Advanced Foundation Engineering
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Lecture No -27
Beams on Elastic Foundation - I

So, this class I will start one new topic that is beams on elastic foundation. So, in my previous lectures I have discussed about the bearing capacity and settlement of shallow foundation and during my discussion as you have seen that we have not taken the foundation soil interaction. That means we are taking that the foundation as a separate part and the soil as a separate part in most of the cases. So, we have not considered the foundation soil interaction.

So, now in this topic we will consider the foundation and the soil interaction. And then as well as another aspect that in the previous problems, we have taken the settlement and bearing capacity. But the shear force and the bending moment those are also required to know during the design of a foundation. So, then how we can determine the settlement and the bearing capacity and the shear force of a foundation by considering the soil and foundation interaction that will be discussed in this topic.

So, in this topic in this section I will concentrate only on the shallow foundation part. But during my discussion on pile foundation there also I will discuss how the beam on elastic foundation concept can be used to analyze the pile foundation or determine the settlement of the pile foundation specifically for laterally loaded pile.

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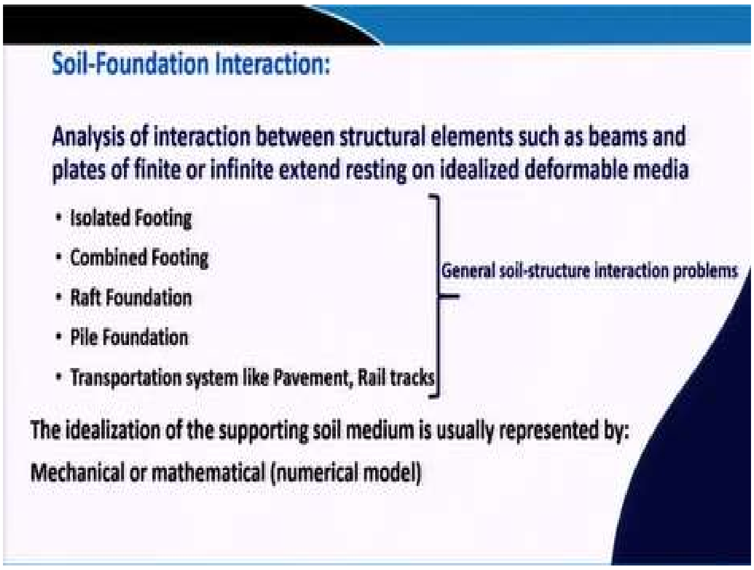

So, now the soil foundation interaction, it is that analysis of interaction between structural elements such as beams and plates of infinite and finite extent resting on idealized deformable medium. That means soil will be replaced by some mechanical elements. And I will consider structural element either beams or plates that are placed but in this course, we will concentrate only on beam. So, plate part will not be discussed. But that can be also considered to model the foundation.

And that beam or plate can be infinite or finite. And I will discuss when you use the infinite plate or beam and when you use the finite beam. So, these interaction problems can be useful for design of isolated footing, combined footing, raft foundation, pile foundation and transportation system, like pavement, railway tracks then your airport runway. So, those can be modeled by using these interaction techniques.

Now the idealization of supporting soil medium is usually represented by mechanical or mathematical model or numerical model. So, here we will consider the mechanical part that means this soil will be idealized by different mechanical components like spring, dashboard or both and then foundation will be idealized either by beams or plates. And then we will consider that interaction between these beams with those mechanical elements. And then this beam can be infinite or finite depending upon which type of problem we are solving.

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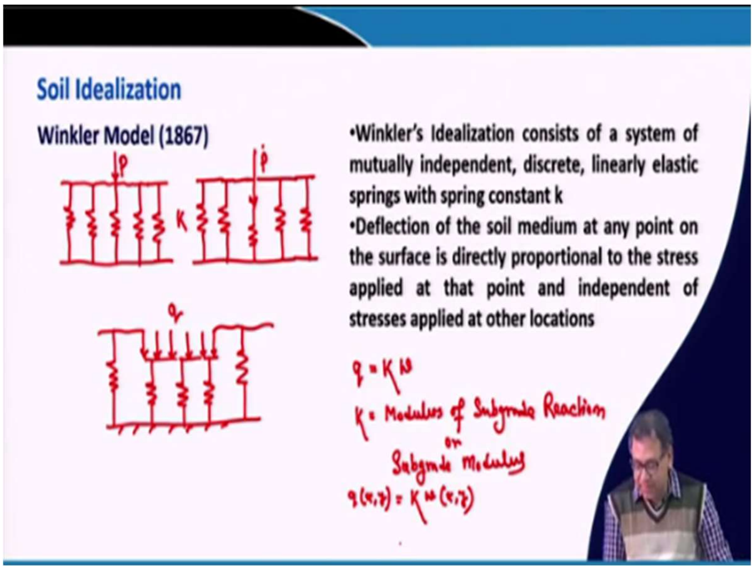

So, now we will discuss the first idealized model that is the Winkler Model. So, Winkler model basically consist of a system of mutually independent discrete linearly elastic springs that means the soil is modelled with springs. So, this is the soil which will be model by spring. This soil layer is modeled by spring whose spring constant is *k*. So, these are modelling spring with spring constant *k*.

So, now in this Winkler model the soil is replaced with the spring and the soil is basically idealized by the springs whose spring constant is *k*. But these springs are not connected to each other. They are discrete and this spring constant is linear or that means it is linearly elastic model. And then this deformation of the soil medium at any point on the surface is directly proportional to the stress applied on that particular point.

And it is independent of the stress applied to the other location. For example, suppose in this middle spring if I consider this middle spring, it will deform due to the stress applied on this middle spring only. Because as there is no connection between the springs even if you apply the stress or the load at any other spring, so that spring will deform but due to the stress applied to the other spring, this middle spring will not be deformed.

Middle spring or any spring will deform due to the stress applied on that particular spring that means the deformation of any particular spring is not influenced due to the stress applied to the other springs. So, that means if I apply a load in this particular spring so only this particular spring will deform but other springs will not deform at all. So, I can say that this is the middle spring where the load P is applied.

This spring will only deform and the other springs will not deform at all. They will remain same because your stress is applied on a particular spring and these springs are not connected. And this deformation is also linear that means in case the stress is increased and thus deformation will increase. So, I can write for q, if your stress is applied over a certain spring, that means this is the stress and those are applied over number of springs. Then those springs will only be deformed.

So, this is your stress which is applied on these three springs. So, only these three springs will deform, but other springs will not deform. This is the Winkler model. So, as I mentioned the deformation of the spring is proportional to the stress applied, that mean if you apply the stress so deformation is w, so that is k times of the w, where k is a spring constant or here it is mentioned that this k is the modulus of subgrade reaction or it is simple called subgrade modulus.

So, if the stress is acting over a rectangular area, that means the x-y plane, then $q(x, y) = kw(x, y)$. That means it is acting over a rectangular area. That means it is given that these springs are not connected, so stress applied on a spring will not influence the deformation of any other spring. So, any spring will not deform due to the stress applied on that particular spring.

So, other springs will not be influenced because of the stress applied to any spring. So, now how I will calculate the subgrade modulus, so that is one problem. So, we know the stress and if you know the k value then you will get the deformation of the soil.

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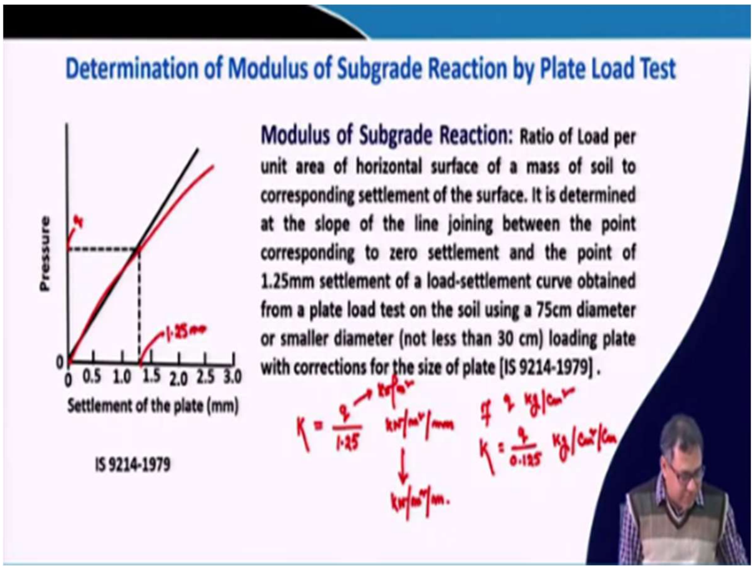

So, now this determination of subgrade modulus can be done by plate load test. So, plate load test I have discussed. So, in the plate load test I will get the load versus settlement plot or pressure versus settlement plot. So, this is the pressure versus settlement plot. And we will determine the *k* value corresponding to the stress, which is applied, so that means the *k* is equal to the stress at 1.25 mm.

So, it is the stress divided by a particular settlement. You can see the ratio of the load per unit area of horizontal surface of a mass of soil to corresponding settlement of the surface. To determine the slope of the line joining between the point corresponding to 0 line a 0 settlement. And the point of 1.25 mm settlement of a load-settlement curve obtained from a plate load test on the soil using 75 cm diameter or smaller diameter, but not less than 30 cm with correction of for the size of plate.

So, as per IS code, this is the definition of the *k*. So, in most of the cases this is the definition but few researchers have suggested some different settlement values and corresponding to that value we have to determine the subgrade modulus. In some cases, one particular stress is recommended that you calculate the settlement for a particular stress then you divide that settlement with that stress that is also *k* value

Sometime the stress is defined or taken and corresponding settlement is calculated or sometimes corresponding to a particular settlement you have to calculate the stress. But the definition is stress divided by that particular settlement. So, but here most common definition is stress corresponding to 1.25 mm settlement divided by 1.25 mm. That means if your stress is in kN/m^2 so, this unit of k will be kN/m^2.

Or if your q is in kg/cm^2, so as per the definition this will be $\frac{q}{0.125}$, this is in cm. So, this unit will be, kg/cm^2 divided by cm or sometime it is called kg/cm^3 also. But I would prefer to represent this as kg/cm^2 divided by cm. Now, ultimately you have to convert it and then you have to represent it in SI unit $kN/m^2/m$.

So, this is the unit of subgrade modulus k, in $kN/m^2/m$. Because this q is a stress and that I will get from this chart. Now, I want to mention that this red one is the original plot. So, red one is the original plot for stress versus settlement curve. So, you can see that this red one is not a straight line. So, to determine the subgrade modulus, what we have to do? We have to consider a 1.25 mm settlement. Then the corresponding point on the settlement curve, you join that point and the zero point and draw a line.

That black line is basically, join this point corresponding to 1.25 mm settlement and the zero line and then you join. And then the line basically the slope of this line will give you this k value. So, that means corresponding q you calculate on that straight line and then you determine the k value. That means here this k is basically the subgrade modulus that means you are joining a particular point corresponding to 1.25 mm and then joining the zero point and then that point and basically the slope of that straight line will give you the k value.

So, that means here this k is $\frac{q}{1.25}$ that means you just calculate this q corresponding to 1.25 mm settlement. That means on the curve you go to this point if it is not straight line, you make a straight line and then you get the value, corresponding to that 1.25 mm settlement, this is my q value and as I mentioned that when you do the plate load test, you do the test for either in between 30 cm to 75 cm diameter plate and do not take a plate size less than 30 cm.

And after 75 cm diameter plate not much variation of the value is observed. So, it is recommended you take 75 cm diameter plate. But you can take smaller size of 75 cm diameter plate, but if you take the smaller size of the plate, then you have to apply some size corrections. So, what are those corrections?

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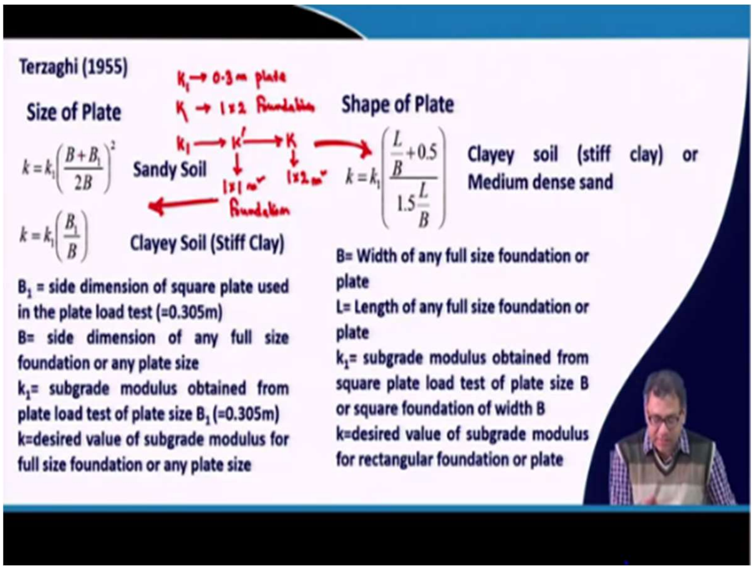

There are number of corrections that you have to apply once you get the k value. So, that the corrections are size of plate, shape of plate and depth of plate. These are the three corrections generally we have to apply. So, Terzaghi suggested in 1955 that for the sandy soil this k is the modulus that you want to determine for any plate size or foundation. And k_1 is the modulus of subgrade reaction that you have determined for a particular plate.

Suppose your plate size is here, Terzaghi has taken 30 cm plate size so that is 30 cm or 0.305 m plate size. So this k_1 value is calculated or determined by using a plate load test where dimension of the plate is 30 cm. So, that is k_1 and B is any plate dimension or obviously greater than 30 cm here because 30 cm is the lowest plate size or any foundation also.

And then you use these correlations to get the k value for the sandy soil and use the second correlation to get the k value for the clayey soil specifically for the stiff clay. So, suppose if you want to determine the k value for 75 cm plate. And you have done the test using 30 cm plates, so that means you get the k_1 which is corresponding to 30 cm plate. Now your B value will be in that case is 75 cm and the B_1 value will be 30 cm and again B value will be 75 cm then you will get the subgrade modulus.

So, that subgrade modulus will give you corresponding to 75 cm plate. And remember that by using this expression also you can see that as your size of the plate increases the subgrade modulus value decreases. So, that is why we have to take the plate size as big as possible. And it is mentioned that 75 cm suggested by IS code and it is observed that after setting 75 cm if you increase further the plate size.

So, not much variation in your result is observed, but those limitations are still there, I mean, I have discussed those limitations that if the soil is a layered soil if it is not a homogeneous soil. Then also you have to use these data with some caution. Then if it is unsaturated soil that means the capillary rise may be possible and clayey soil it says, this is a short-term test and clayey soil, it is a long-term behavior.

So, then also you have to use them with some cautions. So, those limitations that have been discussed will still be applicable here also. And the shape of the plate suppose, if you are doing the test for square plate then this is fine because these expressions are all valid for the square plate. I mean this k value you are getting for the square plate. But your actual foundation is a rectangular foundation.

Then how you convert that value for the rectangular foundation? So, first step is that you have done the test for k_1 equal to say some plate size B then you convert it for the actual foundation, if it is the rectangular footing by using this expression and that is for the clayey soil or medium dense soil. So, more or less for any kind of soil I can use these expressions. So, remember that suppose you have done the test on a 30 cm plate.

And you want to determine what would be the bearing capacity for a 1 m foundation, 1 m by 2 m foundation. So, first you convert 30 cm to 1 m. So, that means here it is observed the after 75 cm not much variation is there. But still there will be some variation. But the rate of variation will decrease if your plate size or the foundation size is beyond 75 cm. So, it means that you can say that it will be always better if you do the test for a real foundation dimension.

But here it is very difficult to conduct a field test with real dimension that we are taking, so you are going for a plate load test. And then it is observed that after 75 cm plate size not much variation is observed in this plate load test data. But still there will be some variation. So, now if you go for real dimension of the foundation, then you can use these theoretical values.

Because theoretically equations are available and you can get the subgrade modulus for any foundation. So, that means if you want to determine what would be the subgrade modulus for 1 m × 2 m foundation. So, in such case you have done the test for 30 cm plate then first convert it to the 1 m × 1 m plate or foundation by using equation for this size of plate. So, that means first this is your measured k value because you have done a test k_1 corresponding to your 0.3 m plate. And you want to determine the k for a 1 m × 2 m foundation.

Then what we will do? First you convert k_1 to k' for 1 m × 1 m square foundation by using this equation. It can be done if it is clayey or the sandy soil. Once you get the k_1 and k' then you convert this k' to k for 1 m × 2 m square foundation by using this expression. So, first you convert the k_1 to k' to get the subgrade modulus for 1 m × 1 m foundation.

But in both the cases this is square. But if your foundation is 1 m × 1 m then fine, that is your actual or the required subgrade modulus. But your foundation is 1 m × 2 m then you use the second conversion, now you convert this square foundation to rectangular foundation. So, in such case, your first conversion is B will be 1 m and B_1 will be 0.3 m and your second conversion is B will be 1 m and B_1 will 2 m. So, this is your conversion for plate size and shape.

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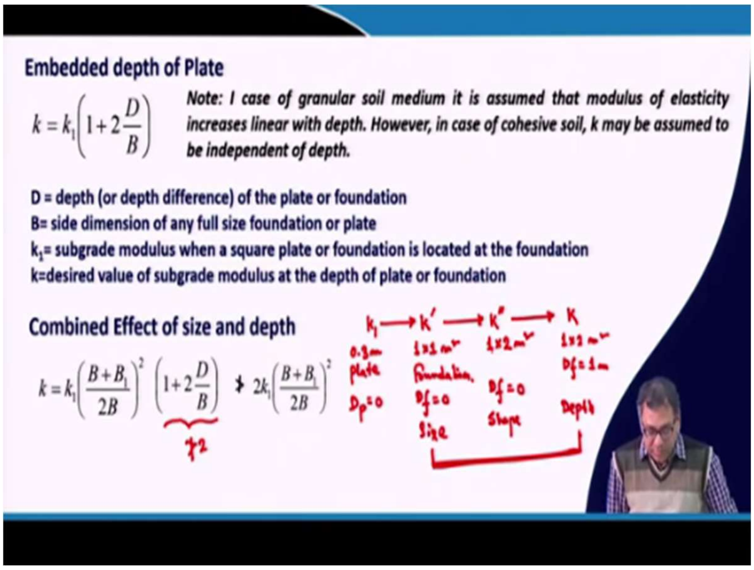

Now the third thing is the depth of the plate. Now ideally your depth of foundation and plate depth should be same. If both are same then no correction is required. But suppose you have done the test on a surface of a soil and you have placed the foundation at a certain depth, then you have to apply the depth correction. So, how we will do that? This is the correction factor. And remember that for the clayey soil, depth is not an important parameter for subgrade modulus.

That means whether you do the test on the surface or at a depth, so this value will not significantly change. But for the sandy soil or granular soil your modulus of elasticity increases linearly with depth. So, that is why k has a very important effect with respect to depth for granular soil. That means for clayey soil depth correction is not required, but for cohesionless soil this depth correction is required.

So, k may be independent of depth for the cohesive soil. So, how will you use this conversion and D is the depth of the foundation and B is the width of the foundation. Again, suppose now there are three cases. Now you have done a plate load test with 30 cm plate and plate is resting on the surface. And now you want to determine the subgrade modulus for a 1 m × 2 m foundation, which is resting at a depth of 1 m.

So, what you will do? So, first you convert this k_1 to k'. So, first it is for 30 cm or 0.3 m plate.

Now you convert it to for 1 m × 1 m square foundation. Then you can convert it to 1 m × 1 m foundation then you can convert it to k'' that is for 1 m × 2 m square foundation, then you convert it for a foundation here all the three cases are for surface foundation.

Then your final k will be 1 m × 2 m square with D_f = 1 m. And all this cases D_f or D_p this D_p is 0 here also D_f is 0 here also D_f is 0. So, you understand that the conversion part that $D_f = D_p = 0$. First you convert it for the square footing for plate size square footing, then the rectangular footing then with some depth. And then this third and fourth k values you can interchange, that mean you can do first depth correction then go for the shape correction that is ok fine.

But you have to first convert it to square footing that means first from plate you have to convert to square footing. All any other plate size suppose you have done a test on 30 cm plate now you want to convert all these things for 75 cm plate that will be 75 cm square. I mean 75 cm square plate. Then do the same thing that means in such case you convert 30 cm plate load test data to 75 cm first then you apply the depth correction, okay?

So, this is the technique by which you can apply three different corrections for shape, size and the embedded depth of the plate. And if you want to combine effect of size and depth correction because in this case it is a combined effect because we have first gone to size and depth. Suppose if this is your size correction, this is shape correction and this is depth correction.

So, if you are applying these two together if you are doing shape and size correction at a time no issue. But if you are using the size and the depth correction at a time then this condition that your depth correction and this is your size correction first part is your size correction that means you have to convert first to square footing then you are going for the depth correction. And when you are doing, remember that you have to satisfy this condition. That means this size correction and the depth correction this part should not be more than $2k_1\left(\frac{B+B_1}{2B}\right)^2$.

That means if you are applying these two corrections together that means this part should not be greater than 2. So, this part should not be if it is greater than 2 you consider twice. So, this is the thing that means this part this second part should not be greater than 2. Once you get this corrected value that means k you have corrected with respect to size and the depth, so that means you remember that the depth correction factor should not be greater than 2. So, when we apply the depth correction factor because of this depth correction because as I mentioned your size of the plate increases as your subgrade modulus decreases.

But as your depth increases your subgrade modulus increases because of this depth effect. But you have to restrict that for any condition these values that mean it will not increase twice, two times more than two times. That means due to depth correction your subgrade modulus value should not increase more than two times. So, that you have to restrict these. That means here first you do the shape corrections.

And then the depth correction at a time but you restrict this thing within two or equal to two then once you get this value then you can apply the shape corrections also, if it is a rectangular footing. So, that means you can go in this sequence also, that means k to k' to k'' to k, but remember that when you are applying this depth correction, if this corrections or correction factor is coming more than two you restrict it to up to two. Otherwise this sequence also you can use. But it should not be more than two, so that you have to remember. Otherwise you can go with this sequence.

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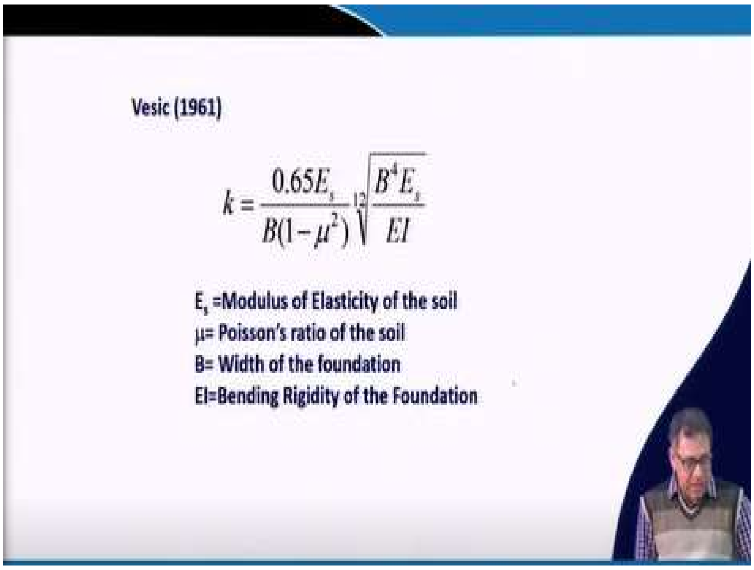

So, next we have different other correlations also. And in the next class what I will do? I will give you some more correlations by which you can get the k value and I will discuss some other methods also by which you can determine the k value and then I will give some ranges by which also you will get the k values for different types of soil, thank you.