Advanced Foundation Engineering
Prof. Kousik Deb
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture No -31
Beams on Elastic Foundation – IV

So, last class I have discussed about the semi-infinite beam subjected to concentrated load. Now today I will first solve two example problems, one is infinite beam subjected to a concentrated load *P* and one is semi-infinite beam subjected to a concentrated load *P*.

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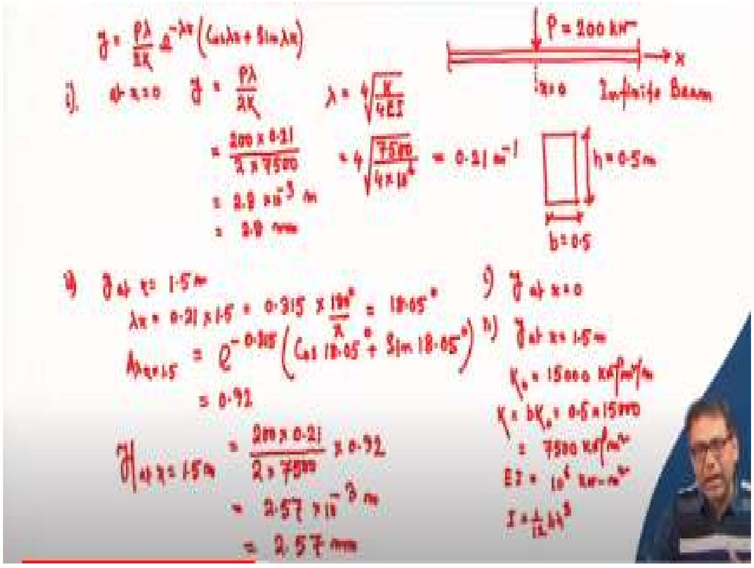

So, first problem that will discuss is an infinite beam. So, this is an infinite beam subjected to a concentrated load *P* = 20 kN and as I mentioned the point where the concentrated load is applied in case of infinite beam is considered at, $x = 0$ point. So, this is $x = 0$ point. So, it is in case of infinite beam. So, the width of the beam is 0.5 m that means *b* = 0.5 m and also, height of the beam, *h* = 0.5 m. So, it is a square cross section.

Now, we have to determine two things; one is settlement at $x = 0$ and another is settlement at $x = 1.5$ m. Now in this case, it is a settlement so, as I have already discussed left side of the loading or right side of the loading does not matter because the settlement will remain same. So, in case of concentrated load in infinite beam settlement will be same whether it is left side of the loading or right side of the loading.

So, the settlement and bending moment are same but the slope and the shear forces are opposite. So, now the k value which is given basically it is k_0 which is given as 15,000 kN/m^2/m. So, that means my $k = b \times k_0$. So, b = 0.5 and k_0 is 15,000. So, this value will be 7500 kN/m^2 and EI value is directly given and that is 10^6 kN/m^2.

So, suppose in this particular problem EI value is directly given but in other cases the E will be only given that means elastic modulus of the beam material but you have to calculate I. So, you know that $I = \frac{1}{12}bh^3$. So, here, you know, the b and h values. So, you will get the I and if you multiply I with E you will get the EI. So, that is also another option. So, now here these properties are given.

So, now the equation for deflection of infinite beam subjected to concentrated load is $\frac{P\lambda}{2k}e^{-\lambda x}(\cos \lambda x + \sin \lambda x)$. Let it be $A_{\lambda x}$. Now at $x = 0$ that is our first case. So, that will be equal to $\frac{P\lambda}{2k}$ only because at $x = 0$ this $\sin 0$ is 0 and $\cos 0$ is 1 and e^0 is 1. So, it will be $\frac{P\lambda}{2k}$. So, we have to calculate the λ value, the λ value is $\sqrt[4]{\frac{k}{4E}}$.

So, this k value is 7500, so $\lambda = \sqrt[4]{\frac{7500}{4\times10^6}}$. So, this is 0.21 m^{-1}. So, we can write that at $x = 0$ P = 200 then $y = \frac{200\times0.21}{2\times7500}$, which is 2.8 × 10^{-3} m. So, that is 2.8 mm. So, this can be positive or negative depending upon where you are calculating the value. Definitely at $x = 0$ settlement will be positive, positive means the downward settlement.

So, and this sign will automatically come where you are taking the value based on your x value. So, now the next problem we will get y at x = 1.5 m. So, at x = 1.5, the λx value = 0.21 × 1.5 so, this is 0.315. So, 0.315 if I multiply it by $\frac{180°}{\pi}$ because this λx when you will use in sine and cos you have to convert it to degree because the λx is in radian. So, we have to convert it to degrees. So, this value is 18.05°.

So, I can write that $A_{\lambda x=1.5} = e^{-0.315}(\cos 18.05° + \sin 18.05°) = 0.92$. So, this value is coming out to be 0.92. So, y at x = 1.5 m. So, this will be $\frac{P\lambda}{2k}$ and this term. So, P is 200, λ is 0.21 divided by 2 then 7500 then $e^{-\lambda x}$. So, that part I have already calculated. So, this $e^{-\lambda x}$ means $e^{-0.315}$ I have already calculated.

So, I have to multiply with 0.92. So, this value is coming out to be 2.57 × 10^{-3} m which is 2.57 mm. So, this way I can calculate or we can calculate the bending moment, shear force or slope at any point from the loading either it is right side or left side. So, all the expressions as I have already discussed for a right side of the loading; that means $x > 0$.

But, if you want to determine the values at left side of the loading; that means $x < 0$ then you have to apply that plus minus sign as I have discussed. So, next problem that I will discuss is for semi-infinite beam, I will take the similar type of problem, but now the beam is semi-infinite beam.

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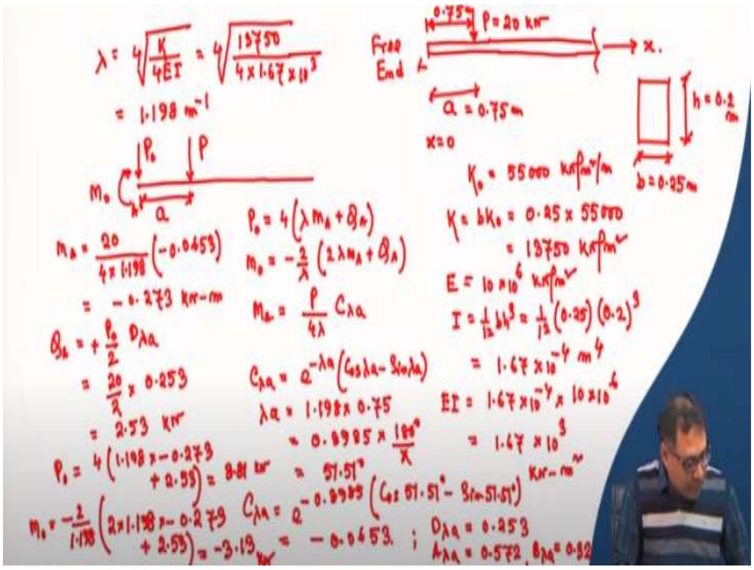

So, this is the semi-infinite beam. So, it is infinite in this direction. So, here it is the free end beam. So, this is free end. So, that free end is A point or we can say it is O point and we are applying one concentrated load P which is 20 kN at a distance of 0.75 m. So, that means a distance, so this is I can say this is a, so a is 0.75 m. So, this point is A where the load is acting but in this case for the semi-infinite beam that free end is considered at $x = 0$.

So, now here again k_0 value is given 55,000 kN/m^2/m then the cross-section area of the beam *b* = 0.25 m and *h* = 0.2 m. So, now we can say that $k = b \times k_0$. So, this is 0.25 × 55,000 = 13750 kN/m^2. So, *E* is given here 10 × 10^6 or 1 × 10^7, we can calculate $I = \frac{1}{12} bh^3$. So, this is $\frac{1}{12}$ *b* is 0.25 then *h* is 0.2 to the power cube.

So, this is 1.67 × 10^{-4} m^4. So, $EI = 1.67 \times 10^{-4} \times 10 \times 10^6$. So, this is 1.67×10^3 kN/m^2. So, the λ value I can calculate $\sqrt[4]{\frac{k}{4EI}}$. So, k = 13750 and *EI* value is 1.67×10^3. So, this value is 1.198 m^{-1}.

So, as I have already discussed in case of infinite beam that if this is the *P* which is acting at a distance of a. So, first we have to calculate M_A and Q_A for free end then apply end conditioning forces such that the net moment and the shear force will be 0 at free end. So, that means here we have to apply one concentrated load P_0 and moment M_0.

So, now the expression of this P_0, I have already derived the expressions, $P_0 = 4(\lambda M_A + Q_A)$ and $M_0 = -\frac{2}{\lambda}(2\lambda M_A + Q_A)$. So, what is M_A and Q_A? M_A is the moment induced due to the load *P* if this beam is considered as infinite beam, Q_A is the shear force induced due to the load *P* if these beams are considered as infinite beam but now it is not actually infinite beam, it is the semi-infinite beam.

So, at free end, we have to apply end conditioning forces such that the M_A and Q_A will vanish because that is the end condition that *P* and end condition moment should be 0 and shear force should be 0. So, now first we have to calculate the M_A due to the load *P* will be equal to $\frac{P}{4\lambda} C_{\lambda a}$. So, I am showing one calculation that how to calculate $C_{\lambda a}$.

Because $C_{\lambda a} = e^{-\lambda a}(\cos \lambda a - \sin \lambda a)$. So, λ is 1.198 and $a = 0.75$. So, $\lambda a = 0.8985$ and that you have to convert to degree, so, this value is 51.51°. Now if I calculate $C_{\lambda a}$ that will be $e^{-0.8985}(\cos 51.51° - \sin 51.51°)$. So, this is equal to -0.0653. Similarly, we can calculate $D_{\lambda a}$ that is = 0.253.

Because $D_{\lambda a} = e^{-\lambda a}\cos\lambda a$. Similarly, we can calculate $A_{\lambda a}$ which is equal to 0.572 and I can calculate $B_{\lambda a}$ also which is equal to 0.32. Similarly, $A_{\lambda a} = e^{-\lambda a}(\cos\lambda a + \sin\lambda a)$ and $B_{\lambda a} = e^{-\lambda a}\sin\lambda a$.

So, if we put all these values then I will get $A_{\lambda a}$, $B_{\lambda a}$, $C_{\lambda a}$, $D_{\lambda a}$. So, now the M_A value is $\frac{P}{4\lambda}C_{\lambda a}$. So, $M_A = \frac{20}{4\times 1.198}\times(-0.0653)$. So, here M_A is - 0.273 kN-m. Now, similarly I can calculate $Q_A = +\frac{P_0}{2}D_{\lambda a}$. Now why it is plus P_0? Because as I mentioned that if you are calculating any value for concentrated load, left side of the loading then for shear force and the slope this sign will be opposite.

What are the given equations? That is for right side of the loading. So, here you can see that A point is this point free end. So, I can write this is free end point, free end point is A. So, free end point is left side to the loading. So, here shear force will be opposite sign and the slope will be opposite sign. So, that is plus $= +\frac{P_0}{2}D_{\lambda a}$. So, it will be P_0 is 20 divided by 2, now $D_{\lambda a}$ is 0.253. So, this value will be 2.53 kN.

So, now I have calculated the M_A and Q_A that is due to this point load *P* because this M_A is due to this point load *P* at a distance *a*. So, we are calculating left side to the loading. So, that is the expression, $M_A = \frac{P}{4\lambda}C_{\lambda a}$ because these expressions are already derived for concentrated load and concentrated moment. So, from there I am getting this value but *Q* will be the opposite sign because it is left side to the loading.

So, now $P_0 = 4(\lambda M_A + Q_A)$. Now λ is 1.198, then M_A is - 0.273, then Q_A is 2.53. So, this value is 8.81 kN. Similarly, $M_0 = -\frac{2}{\lambda}(2\lambda M_A + Q_A)$. Now, λ is 1.198, then M_A is - 0.273, then Q_A is +2.53. So, this is equal to -3.13 kN. So, now we have calculated the end conditioning forces P_0 and M_0. So, these values are 8.81 kN and -3.13 kN, respectively.

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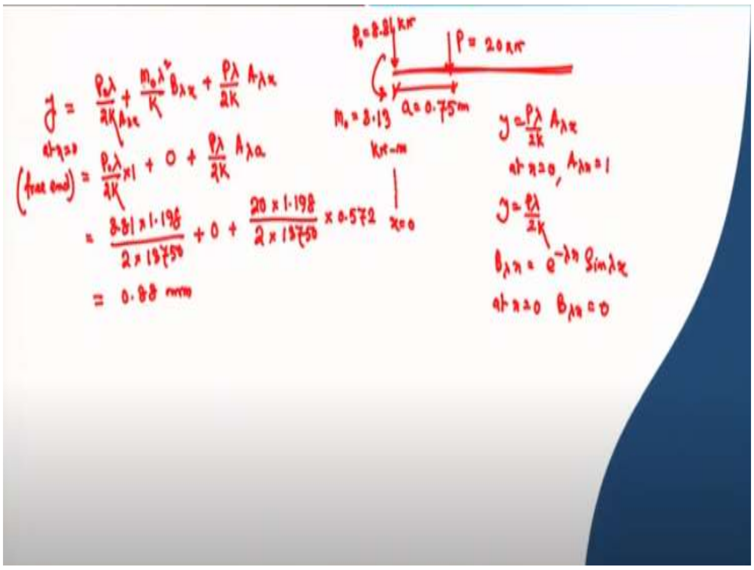

So, now finally, I will get these forces. So, $P = 20$ kN and this distance is a = 0.75 m. Now we have P value, P_0 is 8.81, so P_0 is acting here, that is P_0, which is 8.81 kN and we have M_0, and M_0 is negative. So, that is why I am giving this sign. So, M_0 is - 3.13 kN-m. So, as we discussed that clockwise moment to the left side is positive and anticlockwise moment to the right side is positive but here it is negative moment, so M_0 is -3.13.

So, that is acting on the left side. So, that is why it is shown as a anticlockwise moment. So, now I want to calculate the settlement at $x = 0$. So, that means I have to calculate the settlement at $x = 0$ and $x = 0$ means the free end, so what is the settlement at free end? So, now we have three forces basically two concentrated loads and one concentrated moment and that is also acting in anticlockwise direction.

Because of these three forces, we have to calculate the settlement at free end. So, now one by one, I am calculating this value so for P_0 at $x = 0$, that means the settlement due to P is $\frac{P\lambda}{2k}A_{\lambda x}$. Now at $x = 0$, $A_{\lambda x} = 1$. So, what will be $\frac{P\lambda}{2k}$? So, that means here due to this P_0 this will be $\frac{P_0\lambda}{2k}$ then because of this moment also I can get $\frac{M_0\lambda^2}{k}B_{\lambda x}$ +, so here also I am writing $A_{\lambda x}$. So, later on, I will put $A_{\lambda x} = 1$.

So, but I am not writing general equations. So, for this concentrated load $\frac{P_0\lambda}{2k}$, for this concentrated moment $\frac{M_0\lambda^2}{k}B_{\lambda x}$ and then for this concentrated force P, $\frac{P\lambda}{2k}A_{\lambda x}$. So, now I can write at $x = 0$ because P_0 is acting at $x = 0$ and I want to determine the y at $x = 0$.

So, I can write $\frac{P_0\lambda}{2k} \times 1$ + this is 0, because $B_{\lambda x} = e^{-\lambda x}\sin\lambda x$. Now at $x = 0$ this $B_{\lambda x} = 0$ because $\sin\lambda x$ is 0. So, $\sin 0$ is 0 and then this is $\frac{P\lambda}{2k} \times x$ is a, a is 0.75 m. So, this will be $aA_{\lambda a}$. So, I can write that $y = \frac{8.81\times1.198}{2\times13750} + 0 + \frac{20\times1.198}{2\times13750} \times 0.572$.

So, this value is coming out to be 0.88 mm. So, this is the settlement at $x = 0$ or free end that is 0.88 mm. So, in this way, we can calculate the settlement or bending moment or shear force or the slope at any point but we have to consider these three forces, one is the P which is the external concentrated load that is applied and another two are the end conditioning forces to make the net bending moment and the shear force at free end zero.

So, that means we can calculate what would be the settlement at the point where the load P is acting, so that also we can calculate. So, in that case, you consider the point is on the right side to the P_0 and the moment and point is just below the P. You can calculate any point right side to P also. So, anything you can do by using these equations. Now, remember that one thing I want to mention here the second term is coming as zero.

So, that is why we can just make the whole term 0 but you can see here M_0 is written. So, M_0 when you write you do not write 3.13, if suppose it is not 0, that means $B_{\lambda x}$ is 0 at $x = 0$, it may be some value but in such case M_0 should be -3.13 because this is a general expression and here the M_0 is clockwise moment because when it is a divided expression for moment; so, that expressions are valid for clockwise moment but here your moment is anticlockwise. So, we have to put minus sign here.

So, that is why when you put M_0 do not put 3.13 because that is if the moment is clockwise then it will be 3.13 but the moment is anticlockwise. So, here you have to write -3.13 but in this case, it is 0. So, we are not thinking about the value of M_0, but if the $B_{\lambda x}$ is not 0 then remember that because it is $-M_0$is minus. So, you have to put minus value, clear? So, the next class I will discuss the beam with infinite length.

Then, I will discuss about the beam resting on two-parameter model and then I will discuss about the finite difference method in the coming classes. Thank you.