0\lambda^2}{k_{qd}} K_{\theta M}(at=2\lambda)$

To make slope zero at fixed head

$$\frac{2H\lambda^2}{k_{qd}} K_{\theta H}(2\lambda) = \frac{4M_0\lambda^2}{k_{qd}} K_{\theta M}(2\lambda)$$

$$M_0 = \frac{H}{2\lambda} \left[ \frac{K_{\theta H}(2\lambda)}{K_{\theta M}(2\lambda)} \right]$$

↓  
Due to the fixity of the pile head




So, that you should do and this is the  $\lambda^3$ . So, this is  $\lambda^3$ . So, that is why when you take the  $M_0$  which is the moment due to the fixity of the pile head, so, that amount of moment will be developed in the pile due to application of the  $H$  due to the fixity of the pile, in case of fixed-head pile to make the net slope at the fixed-head 0 because for the fixed boundary condition the slope should be 0. So, that is why we will get  $M_0$  this value. So, now, we will go for the pile as a semi-infinite beam.

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$\lambda L$	$z/L$	$K_{\rho H}$	$K_{\rho V}$	$K_{MM}$	$K_{\theta H}$	$K_{\theta M}$	$K_{\rho M}$	$K_{MM}$	$K_{\rho M}$
4	0.000	1.0008	1.0015	0.0000	1.0000	-1.0015	1.0021	1.0000	0.0000
4	0.0625	0.7550	0.9488	0.1926	0.5616	-0.5624	0.7567	0.9472	0.1929
4	0.1250	0.5323	0.8247	0.2907	0.2411	-0.2409	0.5344	0.8229	0.2910
4	0.1875	0.3452	0.6693	0.3218	0.0234	-0.0220	0.3478	0.6673	0.3219
4	0.2500	0.1979	0.5101	0.3093	-0.1108	0.1136	0.2010	0.5082	0.3090
4	0.3125	0.0890	0.3641	0.2717	-0.1810	0.1855	0.0926	0.3626	0.2705
4	0.3750	0.0140	0.2403	0.2226	-0.2055	0.2118	0.0178	0.2397	0.2204
4	0.4375	-0.0332	0.1419	0.1715	-0.1996	0.2079	-0.0295	0.1430	0.1671
4	0.5000	-0.0590	0.0682	0.1243	-0.1758	0.1858	-0.0558	0.0720	0.1176
4	0.5625	-0.0692	0.0163	0.0843	-0.1432	0.1545	-0.0674	0.0242	0.0749
4	0.6250	-0.0687	-0.0176	0.0529	-0.1084	0.1200	-0.0696	-0.0043	0.0406
4	0.6875	-0.0615	-0.0379	0.0299	-0.0756	0.0858	-0.0665	-0.0178	0.0149
4	0.7500	-0.0505	-0.0488	0.0147	-0.0475	0.0538	-0.0616	-0.0206	-0.0025
4	0.8125	-0.0376	-0.0536	0.0057	-0.0255	0.0242	-0.0568	-0.0166	-0.0122
4	0.8750	-0.0239	-0.0552	0.0014	-0.0101	-0.0033	-0.0535	-0.0096	-0.0148
4	0.9375	-0.0101	-0.0555	0.0001	-0.0016	-0.0296	-0.0520	-0.0029	-0.0106
4	1.0000	0.0038	-0.0555	0.0000	-0.0000	-0.0555	-0.0517	-0.0000	-0.0000

Source : Poulos and Davis (1980)



So, we will go for the pile and as I have already discussed that how we will get this  $K_{\rho H}$  to the  $K_{QM}$  coefficient values with respect to different  $\lambda L$  and  $z/L$ ,  $z$  is any depth and  $L$  is the length of the pile and  $\lambda$  I have already discussed. So, this is  $\lambda L = 2$ , this is  $\lambda L = 3$ , this is for  $\lambda L = 4$ , this is for  $\lambda L = 5$ .

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**Pile as semi-infinite beam**

**For free-headed pile**

$$y = \frac{2H\lambda}{k_d} D\lambda z$$

$$\theta = -\frac{2H\lambda^2}{k_d} A\lambda z$$

$$M = -\frac{H}{\lambda} B\lambda z$$

$$Q = -H C\lambda z$$

At  $z=0$ ,  $A\lambda z = 1$ ,  $B\lambda z = 0$ ,  $C\lambda z = 1$  and  $D\lambda z = 1$

**For fixed-headed pile**

$$y = -\frac{2m_0\lambda^2}{k_d} C\lambda z$$

$$\theta = \frac{4m_0\lambda^3}{k_d} D\lambda z$$

$$M = m_0 A\lambda z$$

$$Q = -2m_0\lambda B\lambda z$$

if  $\lambda L > 2.5$ , then the pile is called long pile, where L is the length of the pile.  
 As per Hetenyi (1946), a beam is called long beam if  $\lambda L > 3.14$ .  
 As per Vesic (1963), a beam is called moderately long beam if  $\lambda L > 2.25$  and  $\lambda L < 5$  and if the  $\lambda L > 5$  then the beam is called long beam.

Now, we will go for the pile as a semi-infinite beam. So, when the pile is called the long pile, so, if the  $\lambda L > 2.5$ , then the pile can be called as a long pile where L is the length of the pile. So as per Hetenyi a beam is called the long beam if  $\lambda L > 3.14$  as per Vesic the beam is called moderately long beam if  $\lambda L > 2.25$  and  $\lambda L < 5$  and if the  $\lambda L > 5$  then the beam is called long beam.

So that means as per Hetenyi, because it is Hetenyi beam concept we are using. So, if it less than 3.14 then we can go for a long pile so that we will see  $\lambda L > 3.14$  then you can go for pile as a semi-infinite beam, if the  $\lambda L < 3.14$  then you can go for pile as a finite beam. So, and from this study, I have observed that if  $\lambda L > 2.5$  so more or less your semi-infinite beam and the finite beam will give more or less same value.

So that means which I want to say that if the  $\lambda L < 3.14$  or  $\lambda L < 2.5$  so definitely you have to go for pile with finite beam. But pile as a finite beam, but if the  $\lambda L > 3.14$  or  $\lambda L > 2.5$  then you can consider pile as semi-finite beam or you can consider pile as a finite beam also, but if it is less than that, then definitely have to go for pile as finite beam, but if you can see your finite beam case this  $\lambda L$  value is given up to 5.

So, if it is greater than 5, so definitely you have to go for semi-infinite beam. So, that means, what does it mean that if the  $\lambda L$  is less than 3.14 or 2.5 then you can go for pile as finite beam if it is 2.5 to 5 then you can either use finite beam or semi-infinite beam if it is greater than 5 then you have to use it as semi-infinite beam. So, that semi-infinite beam, the similar type of concept like finite beam we are using here.

So, here semi-infinite beam that we are using that this  $H$  is acting here. So, that means, in such case this is for the free-headed pile the equations are similar over here also the two ends are there is one end you have to apply the end conditioning forces  $P_{0A}$  and  $m_{0A}$  are the end conditioning forces you have to apply. So, here your springs are in this direction. So, you can because this is the way we have to derive the equations if you change the direction your equation sign will also change.

So, for this condition we are deriving these equations, so, we have to apply end conditioning forces and this is infinite. So, this beam is infinite along the depth. So, that means, after solving these we will get deflection,  $y = \frac{2H\lambda}{k_h d} \times D_{\lambda z}$ , slope,  $\theta = -\frac{2H\lambda^2}{k_h d} A_{\lambda z}$ , moment,  $M = -\frac{H}{\lambda} B_{\lambda z}$  and shear force,  $Q = -HC_{\lambda z}$ .

Similarly, for the moment this is infinite direction and  $H$  is acting this direction, so, moment is acting this side. So, for the moment we can apply this is for  $H$  and this is for  $m_0$  the equations are  $y = -\frac{2m_0\lambda^2}{k_h d} C_{\lambda z}$ ,  $\theta = \frac{4m_0\lambda^3}{k_h d} D_{\lambda z}$ , bending moment  $M = m_0 A_{\lambda z}$  and the shear force  $Q = -2m_0\lambda B_{\lambda z}$ .

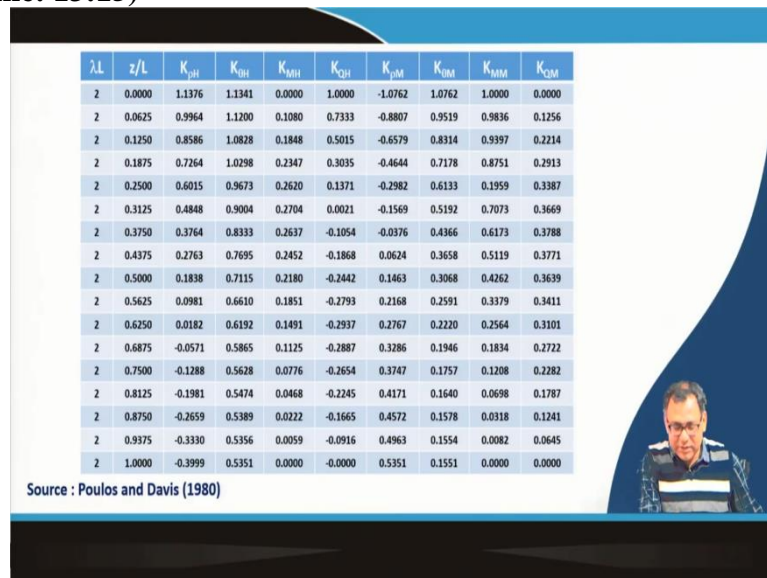
So, as you know that  $A_{\lambda z} = e^{-\lambda z}(\cos \lambda z + \sin \lambda z)$  and  $z$  is this direction then  $B_{\lambda z} = e^{-\lambda z} \sin \lambda z$   $C_{\lambda z} = e^{-\lambda z}(\cos \lambda z - \sin \lambda z)$  and  $D_{\lambda z} = e^{-\lambda z} \cos \lambda z$ . So, at  $z = 0$   $A_{\lambda z} = 1$ ,  $B_{\lambda z} = 0$ ,  $C_{\lambda z} = 1$  and  $D_{\lambda z} = 1$ .

So, this is the equation which is slightly easier than the beam equations where beam has finite length because where I have given those coefficients as a tabular form, but here the coefficient equations are available. So, you can put directly the desired value and you will get the deflection, shear force, bending moment and the slope at any point. So, now, these equations are valid for free-headed pile.

So, again for fixed-headed pile, how we can determine these values. So, I am writing here this is for fixed-headed pile. So, again the net slope at  $z = 0$  will be 0. So, that slope will be due to the  $H$  and due to  $m_0$  and because this  $m_0$  is the moment due to the fixity of the fixed end, net slope will be 0. So, that slope minus out again as I mentioned, we will not consider the sign we will consider only the magnitude because  $H$  and moment are acting in opposite direction.

Which is the actually in that case only the net slope will be 0. So, that is why we will take only the magnitude. So, that magnitude if I take so, that will be  $\frac{2H\lambda^2}{k_h d} A_{\lambda z} = \frac{4m_0\lambda^3}{k_h d} D_{\lambda z}$ . So, the  $m = m_0$  the moment due to fixity. So, that will be equal to  $H$  this is by  $2\lambda$  this  $\lambda^3 \lambda^2$  and then  $A_{\lambda z}$  at  $z = 0$ . And  $D_{\lambda z}$  at  $z = 0$  so, as I mentioned at  $z = 0$   $A_{\lambda z} = 1$   $D_{\lambda z} = 1$  so, it will cancel out. So, finally, the  $m_0$  the moment due to fixity will be  $\frac{H}{2\lambda}$ . So, this is the equation.

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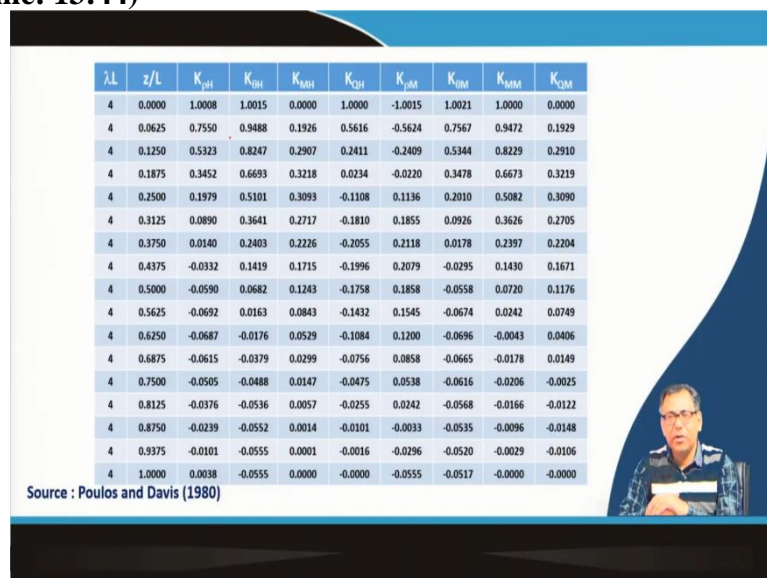


$\lambda L$	$z/L$	$K_{\theta H}$	$K_{\theta M}$	$K_{\theta H}$	$K_{\theta M}$	$K_{\theta H}$	$K_{\theta M}$	$K_{\theta H}$	$K_{\theta M}$
2	0.0000	1.1376	1.1341	0.0000	1.0000	-1.0762	1.0762	1.0000	0.0000
2	0.0625	0.9964	1.1200	0.1080	0.7333	-0.8807	0.9519	0.9836	0.1256
2	0.1250	0.8586	1.0828	0.1848	0.5015	-0.6579	0.8314	0.9397	0.2214
2	0.1875	0.7264	1.0298	0.2347	0.3035	-0.4644	0.7178	0.8751	0.2913
2	0.2500	0.6015	0.9673	0.2620	0.1371	-0.2982	0.6133	0.1959	0.3387
2	0.3125	0.4848	0.9004	0.2704	0.0021	-0.1569	0.5192	0.7073	0.3669
2	0.3750	0.3764	0.8333	0.2637	-0.1054	-0.0376	0.4366	0.6173	0.3788
2	0.4375	0.2763	0.7695	0.2452	-0.1868	0.0624	0.3658	0.5119	0.3771
2	0.5000	0.1838	0.7115	0.2180	-0.2442	0.1463	0.3068	0.4262	0.3639
2	0.5625	0.0981	0.6610	0.1851	-0.2793	0.2168	0.2591	0.3379	0.3411
2	0.6250	0.0182	0.6192	0.1491	-0.2937	0.2767	0.2220	0.2564	0.3101
2	0.6875	-0.0571	0.5865	0.1125	-0.2887	0.3286	0.1946	0.1834	0.2722
2	0.7500	-0.1288	0.5628	0.0776	-0.2654	0.3747	0.1757	0.1208	0.2282
2	0.8125	-0.1981	0.5474	0.0468	-0.2245	0.4171	0.1640	0.0698	0.1787
2	0.8750	-0.2659	0.5389	0.0222	-0.1665	0.4572	0.1578	0.0318	0.1241
2	0.9375	-0.3330	0.5356	0.0059	-0.0916	0.4963	0.1554	0.0082	0.0645
2	1.0000	-0.3999	0.5351	0.0000	-0.0000	0.5351	0.1551	0.0000	0.0000

Source : Poulos and Davis (1980)

Which is similar to the equation for finite beam. So, but here that  $K_{\theta H} = 0$   $K_{\theta M}$  at  $z = 0$  that is also close to  $K_{\theta H}$  so, which is not 0 actually  $K_{\theta H}$  is not 0 at  $z = 0$  this is  $z = 0$  so, that is that is why you have to calculate or you have to put these values depending upon what is your  $\lambda L$ .

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$\lambda L$	$z/L$	$K_{\theta H}$	$K_{\theta M}$	$K_{\theta H}$	$K_{\theta M}$	$K_{\theta H}$	$K_{\theta M}$	$K_{\theta H}$	$K_{\theta M}$
4	0.0000	1.0008	1.0015	0.0000	1.0000	-1.0015	1.0021	1.0000	0.0000
4	0.0625	0.7550	0.9488	0.1926	0.5616	-0.5624	0.7567	0.9472	0.1929
4	0.1250	0.5323	0.8247	0.2907	0.2411	-0.2409	0.5344	0.8229	0.2910
4	0.1875	0.3452	0.6693	0.3218	0.0234	-0.0220	0.3478	0.6673	0.3219
4	0.2500	0.1979	0.5101	0.3093	-0.1108	0.1136	0.2010	0.5082	0.3090
4	0.3125	0.0890	0.3641	0.2717	-0.1810	0.1855	0.0926	0.3626	0.2705
4	0.3750	0.0140	0.2403	0.2226	-0.2055	0.2118	0.0178	0.2397	0.2204
4	0.4375	-0.0332	0.1419	0.1715	-0.1996	0.2079	-0.0295	0.1430	0.1671
4	0.5000	-0.0590	0.0682	0.1243	-0.1758	0.1858	-0.0558	0.0720	0.1176
4	0.5625	-0.0692	0.0163	0.0843	-0.1432	0.1545	-0.0674	0.0242	0.0749
4	0.6250	-0.0687	-0.0176	0.0529	-0.1084	0.1200	-0.0696	-0.0043	0.0406
4	0.6875	-0.0615	-0.0379	0.0299	-0.0756	0.0858	-0.0665	-0.0178	0.0149
4	0.7500	-0.0505	-0.0488	0.0147	-0.0475	0.0538	-0.0616	-0.0206	-0.0025
4	0.8125	-0.0376	-0.0536	0.0057	-0.0255	0.0242	-0.0568	-0.0166	-0.0122
4	0.8750	-0.0239	-0.0552	0.0014	-0.0101	-0.0033	-0.0535	-0.0096	-0.0148
4	0.9375	-0.0101	-0.0555	0.0001	-0.0016	-0.0296	-0.0520	-0.0029	-0.0106
4	1.0000	0.0038	-0.0555	0.0000	-0.0000	-0.0555	-0.0517	-0.0000	-0.0000

Source : Poulos and Davis (1980)

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$\lambda L$	$z/L$	$K_{yH}$	$K_{\theta H}$	$K_{MH}$	$K_{CH}$	$K_{yM}$	$K_{\theta M}$	$K_{MM}$	$K_{CM}$
5	0.0000	1.0003	1.0003	0.0000	1.0000	-1.0003	1.0002	1.0000	0.0000
5	0.0625	0.6964	0.9214	0.2249	0.4711	-0.4715	0.6964	0.9211	0.2250
5	0.1250	0.4342	0.7476	0.3131	0.1206	-0.1210	0.4343	0.7472	0.3133
5	0.1875	0.2317	0.5479	0.3155	-0.0842	0.0840	0.2320	0.5472	0.3158
5	0.2500	0.0910	0.3628	0.2716	-0.1817	0.1818	0.0907	0.3620	0.2720
5	0.3125	0.0013	0.2121	0.2093	-0.2079	0.02084	0.0022	0.2111	0.2095
5	0.3750	-0.0466	0.1013	0.1461	-0.1919	0.1930	-0.0455	0.1002	0.1461
5	0.4375	-0.0659	0.0277	0.0915	-0.1556	0.1575	-0.0644	0.0267	0.0910
5	0.5000	-0.0671	-0.0157	0.0494	-0.1133	0.1163	-0.0654	-0.0161	0.0482
5	0.5625	-0.0584	-0.0368	0.0203	-0.0738	0.0778	-0.0567	-0.0361	0.0180
5	0.6250	-0.0456	-0.0435	0.0026	-0.0412	0.0461	-0.0444	-0.0409	-0.0012
5	0.6875	-0.0321	-0.0419	-0.0063	-0.0169	0.0223	-0.0321	-0.0365	-0.0117
5	0.7500	-0.0197	-0.0369	-0.0088	-0.0008	0.0055	-0.0221	-0.0276	-0.0159
5	0.8125	-0.0090	-0.0317	-0.0075	0.0081	-0.0059	-0.0150	-0.0175	-0.0157
5	0.8750	0.0002	-0.0279	-0.0044	0.0108	-0.0139	-0.0110	-0.0086	-0.0125
5	0.9375	0.0086	-0.0261	-0.0014	0.0079	-0.0201	-0.0094	-0.0023	-0.0072
5	1.0000	0.0167	-0.0259	0.0000	0.0000	-0.0259	-0.0091	-0.0000	0.0000

Source : Poulos and Davis (1980)

So, you can see as  $\lambda L$  value increases these  $K_{\theta H}$  value is close to 0, so this is 1.0003 for  $\lambda L = 5$  then 1.0015 for  $\lambda L = 4$  then 1.004 for  $\lambda L = 3$  then 1.1341 for  $\lambda L = 2$ . So, that is why for our other case for  $\lambda L$  more than 2, 3 this is close to 0 then 4 it is also close to 1 then 5 it is also close to 1 so that is I want to say that if your  $\lambda L$  value is less than 2.5 so definitely you have to model your pile as finite beam and if it is more than that, then you can model it as infinite beam or the semi in finite beam.

So, you will get more or less same value. But it is up to 5 if it is more than 5  $\lambda L$  values more than 5 so definitely you have to go for pile as semi-infinite beam.

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Constant  $k_h$  (Uniform  $k_h$ )

Determine the deflection of a pile at ground surface ( $z=0$ ) if  $H = 50 \text{ kN}$ ,  $k_h = 70 \text{ MN/m}^3/\text{m}$

$E_p I_p = 37000 \text{ kN-m}^2$  (i) Free-headed pile (ii) Fixed-headed pile.  $d = 0.4 \text{ m}$   
 $L = 7.5 \text{ m}$

$$\lambda = \frac{4 \sqrt{k_h d}}{\sqrt{E_p I_p}} = \frac{4 \sqrt{70 \times 10^6 \times 0.4}}{\sqrt{4 \times 37000}} = 0.66 \text{ m}^{-1}$$

$$\lambda L = 0.66 \times 7.5 = 4.95 \approx 5$$

(i) Free-headed pile.  
 $y = \frac{2H\lambda}{k_h d} \times K_{yH} = \frac{2 \times 50 \times 0.66}{70 \times 10^6 \times 0.4} \times 1.0003 = 2.96 \text{ mm}$  Pile as finite beam

(ii) Fixed-headed pile  
 $M_0 = \frac{H}{2\lambda} \left[ \frac{K_{\theta H}(z=0)}{K_{\theta M}(z=0)} \right] = \frac{50}{2 \times 0.66} \left[ \frac{1.0003}{1.0002} \right] = 37.9 \text{ kN-m}$   
 $y(\text{Due } M_0) = \frac{2M_0}{k_h d} \lambda^3 \times K_{yM} = \frac{2 \times 37.9 \times (0.66)^3}{70 \times 10^6 \times 0.4} \times (-1.0003)$   
 Net deflection =  $2.96 - 1.18 = 1.78 \text{ mm}$

So now let us solve 1 problem that how we can use this concept that I have discussed. So, that means the example problem where it is constant  $H k_h$  value so, it is constant  $k_h$  or uniform  $k_h$ .

So, determine the deflection of a pile at ground surface. So, that means  $z = 0$  if horizontal force acting is 50 kN and  $k_h$  is 70 MN/m<sup>2</sup>/m and  $E_P I_P$  is given as 3700 kN/m<sup>2</sup>.

So, two cases one is for free-headed pile and another case is for fixed-headed pile. So, first one we will consider that pile let me check what is the  $L/d$  value. So,  $d$  value is given as 0.75 m and  $L$  is given as 7.5 m. So,  $L$  is given 7.5 m and your  $d$  is 0.75 m or here it is given 0.4 m you should say so, because this is the actual dimension  $d$  is 0.4 m and  $L$  is 7.5 m.

So, let us see which equation or which condition you should apply for pile as a finite beam or pile as a semi-infinite beam. So, for that first we have to calculate the  $\lambda$  value. So,  $\lambda$  is  $\sqrt[4]{\frac{k_h d}{E_P I_P}}$ .

So, this is equal to  $\sqrt[4]{\frac{70 \times 10^3 \times 0.4}{4 \times 3700}}$ . So, this is equal to  $0.66 \text{ m}^{-1}$ .

So, the  $\lambda L$  value is  $0.66 \times 7.5$  so, this is 4.95 so, it is close to 5 so, as I mentioned if it is within 2.5 or 3.1425 or I should say the 2.5 to 5 then you can use the pile as semi-infinite or more pile finite beam. But if it is greater than 5 then you have to go for pile as semi-infinite beam but here it is exactly 5. So, you will use both concepts we will use pile as finite beam and pile as semi-infinite beam.

And let us see whether there will be any difference or not. So, first case that this is for free-headed pile and first we are considering the pile as a finite beam. So, pile as finite beam what is the equation. So, this is the equation, deflection is  $\frac{2H\lambda}{k_h d} \times K_{\rho H}$  and you have to determine at  $z = 0$ . So, this is  $\lambda L = 5$ .

So, at  $\lambda L = 5$  and  $z/L = 0$ ,  $K_{\rho H}$  is 1.0003. So, here we are calculating it for  $z = 0$  but if we are using it for at any  $z$ . So, according to that value or that coefficient value you have to consider. So according to that coefficient value you have to consider so here it is 1.0003. So now I will put this value this is for free-headed pile  $\frac{2H\lambda}{k_h d} \times K_{\rho H}$  and  $K_{\rho H} = 1.0003$  at  $z = 0$ .

Because we have to calculate its settlement at  $z = 0$ , we have calculated as 1.0003. So, we will get this is 2 this is 50 kN  $\lambda$  is 0.66  $k_h$  is  $70 \times 10^3 \times d$  is  $0.4 \times 1.0003$ . So, this value is 2.36

mm. So, the deflection of the pile head for free-headed pile is 2.36 mm by considering this is beam as finite beam. So, now, next one we will calculate for fixed-headed pile.

The same condition if the pile head is fixed, then how we will calculate that, now we will calculate what will be the moment that will be generated due to the fixity. So, moment due to fixity. So, that moment expression is given as  $\frac{H}{2\lambda} \left[ \frac{K_{\theta H(z=0)}}{K_{\theta M(z=0)}} \right]$ . Now, at  $K_{\theta H}$  at  $z = 0$  for  $\lambda L = 5$  this is 1.0003 and  $K_{\theta M}$  is 1.0002 at  $z = 0$ .

So, that we will put here. So, this  $H = 50$ ,  $\lambda$  is 0.66, this is 1.0003 this is 1.0002. So, that moment will be developed due to the fixity is 37.9 kN-m. Now, the next step is that, if the pile is free, then already deformation is 2.36 mm for free-headed pile, but now pile is fixed and due to the fixity, counter moment is developed and that value is 37.9 kN-m.

So, now, we have to calculate what would be the deformation for this moment, which is developed due to the fixity and if you subtract these moments, this deformation which is developed due to the fixity from the free-headed pile deformation, then I will get the deformation for the fixed-headed pile. Because that much moment  $m_0$  is developed due to the fixity and that is acting in the opposite direction to the  $H$ .

So, basically that is reducing the deformation of the pile head. So, definitely the deformation of the pile head under fixed-head condition will be less than the deformation of the pile head under free-head condition. So, that means, the deformation reduction due to the movement which is generated due to the fixity of the pile head. So, first you will calculate what is the deformation due to  $m_0$ ?

So, we have given the equation for the moment for this pile as finite beam and that deformation is  $\frac{2m_0\lambda^2}{k_h d} K_{\rho M}$  and for pile  $\lambda L = 5$  and  $K_{\rho M}$  is -1.0003. So, now we will put this value here. So, due to  $m_0$ , the equation is  $y = \frac{2m_0\lambda^2}{k_h d} K_{\rho M}$  and  $K_{\rho M}$  is -1.0003 so, it will be  $\frac{2 \times 37.9 \times 0.66^2}{70 \times 10^3 \times 0.4} \times (-1.0003)$ .



So, this value is - 1.18 mm. So, the net deflection that will be deflection due to the free end 2.36 mm and now, the reduction of the deflection due to the moment which is developed due to the fixity so, that is 1.18 mm.

So, the deflection of the fixed-headed pile at  $z = 0$  is 1.18 mm. So, this is the deflection of the pile head at  $z = 0$  for free-headed pile and this is the deflection at  $z = 0$  for fixed-headed pile. So, this can be done for any depth remember that, but when you calculate  $m_0$  these  $K_{\theta H}$   $K_{\theta M}$  can be anything depending on  $z$  value suppose you are doing at  $z = 1$  m.

But still you determine the  $m_0$  considering  $K_{\theta H}$   $K_{\theta M}$  at  $z = 0$ , but when you calculate the coefficient for the required depth, because the table is given for  $z/L$ . So, that if  $z = 1$  and  $L = 7.5$  then  $1/7.5$  you can determine for any depth, but these  $m_0$  you have to determine at  $z = 0$  remember that if I consider pile as finite beam.

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Pile as semi-infinite beam

i) Free-headed Pile

$$y = \frac{2H\lambda}{k_h d} D_{\lambda z} = \frac{2 \times 50 \times 0.66}{70 \times 10^3 \times 0.4} \times 1 = 2.36 \text{ mm}$$

ii) Fixed-headed Pile

$$m_0 = \frac{H}{2\lambda} = \frac{50}{2 \times 0.66} = 37.9 \text{ kN-m}$$

$$y(\text{due to } m_0) = -\frac{2m_0\lambda^2}{k_h d} C_{\lambda z} = -\frac{2 \times 37.9 \times (0.66)^2}{70 \times 10^3 \times 0.4} \times 1 = -1.18 \text{ mm}$$

$$\text{Net deflection} = 2.36 - 1.18 = 1.18 \text{ mm}$$

Now, if the next pile is as semi-infinite beam: The pile as semi-infinite beam so, you use the same problem, but here again for free-headed pile the equation of  $y = \frac{2H\lambda}{k_h d} \times D_{\lambda z}$  so,  $D_{\lambda z} = 1$  at  $z = 0$  but for any other  $z$ , I have discussed that how to calculate this coefficient for any depth or any distance. So, you have to convert this in radian.

So, that will be in radian and you have to convert it to degree when you use them. So, remember that, so, that means here now this is  $z = 1$ . So, I can put this is equal to 2,  $H = 50$  and  $\lambda$  is again 0.66 because that will not change whether be it is a semi-infinite beam or finite beam. So, lambda value is 0.66,  $k_h$  is  $70 \times 10^3$  and  $d = 0.4$  and  $D_{\lambda z} = 1$ .

So, these will be 2.36 mm which is the same value that we obtained when you consider pile as a finite beam. So, because as I mentioned if it is more than 2.5 then you will get more or less similar value. Now, if it is fixed-headed pile then the moment due to the fixity  $m_0$  is already been developed that is  $\frac{H}{2\lambda}$ . So, this is  $\frac{H}{2\lambda}$ . So,  $H$  is 50,  $\lambda$  is 0.66. So, this is 37.9 kN-m.

So, the deflection due to  $m_0$  will be minus that deflection due to  $m$  is given by  $-\frac{2m_0\lambda^2}{k_h d} C_{\lambda z}$ .

So,  $C_{\lambda z} = 1$  at  $z = 0$ . So, this will be  $-\frac{2 \times 37.9 \times 0.66^2}{70 \times 10^3 \times 0.4} \times 1$  and that is equal to - 1.18 mm.

So, the net deflection will be 2.36 - 1.18 that is 1.18 mm. So, we are getting the same value if we consider the pile as a semi-infinite beam or if we consider the pile as a finite beam because here the  $\lambda L$  value is 5. Now, you can check the same problem if the  $\lambda L$  value is 2 and then you solve it with semi-infinite beam and the finite beam then you will find there will be a difference between the deflection that you are getting for finite beam and the semi-infinite beam.

That you can check because in that case you have the result that you are getting by considering the pile as a finite beam that is the proper result because in that case you cannot model pile as semi-infinite beam. So, but both the cases whether the pile is finite beam or the semi-infinite beam where  $k_h$  is uniform. So, that mean this concept is more or less useful for cohesive soil where the  $k_h$  value is uniform.

So, in the next class I will discuss that how we can determine the settlement and other quantities like bending moment, shear force and slope for a pile if  $k_h$  value is not uniform, it is varying with depth. So, that will be discussed in the next class.