Advanced Foundation Engineering
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Lecture No -30
Beams on Elastic Foundation – IV

So, last class I have discussed that how you will get the shear force, bending moment, settlement and slope, when infinite beam is subjected to a concentrated load. Now today I will discuss if the infinite beam is subjected to a concentrated moment then how I will get all these equations. So, quickly I will discuss that because this moment and this concentrated load these are very important.

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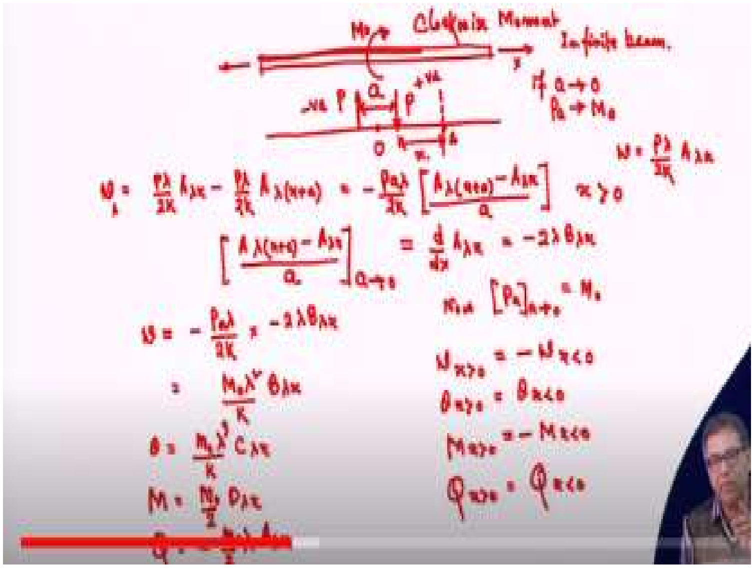

By using these two equations, I will get the other equations also for different conditions, like finite beams, semi-infinite beam cases. So, suppose this is the infinite beam which is subjected to a concentrated moment M_0 or M_0 is acting. So, this is *x* axis and this is also infinite beam. So, now this case can be drawn or can be represented in this form also because this is a concentrated force *P* whose distance is *a*.

So, I can write say suppose this is your O point, I can write that if a tends to 0, then Pa tends to M_0 that means I am representing this moment into this form. So, now what will be the deformation due to this condition? So, deformation *w* will be because now it is an infinite beam.

So, I am considering where this positive this lower direction force is acting then that is equal to or so I can write this distance of this point.

At any point say this is the point where *P* is acting and the distance from this point to another *P* concentrated load point is a but one case it is acting downward direction another case it is acting upward direction. So, now suppose we have this distance from this downward load acting force is *x* and I want to determine the deflection at this point. So, deflection at this point, say A. So, deflection at A point will be what?

So, deflection equation, you know the deflection equation for a concentrated load is $\frac{P\lambda}{2K}A_{\lambda x}$ that is the equation. So, here one point is $\frac{P\lambda}{2K}$ and this is $\frac{P\lambda}{2K}$ and upward acting concentrated load is considered as positive, sorry downward acting concentrated load is considered as positive this is downward and upward acting is considered as negative.

So, moment I have given what is the sign convention that the clockwise moment in left side is positive whereas the anticlockwise moment in your right side is considered positive. And the upward acting concentrated force is negative and downward acting concentrated force is positive. And remember that all the equations that will be derived in this way, is valid for clockwise moment. So, this is for clockwise moment.

And then, so that means here it is upward acting so it will be minus. So, $\frac{P\lambda}{2K}$, now this will be $A_{\lambda(x+a)}$, because that is *x* distance of $x + a$. So, I can write this equation as $-\frac{P\lambda}{2K}$. So, if I take basically $\frac{P\lambda}{2K}$, then this will be $-k$ have taken $A_{\lambda(x+a)} - A_{\lambda x}$ are taken a; so, we have to divide by a, this is the equation. Again, these equations are valid for $x = 0$ that means for the right side.

So, now I can write that for $\frac{A_{\lambda(x+a)}-A_{\lambda x}}{a}$, if a tends to 0 that means this will be equal to $\frac{dA_{\lambda x}}{dx}$ and $A_{\lambda x} = e^{-\lambda x}(\cos \lambda x + \sin \lambda x)$. So, if you differentiate then you will get this is equal to $-2\lambda B_{\lambda x}$. So, this $A_{\lambda x}$, $B_{\lambda x}$, $C_{\lambda x}$ and $D_{\lambda x}$ all are given. So, this is $B_{\lambda x}$.

Finally, I can write that your w at any point because initially I have taken one point. Now it is any point will be equal to $-\frac{Pa\lambda}{2K} = -2\lambda B_{\lambda x}$. Now, for Pa tends to M_0 as a tends to 0, I mentioned. So, this 2 and this 2 will cancel out and this will be $\frac{M_0\lambda^2}{k} B_{\lambda x}$. So, this is the equation of deflection for the concentrated clockwise moment in right side.

Similarly, I can write the slope equation and that is $\frac{M_0\lambda^3}{k} C_{\lambda x}$ and bending moment is equal to $\frac{M_0}{2} D_{\lambda x}$, shear force is equal to $-\frac{M_0\lambda}{2} A_{\lambda x}$. So, these are the equations for concentrated clockwise moment. So, you can get these equations as I have discussed in concentrated load case, thus if you do $\frac{dw}{dx}$, you will get the slope then $\frac{d^2w}{dx^2}$ will give you moment then $\frac{dM}{dx}$ is the shear force.

Now again, all these equations are valid for $x \geq 0$. Now in this case your $w_{x>0} = -w_{x<0}$ then if it is other side now previous case concentrated load case the w and bending moment, slope, there is no change but now there is the opposite sign in case of slope, in case of deflection and the bending moment because previously in deflection and bending moment, there is no change.

Now in deflection and bending moment there will be opposite sign but for the slope and the shear force they will remain same. So, now this is the case and for the slope $\theta_{x>0} = \theta_{x<0}$ and bending moment $M_{x>0} = -M_{x<0}$ and $Q_{x>0} = Q_{x<0}$. So, now these are very important. So, I have discussed the bending moment and concentrated force.

So, concentrate force upward direction and bending moment in clockwise direction. Now if it is opposite suppose concentrated force is acting in downward direction are given and the bending moment is given as clockwise moment. Now if it is an opposite concentrated moment, load is acting in upward direction and bending moment is acting in anticlockwise direction then all the equation you have to put minus sign, that means in such case $w = -\frac{M_0\lambda^2}{k} B_{\lambda x}$if $x \geq 0$.

Now accordingly for $x \leq 0$, then you have to modify your equations like the cases I have explained. So, only if the concentrated load is acting in upward direction or moment is acting in anticlockwise direction. So, in case of P or P_0, it will be $-P$ or $-P_0$ or in case of M_0, it will be

$-M_0$ that is it.

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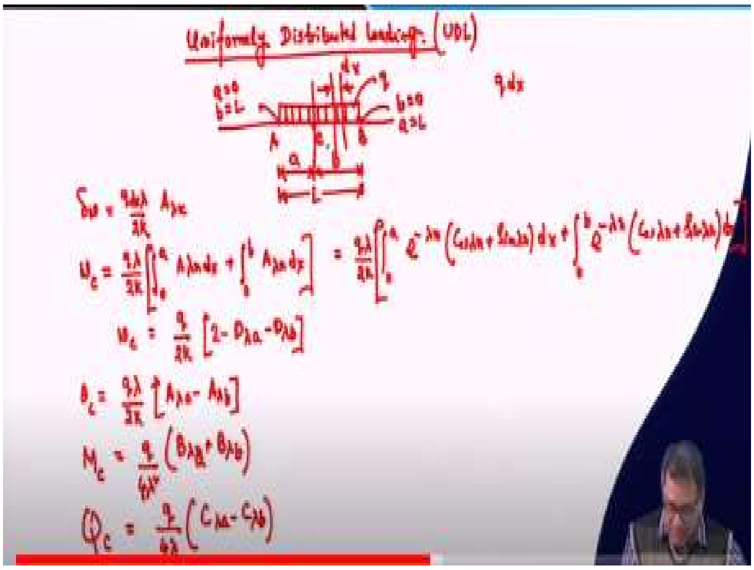

Now I will give the idea if it is a uniformly distributed load. So, uniformly distributed load case also you can calculate if you have a uniformly distributed load here. Now, if you have points A and B and your point of interest is C at this point which is at a distance of say *a* from the A point and *b* from the B point and this uniformly distributed load is *q* and you have to take a small segment of *dx*.

So, because of this *dx* there will be concentrated force acting and that will be $q \times dx$. So, that means this UDL is considered as a number of concentrated forces of $q \times dx$ because they have taken the number of dx elements. So, that means number of $q \times dx$ is acting over there. So, all *n* number of concentrated loads is acting over this UDL.

Now, you know the concentrated load for this small segment and I will get the δw, this δw is the small deflection and that will be equal to $\frac{qdx\lambda}{2k}A_{\lambda x}$ at any point due to this small concentrated load $q \times dx$ this is the equation. Now this p we have to replace by $q \times dx$. So, similarly now I can get the deflection at C point.

If I want to get the deflection of C point *w* at C point then I have to do the integration. So, there are two regions, one from 0 to *b* and another from 0 to *a*, whether it is the left side or the right side does not matter or we have to add them and there are n number of concentrated loads. So, due to this n number of concentrated loads I will get the deflection at C point in such case all the contribution you have to add, if there are n numbers of concentrated loads.

So, we have to add all the settlements due to all particular concentrated loads. For example, at this C point there will be settlement due to n number of concentrated loads because there is a concentrated load 1, 2, 3. So, due to 1 there will be a settlement at C, due to 2 there will be a settlement at C, due to n there will be a settlement at C. So, we have to add those settlements and then I will get the settlement at C point.

That means, integrate and this P is replaced by qdx. So, this is $\frac{q\lambda}{2k}\left[\int_0^a A_{\lambda x}dx + \int_0^b A_{\lambda x}dx\right]$. So, this $A_{\lambda x}$, you know that $\frac{q\lambda}{2k}$ this will be 0 to a, now $A_{\lambda x} = e^{-\lambda x}(\cos\lambda x + \sin\lambda x)$. So, that is $dx +$ 0 to b, now this is whole third bracket 0 to b, $e^{-\lambda x}(\cos\lambda x + \sin\lambda x)$ this is dx then over third bracket.

After doing this finally, I will get the equation for the deflection at point C. So, that deflection at point C will be, $w_C = \frac{q}{2k}[2 - D_{\lambda a} - D_{\lambda b}]$, where a and b are the distances from point C to the edge of the loading. Similarly, I will get the θ in similar way. So, I am giving the final expression that is $\theta_C = \frac{q\lambda}{2k}[A_{\lambda a} - A_{\lambda b}]$.

And the bending moment at C is $M_C = \frac{q}{4\lambda^2}[B_{\lambda a} + B_{\lambda b}]$ and the shear force at C is $Q_C = \frac{q}{4\lambda}[C_{\lambda a} - C_{\lambda b}]$. So, these are the equations for the UDL. Now this C point may lie outside of the left side of the loading and C point can be on the right side of the loading, so in such case you have to take the integration accordingly.

And when you do this integration put this value remember that first deflection, left side, it will be the minus sign and for the deflection and the moment there will be no change but for the shear force and the slope there is the minus sign when you do the integration for concentrated load. Remember, that I have discussed and for the moment that deflection and moment will change if it is in the left side and other slope and shear force will remain same.

So, depending upon the position of your point of interest, now the point of interest is within the loaded region so, I have derived like this. The point of interest may be on left side of the load and it may be on the right side of the load, the outside of the loading either it can be left side or right side. So, according to that you have to derive the equation. Another thing in this equation you have taken a and b.

Suppose, if $a = 0$, $b = l$ then it is at point A, at point A; your $a = 0$, $b = l$ and at point B your $b = 0$, $a = l$. If you put these values you will get the deflection and the other quantities at the edges of the loading and also these are the cases for different loading conditions.

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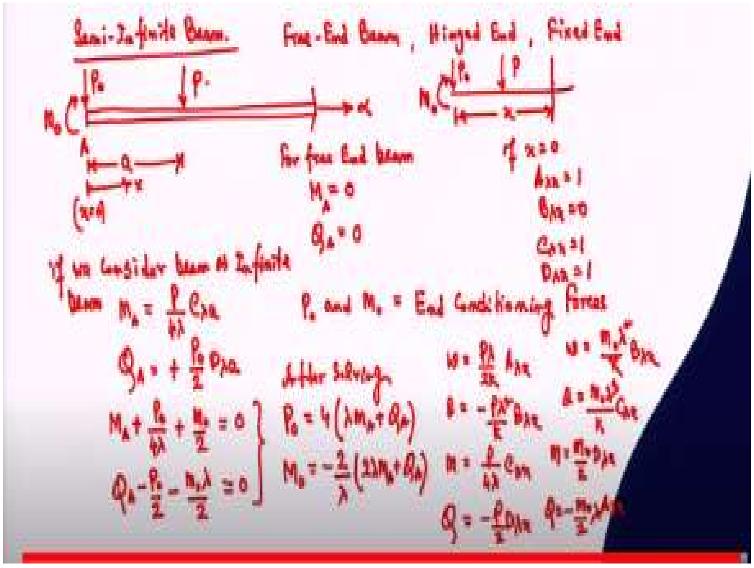

Now I will go for the semi-infinite beam and then I will give the idea how I will go for the finite beam also by using the same concept. Now this is the semi-infinite beam and remember that I have given concentrated load and moment and then UDL, and there will be other loading options also. That means if there is a triangular loading then also you can get the deflection, bending moment and all other parameters for infinite beam.

Now three cases can be considered for the semi-infinite beams, one is your free end beam, then the next can be your hinged end beam, then that can be your fixed end beam. So, that means this is a free end beam, hinged end beam and fixed end beam. So, I am discussing the free end beam, so similarly you can get for hinged end and the fixed end beam. The free end beam I am discussing. So, this is the free end beam.

So, actually, this side is infinite, but this side is finite; that means there is a free end at point A say. So, there is an end condition it is not infinite and there is a load which is acting say P at a distance of say a on this free end. So, here what to do, how you can solve this problem and what are the boundary conditions that we have to apply at the free end?

Now, for free end beam your bending moment will be 0 that means bending moment at A will be 0 and shear force at A will also be 0. Because this is the boundary condition for this free end beam because only one end is free here. So, this is the boundary condition that $M_A = 0$, $Q_A = 0$. Now, we will consider this beam as infinite beam. So, first step, we will consider that this beam is infinite beam and by using the infinite beam concept I will determine the moment and shear force that will be developed at point A.

So, now if we consider beam as infinite beam then the moment at point A will be 0 due to this point load and that will be equal to $\frac{P}{4\lambda} C_{\lambda x}$. Because this is the moment at point A due to this point load and the shear force at point A due to this point load that will be minus because actual condition is $-\frac{P_0}{2} D_{\lambda x}$ but remember that, now your point of interest is A and which is left side to the load.

So, bending moment there will be no change but the shear force this will be positive because this is negative this will be positive. So, that means you will calculate this bending moment and the shear force at point A. So, now because of this as we consider this beam as infinite and because of this point load there will be a bending moment and shear force at point A but actually, this is not infinite beam, it is a semi-infinite beam.

So, we have to apply a counter moment and shear force so that the net bending moment and shear force at point A is 0 because that is the boundary condition. So, that means we have to because of this P as we consider the infinite beam there will be a moment and this shear force but actually, at that point the moment and shear force will be 0 as per the boundary condition.

So, that means we have to provide a counter force and the moment such that the net moment at this point will be 0 and the net shear force at this point will be 0. So, then it will be a semi-infinite beam with free end and free end boundary condition will also be satisfied. So, that means we have to apply force P_0 and the moment M_0 such that the net force and the shear force and the moment will be 0 for this particular case.

So, this M_0 and P_0 these are called end conditioning moment and end conditioning load, respectively. So, one is moment and another one is concentrated force but these are called end conditioning forces. So, this P_0, M_0 are called end conditioning forces because one is moment, but another is concentrated load. So, now finally if I write that at this point bending moment should be 0.

So, finally, I can write that M_A, that is the moment due to point P and the moment because we have now applied like a concentrated load P_0 and the moment M_0. So, due to the concentrated load P_0 there will be a moment at that point and due to this concentrated moment, there will be a moment at this point. So, that means there will be three moments what are the three moments? Now we have to make this bending moment and next bending moment and shear put 0, we have applied a concentrated force P_0 and applied concentrated moment M_0.

So, at A point there will be moment due to three forces, one is due to P, one is due to P_0 and one is due to M_0. So, the moment due to P is Pa whose expression is given. So, moment due to P_0 will be equal to $\frac{P_0}{4\lambda}$ because your concentrated moment due to concentrated load I can write this equation that the w due to concentrated load is $\frac{P\lambda}{2k}A_{\lambda x}$, and slope is $-\frac{P\lambda^2}{k}B_{\lambda x}$.

Now $M = \frac{P}{4\lambda}C_{\lambda x}$ and shear force, $Q = -\frac{P}{2}D_{\lambda x}$ this is due to the concentrated load and due to the concentrated moment your $w = \frac{M_0\lambda^2}{k}B_{\lambda x}$. Then, $\theta = \frac{M_0\lambda^3}{k}C_{\lambda x}$, then bending moment is $M =$

$\frac{M_0}{2} D_{\lambda x}$, then this is $Q = -\frac{M_0 \lambda}{2} A_{\lambda x}$. So, these four cases I have derived, so you should remember these equations are very important.

So, for all the semi-infinite, infinite cases these equations will be used, that means the moment due to concentrated load P_0 will be $\frac{P}{4\lambda} C_{\lambda x}$. Now, your point of interest is this one, so always remember your $x = 0$ will be point of interest. So, now here point of interest this is $x = 0$ and see *x* will start from here. So, your *x* is starting from here at this point $x = 0$, so at this point $x = a$.

So, that means this particular case your point of interest is $x = 0$ not only the always 0 interest will be $x = 0$ that depends on only determining your value. But for this particular case your point of interest $x = 0$, so $x = 0$ means $C_{\lambda x}$ will be 1 because $C_{\lambda x}$ is $e^{-\lambda x}(\cos \lambda x - \sin \lambda x)$, so $\sin 0 = 0$, $\cos \lambda x$ is 1. So, this is equal to 0. So, now if $x = 0$ always remember that $A_{\lambda x} = 1$, $B_{\lambda x} = 0$, $C_{\lambda x} = 1$, $D_{\lambda x} = 1$. If your $x = 0$ always, remember that.

So, that means we will see this will be $\frac{P}{4\lambda}$ and due to this concentrated moment, the moment will be your $\frac{M_0}{2}$ again $D_{\lambda x} = 1$. So, this will be $\frac{M_0}{2}$ and that will be equal to 0 as per the boundary condition. Similarly, for the shear force also I should write that your Q_A is a shear force due to the *P*, then the shear force due to the concentrated load P_0 will be $-\frac{P_0}{2}$ because $D_{\lambda x}$ will be 1, this will be $\frac{P_0}{2}$ then for this moment the shear force will be $-\frac{M_0}{2}\lambda$ as $A_{\lambda x}$ will be 1.

So, this will be $-\frac{M_0}{2}\lambda$, that is will also be equal to 0 as per the boundary condition. So, now after solving the end conditioning force will be $P_0 = 4(\lambda M_A + Q_A)$ and $M_0 = -\frac{2}{\lambda}(2\lambda M_A + Q_A)$. So, M_A and Q_A we will get from these equations. So, and this is basically I should specifically write this is λa because the distance is *a* and this one is also I can write specifically this is equal to *a*, because the distance is *a*.

So, I will get this equation from M_A and Q_A by using these two equations and this is the end conditioning force. So, that means this much of P_0 and M_0, we have to apply to make net bending moment and shear force equal to 0. So, that means now if I want to determine deflection at any point, so now this is the case. There will be a P_0, M_0 and there will be a P. So, at any distance from this point X, I can determine the deflection, bending moment, shear force because now we have three loading conditions.

And we have to add or subtract them depending upon its position and the values that the equation has given. So, the next class I will solve one particular problem for this semi-infinite beam and then I will give you idea how you can use this concept for infinite finite beam also. Then I will go for the beams on two-parameter model and then I will discuss about closed form solution I am talking about and I will discuss how this beam problem can be solved by using finite difference technique. Thank you.