

Advanced Foundation Engineering
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Lecture - 54
Pile Foundation: Under Lateral Load and Uplift - IV

So, last class I have discussed that how we can determine the H_u value for fixed-headed pile in cohesionless soil.

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Piles in Cohesionless Soils: Fixed-Headed Piles

Short Pile:

Diagram shows a short pile fixed at the bottom and free at the top. A lateral load H_u is applied at the top. The soil reaction is shown as a triangular distribution with a maximum value of $3\ell^2 d^3 k_p$ at the bottom. The bending moment diagram shows a maximum moment M_{max} at the surface.

$$H_u = \frac{1}{2} \times 3\ell^2 d^3 k_p \times L$$

$$= \frac{3}{2} \ell^2 d^3 k_p \quad \text{--- (1)}$$

$$M_{max} = \frac{1}{2} \times 3\ell^2 d^3 k_p \times L \times \frac{2}{3} L$$

(Taking moment w.r.t surface)

$$M_{max} = \frac{2}{3} H_u L \quad \text{--- (2)}$$

And I have discussed 2 cases short pile and the intermediate piles and this is the short pile. So, these are the 2 equations by which we can determine the H_u and the maximum moment and in this case maximum moment will be at the surface.

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Intermediate Pile:

Diagram shows an intermediate pile fixed at the bottom and free at the top. A lateral load H_u is applied at the top. The soil reaction is shown as a triangular distribution with a maximum value of $3\ell^2 d^3 k_p$ at the bottom. The bending moment diagram shows a maximum moment M_{max} at the surface and a yield moment M_y at the bottom.

$$H_u = \frac{1}{2} \times 3\ell^2 d^3 k_p \times f$$

$$f = 0.82 \sqrt{\frac{H_u}{\ell^2 d^3 k_p}}$$

$$F = \frac{1}{2} \times 3\ell^2 d^3 k_p L - H_u$$

$$= \left(\frac{3}{2} \ell^2 d^3 k_p \right) - H_u \quad \text{--- (1)}$$

Taking moment w.r.t surface

$$F \times L - \frac{1}{2} \times 3\ell^2 d^3 k_p L \times \frac{2}{3} L = M_y$$

(Intermediate pile) (Yield moment)

other case it is long pile.

$$M_{max} < M_y$$

(Intermediate pile)

$$M_{max} > M_y$$

(Long pile)

$$f = 0.82 \sqrt{\frac{H_u}{\ell^2 d^3 k_p}} \quad \text{--- (3)}$$

$$M_y = \left(\frac{3}{2} \ell^2 d^3 k_p - H_u \right) L - \ell^2 d^3 k_p$$

$$= 1.5 \ell^2 d^3 k_p - H_u L - \ell^2 d^3 k_p$$

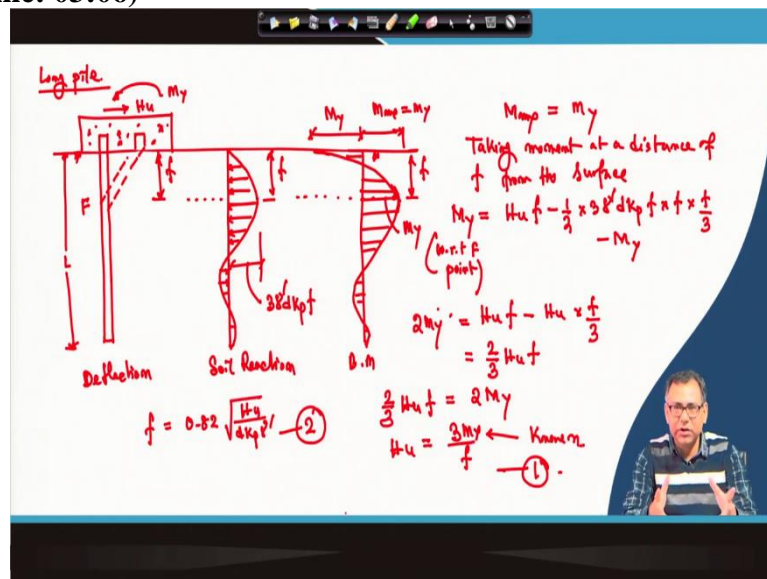
$$= 0.5 \ell^2 d^3 k_p - H_u L \quad \text{--- (2)}$$

And this is the condition by which we can determine what is the maximum lateral load capacity of the pile and we can determine the M_{\max} also which will act at the depth f and then as we know we can determine f because f expression was already given in previous classes, so, that f expression is which is for free-headed pile part that is $f = 0.82 \sqrt{\frac{H_u}{\gamma' d K_p}}$.

So, how we are getting that f expression by equating these equations because we are taking this value here because that we did for the free-headed file. So, this is nothing but $3\gamma' f d K_p \times f$. So, that force will be equal to H_u . So, that $H_u = 3\gamma' f d K_p \times f$. So, here we can write that f nothing but $0.82 \sqrt{\frac{H_u}{\gamma' d K_p}}$.

So, by using this equation I can determine once I get a H_u by equation 2 then I can put these H_u in this equation 3 I will get the f now once I know the f now with respect to this F point I can calculate the M_{\max} what is the M_{\max} then I will check whether it is less than M_y or not if it is less than M_y then fine intermediate pile otherwise it is a long pile.

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So, now, I will go to the long pile case. So, for the fixed-headed long pile case this is the ground surface and this is the fixity of the pile and this is the long pile and in case of long pile there will be yielding at a point and this is the pile cap this is L and the yielding occurs at a distance of f and then the soil reaction. So, again this will be something like that.

This is the value which is $3\gamma' f d K_p$ because this is f and these values basically at F where the

maximum bending moment also will occur at F and now there will be a yield moment at the surface as well as at the point F this is the maximum yield moment. So, I should write this is the point. So, this is the bending moment diagram. So, this will be M_y this is a long pile and this is also M_{\max} which is equal to M_y and this is acting at a depth f . So, this is the deflection.

This is a soil reaction and this is the bending moment this is M_{\max} . So, this is bending moment this is the deflection and this is soil reaction. So, these are the different diagrams, so, here also I have drawn all the cases. So, now for the fixed-head case, what are the values, I should write that I have to calculate the H_u and there will be a yield moment M_y in this fixed-head case and then now for the fixed-head case M_{\max} at the point F definitely that will be equal to M_y .

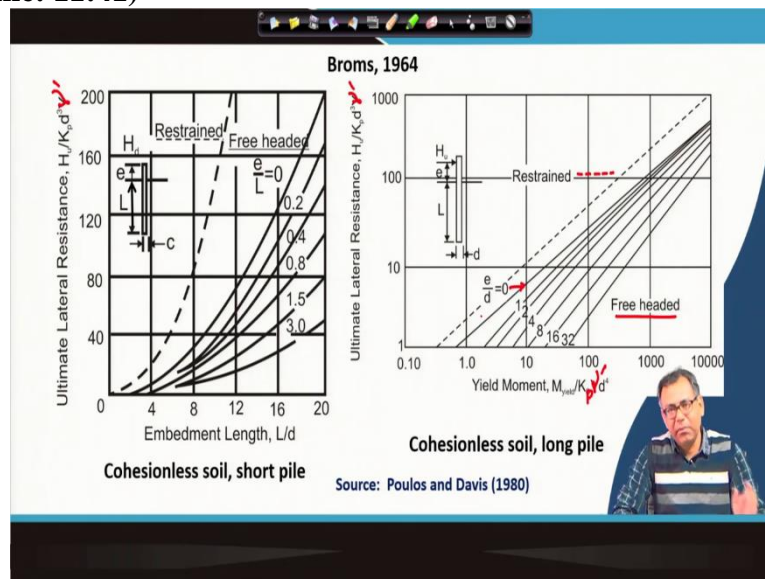
Now, we are taking the moment M_y at a distance f from the surface. So, H_u is now it is at a distance of f and at that point this M is nothing but M_y . So, M_y at that point will be equal to $H_u \times f$ then the negative contribution and again we assumed that up to F point this is the linear portion and that point is total force will be $\frac{1}{2} \times 3\gamma' f dK_p \times f$ and we are taking from this point F.

So, if I considered it is a triangle above the F point so, then this will be $f/3$ then the moment M_y is acting at the surface is $-M_y$. So, this is with respect to F point. So, this is the moment with respect to F point and that is also yield moment. So, we can write this will be $2M_y = H_u f - H_u \times \frac{f}{3}$.

So, $\frac{3}{2}\gamma' dK_p f^2$ this is equal to H_u , so, this will be $\frac{2}{3}H_u f$. So, I can write that $\frac{2}{3}H_u f = 2M_y$ or I can write $H_u = \frac{3M_y}{f}$ where M_y is again known. So now how we will get the f value again f value is equal to the same expression because here also we assumed this is the linear above the F point.

So, soil reaction is linear. So again, $= 0.82 \sqrt{\frac{H_u}{\gamma' dK_p}}$. So, this equation 1 will give us the H_u equation 2 also give us the f value. So, we have 2 unknowns, f is one unknown, H_u is another unknown when we have 2 equations we will solve and we will get the H_u and maximum moment is now the yield moment, so that is known. So, this way, we can determine H_u and the

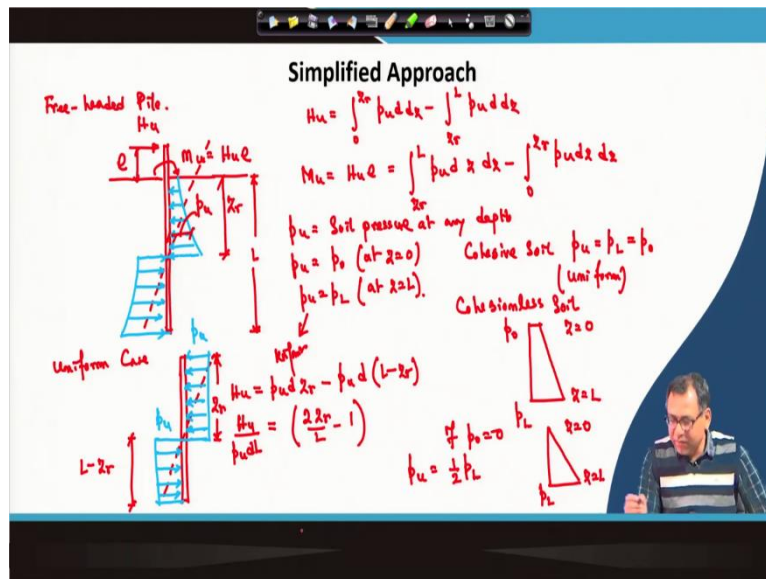
position of maximum moment for long pile and so that means here also I have derived for cohesionless soil. I have discussed the equations for the 5 cases what are the possible equations
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So, either you can use those equations and based on those equations, again, the charts are given. So, you can use these charts also. That means for the fixed-headed pile, this is the dotted line, free-headed pile firm lines are there L/d again less than or equal to 20 then I will get $\frac{H_u}{K_p d^3 \gamma'}$. So, this is $\frac{H_u}{K_p d^3 \gamma'}$. Here also we should know the K_p value.

So, here also I will get $H_u K_p d^3 \gamma'$ and this dotted line is for the fixed-headed pile and firm line for the free-headed pile. This is the firm line. So, this is 0 and this is 1. So, 0, 1, 2, 4, 8, 16, 32 for free-headed pile and fixed-headed pile. This is dotted line. So, either you can use these charts or the equations. So, we can determine the ultimate lateral load carrying capacity of piles and the moments and the positions of the maximum moment by using Broms's approach.

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Then I will discuss another approach which is called the simplified approach. So, in this simplified approach, which is very I mean obviously, it is simplified means is very simple, what is that approach and that is for free-headed pile. So, that simplified approach suppose this is a free-headed pile, this pile is subjected to a lateral load H_u with eccentricity e . So, there will be a moment also which is M_u which is nothing but $H_u \times e$.

So, it is assumed that this simplified approach then this pile will definitely rotate because it is a free-headed pile. So, it will rotate like this. So upper portion there will be a passive resistance the lower portion there will be passive resistance so that we will see here the distribution is something like that. So, at this point there will be a change of the distribution. So, this is the distribution and for that this is the distribution.

So, only this distribution is assumed. So, based on that we will calculate what would be the ultimate load carrying capacity of the pile. How we will calculate by this distribution and showing one particular case that H_u will be equal to this net reaction. So, suppose this is the Z_r , where the Z_r is the point of rotation where the soil distribution changes from this passive resistance in the upper portion or within the Z_r portion.

It is acting from right to left and below it is acting from left to right this is the other passive resistance. So, we can write that this is equal to $\int_0^{Z_r} p_u dz$ and then - because this is in opposite direction it is $\int_{Z_r}^L p_u dz$ and similarly, I can write that maximum moment, $M_u = H_u \times e$ because now moment we are taking in the opposite sign so, it will be - or I can take this is first.

So, $\int_0^{Z_r} p_u dz - \int_{Z_r}^L p_u dz$. Now, what is p_u ? p_u is the soil pressure and it varies from p_0 to some value. So, that means here p_u is the soil pressure at any depth. So, $p_u = p_0$ at $z = 0$ and $p_u = p_L$ at $z = L$. So, at any point this is p_u which is soil pressure and then with that this unit is kN/m^2 and if I multiply with d it will be kN/m that we are doing for all the cases.

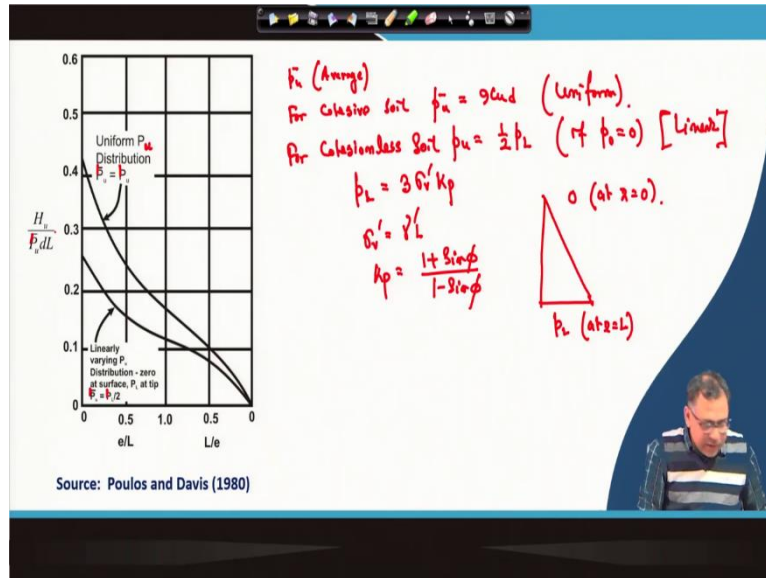
Now, this p_u distribution as I have discussed already that for cohesive soil this $p_u = p_L$ that means it is uniform or you can write p_0 also. That means, it is uniform, but for cohesionless soil this is not uniform. This is varying linearly from p_0 to p_L . So, p_u varies linearly from p_0 to p_L . So, p_0 at $z = 0$ and p_L at $z = L$. So, p_u is varying linearly with length of the pile and so, that means, if $p_0 = 0$ then the variation will be something like this, this is p_L .

So, in such case p_u will be the average pressure because this is 0. Now, $p_u = \frac{1}{2}p_L$ showing that case if this is $z = 0$ this is $z = L$. So, now for uniform case, so, that means for uniform case, the distribution of the pressure will be something like this. This is the point of rotation distribution will be like this and it is not in scale definitely. So, this will be the distribution for uniform case.

This is always p_u , this is also p_u , so it is also for this I should write that. For uniform case how we can calculate the H_u expression. So, you can calculate the H_u will be equal to that this is Z_r . So, this will be your $p_u \times d \times Z_r - p_u \times d \times (L - Z_r)$. So, that is the total net reaction and that is equal to H_u . So, that means we have $L - Z_r$. So, finally I will get our expression in terms of $\frac{H_u}{p_u d L}$.

So, that is equal to $\frac{2Z_r}{L} - 1$. So, this way if I know the Z_r and if I know the point of rotation then I can calculate the H_u now, to know the depth of point of rotation Z_r from the top, we have to take the moment. So, in this way we can do it for uniform distribution case we can do it for the linear case also.

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So, directly I have already given the linear case distribution. So, that will also shift from this side to that side. So, that means for the linear case so, distribution will be so, it will go something like this then it will be like something like that. So, that means here this is Z_r and this is p_L and this is the soil reaction. So, for this particular case, we can get the net reaction and that will give me the H_u and for determine Z_r we have to take the moment.

So, and that equations are given. So, based on those equations, I have just discussed about the concept that how simplify approach is used and then I have not given the equation because I have discussed the concept. So, based on that concept equations are derived and then based on those equations, these charts are developed. So, we can use this chart and then we can get the H_u this \bar{p}_u is basically the average.

Now for cohesive soil \bar{p}_u which is uniform and that value is given $9c_u d$ that is for cohesive soil which is uniform. And for cohesionless soil \bar{p}_u is nothing but $\frac{1}{2} \times p_L$ as I mentioned that if your $p_0 = 0$ and $p_L = 3\sigma'_v K_p$ because that is assumed for the cohesionless soil and similarly, I can write that $\sigma'_v = \gamma' \times L$ and $K_p = \frac{1 + \sin\phi}{1 - \sin\phi}$.

So, this is linear. So, you will get these values. So, once I get this value, I will get the H_u value because I have to calculate \bar{p}_u . So, a uniform, \bar{p}_u distribution, and this curve is for linear distribution. So, distribution is 0 at the surface to p_L at the tip so, that means the distribution is something like this. So, that means, this is p_L at $z = L$ and this is 0 at $z = 0$. So, this is the distribution.

So, \bar{p}_u will be $\frac{1}{2} p_L$. So, p_L I will get by using these equations and then once I get the e/L value, then for different cases where the uniform or the linear I will get $H_u \bar{p}_u d L$ I know L I know \bar{p}_u I will get from these equations, so, I will get the H_u which is simple.

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A H steel pile (HP) of following properties and length 20m is embedded in (a) granular soil and (b) clayey soil. Yield stress of the pile is 340 MN/m² (F_y). The water table is 2m below the ground surface. Unit weight of the soil above and below water table are 18 kN/m³ and 20 kN/m³. (a) $\phi=40^\circ$ and $c=0$ (b) $\phi=0$ and $c=100$ kN/m². $e=0$. Determine the H_u .

$d = 250 \text{ mm}$, $L = 20 \text{ m}$ $\frac{1}{d} = \frac{20}{0.25} = 80$
 Long pile.

$d_1 = 246 \text{ mm}$
 $d_2 = 256 \text{ mm}$
 $I_{xx} = 87.5 \times 10^{-6} \text{ m}^4$
 (given)
 Size = 250 mm
 Weight = 0.608 kN/m

So, let us see which approach is giving a better result or the result which we can use for our design. So, this is actually H_u . So, the example problem that I have chosen that a H section steel pile of following properties and the length of 20 m is embedded in a granular soil two cases that this H section pile is embedded in a granular soil and in a clayey soil and yield stress of the pile is 340 MN/m².

So, for every material pile this yield stress is given. So, yield stress is known, water table is 2 m below the ground surface unit weight of the soil above and below water table are 18 kN/m³ and 20 kN/m³, respectively. So, for the case a friction angle is 40° and cohesion is 0 case b friction angle is 0 cohesion is 100 kPa eccentricity is 0 determine the H_u .

So, for this H section which is given, so, I am giving the details of that section. So, it is a steel pile with H section, this is the H section this is the Y-Y axis on this is the X-X axis. Now, there are different terminologies. So, this is the d_1 which is the depth of this section and this is the d_2 which is width of this section. So, these d_1 and d_2 these values are available depending upon which section you are using.

So, for this particular section d_1 value is equal to 246 mm and d_2 value is equal to 256 mm

flange width is also given. So, this is the width so, that is 10.6 mm, all the values are given for a particular section now, the moment of inertia of this section about the X-X axis that is also given $87.5 \times 10^{-6} \text{ m}^4$.

So, this is also given. So, all the values that are written are all given. Now, the size of this section, equivalent size of this section that is also given and that is 250 mm. Now, weight is also given and weight is 0.608 kN/m. So, these all values are given now, we have to determine the ultimate lateral load, H_u this pile can take. So, this is the section and these are the details of this section in the question the soil properties are given.

So, this side all the values are given. Now, we have to calculate the values that we are asked to determine. So, that is H_u . Now, before I calculate the H_u . So, we should know what is the yield moment? Because that if it is a long pile or short pile based on that you have to use the yield moment. So, if the size is given 250 mm. So, that means diameter we will take as 250 mm because size means the equivalent width or the diameter that we are taking that 250 mm and L is given here 20 m.

So, $L/d=20/0.25$ and hence it is greater than 20. So, these will be around 80. So, 80 definitely it will be a long pile and if it is close to 20 or 30 or something then also, we can consider intermediate pile but this is 80 means definitely it will be a very long pile, so, this is a long pile. So, we have to calculate the yield moment, because as I mentioned in during my derivation also the long pile primarily the ultimate load carrying capacity depends on the yield moment of the past.

So, that yield moment you have to determine, so, we have written all the known values. So, in next class from these known values, I will determine the yield moment because it is a long pile and based on that yield moment, I will determine what will be the value of H_u for different cases granular soil as well as the clayey soil and I will use both approaches Broms's theory as well as the simplified approach. Thank you.