

Advanced Foundation Engineering
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Lecture-22
Shallow Foundation: Bearing Capacity-XVI

So, last class I was discussing about the bearing capacity of layered soil and I solve the second case that is the stronger sand and the bottom layer is the weaker clay.

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A rectangular footing of size 2m X 3m is founded at a depth of 1m in a stronger clay. The thickness of the stronger layer is 2m. The water table is at the ground level. A softer clay layer is located below the stronger layer. The undrained cohesion of the stronger and softer layer are 100 kPa and 40 kPa, respectively. The saturated unit weight of the stronger and softer layer is 20 kN/m³ and 18 kN/m³ respectively. Determine the ultimate bearing capacity. Unit weight of the water is 10 kN/m³.

Calculation:

$$q_u = q_b + (1 + \frac{B}{L}) \left(\frac{2c_u H}{B} \right) + \gamma' H \left(1 + \frac{B}{L} \right) \left(1 + \frac{2e_f}{H} \right) \left(\frac{K_{st} \tan \phi}{B} \right) 2m$$

$$q_b = c_2 N_{c2} + \gamma'_2 (D_f + H) N_{q2} + \frac{1}{2} \gamma'_2 B N_{\gamma 2}$$

For $\phi = 0$, $N_{c2} = 5.14$, $N_{q2} = 1$, $N_{\gamma 2} = 0$

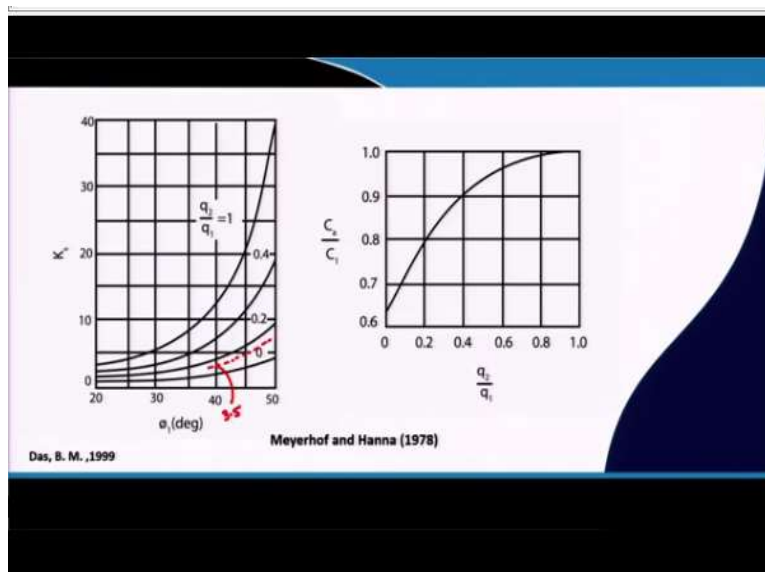
$$q_b = 40 \times 5.14 + (20 - 10) (1 + 1) = 253 \text{ kN/m}^2$$

For $\phi = 0$, $K_p = 1$, $S_{\gamma 2} = 1$

$$q_u = 253 + (1 + \frac{2}{3}) \left(\frac{2 \times 100 \times 2}{2} \right) + (20 - 10) (1 + 1) = 400 \text{ kN/m}^2$$

Now, this class I will solve the third case, this is the case 1 that I have solved stronger clay versus weaker clay.

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$q_u = 5.14 c_1 \left(1 + 0.2 \frac{D_f}{B}\right) + \gamma_1 D_f = 5.14 \times 100 \left(1 + 0.2 \times \frac{3}{3}\right) + (20-10) \times 1 = 592.5 \text{ kN/m}^2$
 $q_u = 253 + \left(1 + \frac{3}{3}\right) \left(\frac{2 \times 20 \times 1}{2}\right) + 0 - (20-10) \times 1 = 393 \text{ kN/m}^2 < 592.5 \text{ kN/m}^2$
 $\frac{q_s}{q_1} = \frac{C_2 N_c(\phi_2)}{C_1 N_c(\phi_1)} = \frac{C_2}{C_1} = \frac{40}{100} = 0.4$
 $\frac{C_2}{C_1} = 0.9 \therefore C_2 = 0.9 \times C_1 = 0.9 \times 100 = 90 \text{ kN/m}^2$
 $q_u = 393 \text{ kN/m}^2$
 Case ϕ -c Stronger Sand layer of $\phi = 40^\circ$, $\gamma_{sat} = 20 \text{ kN/m}^3$, $\gamma'_{sat} = 18 \text{ kN/m}^3$. Water table is at the base of the foundation. $\gamma_w = 10 \text{ kN/m}^3$
 $q_b = 40 \times 5.14 \left(1 + 0.2 \times \frac{3}{3}\right) + 18 \times 1 + (20-10) \times 1 = 261 \text{ kN/m}^2$
 $\frac{1}{2} H \left[\frac{\gamma_1 \gamma_2 + \gamma_2 \gamma_3 + \gamma_3 \gamma_4}{\gamma_1 \gamma_2} \right]$
 $\frac{1}{2} H \left[\frac{10 \times 20 + 20 \times 18 + 18 \times 10}{10 \times 20} \right]$
 $\frac{1}{2} \times 2 \times \left[\frac{10 \times 20 + 20 \times 18 + 18 \times 10}{10 \times 20} \right] = 28 \text{ kN/m}^2$
 $q_u = 261 + 28 = 289 \text{ kN/m}^2$

Then stronger sand versus weaker clay.

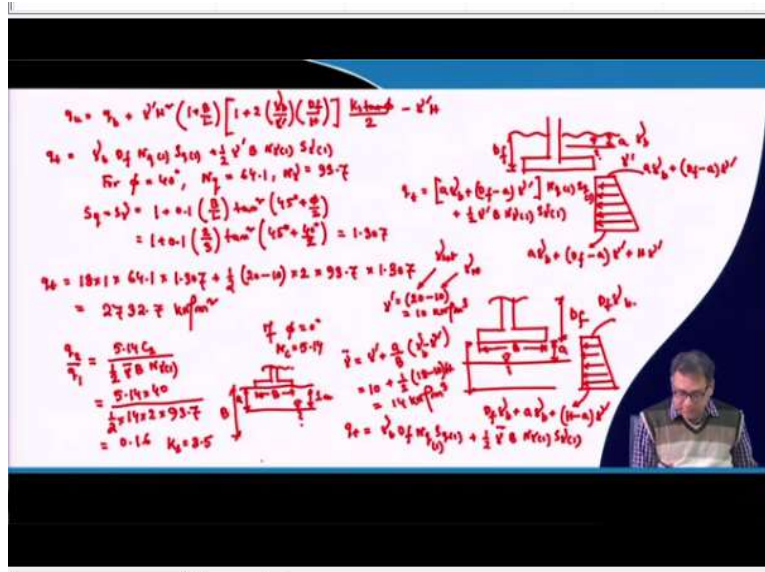
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$q_u = 261 + 10 \times 1 = \left(1 + 2 \left(\frac{1}{3}\right) \left(\frac{18}{10}\right)\right) \frac{3.5 \tan 40^\circ}{2} - 10 \times 1 = 363 \text{ kN/m}^2 < q_b$
 Case III ϕ - ϕ $\gamma_{sat} = 20 \text{ kN/m}^3$, $\gamma'_{sat} = 18 \text{ kN/m}^3$, $c_1 = 0$, $\phi_1 = 40^\circ$, $\gamma_{sat} = 19 \text{ kN/m}^3$, $c_2 = 0$, $\phi_2 = 25^\circ$
 $q_u = q_b + \gamma' H \left(1 + \frac{3}{3}\right) \left[\frac{K_2 \tan \phi_2}{H \gamma'} \right] - \gamma_w H = 0$
 $q_b = \gamma_1 (2 \times 3 + 1) N_c(\phi_1) + \frac{1}{2} \gamma_2 B N_c(\phi_2) \frac{H}{B}$
 $\phi = 25^\circ$, $N_c = 10.7$, $N_q = 4.8$
 $S_1 = S_2 = 1 + 0.1 \left(\frac{3}{3}\right) \tan^2 \left(45^\circ + \frac{25^\circ}{2}\right) = 1 + 0.1 \left(\frac{3}{3}\right) \tan^2 \left(45^\circ + \frac{25^\circ}{2}\right) = 1.164$
 $q_b = \left\{ 1 \times 18 + (20-10) \times 1 \right\} \times 10.7 \times 1.164 + \frac{1}{2} \times (19-10) \times 2 \times 4.8 = 1.164$
 $= 420 \text{ kN/m}^2$
 $q_u = 2732.7 \text{ kN/m}^2$
 $\frac{q_s}{q_1} = \frac{\frac{1}{2} \gamma_2 B N_c(\phi_2)}{\frac{1}{2} \gamma_1 B N_c(\phi_1)} = \frac{\gamma_2 N_c(\phi_2)}{\gamma_1 N_c(\phi_1)} = \frac{(19-10) \times 4.8}{14 \times 35.7} = 0.044$
 $K_u = 2$

Now, the third case, this is case 3 which is ϕ versus ϕ , that means, we are taking a stronger sand the same properties that my γ_1 is 20 kN/m^3 , then γ_2 is 18 kN/m^3 . Then c_1 is 0 and ϕ_1 is 40° , water table is again at the base of the foundation, rectangular foundation is $2 \text{ m} \times 3 \text{ m}$. Now, you are given the same layer which is definitely below the water table.

So, γ_2 will be equal to 18 kN/m^3 and c_2 is 0, ϕ_2 is 25° . So, ϕ_1 is given as 25° . Now, we have to again determine the $q_{ultimate}$.

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The same basic equation will be also valid for this case, because we have not changed the other properties and the top layer is also sand. So, that is why c_1 and c_2 both are 0. So, this basic equation again we can apply. So, that basic equation is $q_u = q_b + \gamma' H^2 \left(1 + \frac{B}{L} \right) \left[1 + 2 \left(\frac{\gamma_{bulk}}{\gamma_{sub}} \right) \times \left(\frac{D_f}{H} \right) \right]$.

Then $\frac{K_s \tan \phi}{2} - \gamma_1 H$. So, this part I will discuss later on. But right now I am writing $\gamma_1 H$. So, now, the q_b is the bearing capacity of the bottom layer. So, the first term will be definitely 0 because now it is cohesionless soil, will be second term in the third term. So, second term γ_1 I am writing the basic equation, later on I will modify that.

The $\gamma_1 (D_f + H)$ and then N_{q2} , then $s_{q2} + \frac{1}{2} \gamma_2$, because this is $\gamma_1, \gamma_2, BN_{\gamma 2} s_{\gamma 2}$. Now for $\phi = 25^\circ$, $N_q = 10.7$ and $N_\gamma = 6.8$ okay, that we are getting. So, s_q and s_γ , I will get $\left(1 + 0.1 \frac{B}{L} \right) \tan^2 \left(45^\circ + \frac{\phi}{2} \right)$. So, that is equal to $1 + 0.1 B$ is 2, $L = 3$, then $\tan^2 \left(45^\circ + \frac{25^\circ}{2} \right)$.

So, this is coming as 1.164. So, now, I will calculate the q_b . Now, as I mentioned in this equation I have written the general way, but now the γ 's is different, or this 18 and for the D_f part there is one unit weight and for H part there is another unit weight. So, that is why I am writing in different form. So, that is equal to $1 \times 18 + (20 - 10)$ or 10×1 . So, that is the surcharge, q and then N_{q2} .

N_{q2} is $10.7 \times s_{q2}$ is $1.164 + \frac{1}{2}$, then q and the unit weight will be γ below the water table i.e. γ_{sub} . So, this is $19 - 10 = 9 \text{ kN/m}^3$. So, width of the footing is 2×6.8 bearing capacity factor, again 1.164. So, q_b is coming out to be 420 kN/m^2 . Now, q_t you are calculating and q_t value will be the initial q_t value that was 2732.7.

Because here also it will be again 2732.7, because all the values are remaining same, because it is not changed and the position of the water table also has not been changed. So, q_t value will be 2732.7 again, because initial case also 2732.7. So, the q_t value is not changing because we are using the same sand and the same water table position. Now the $\frac{q_2}{q_1}$, that will slightly change now.

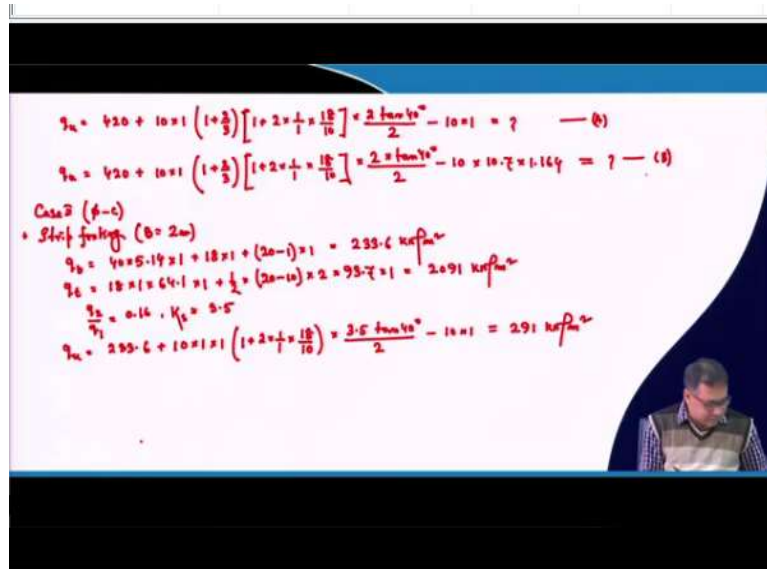
Now what is q_2 ? So, q_2 value will be both are sand and on the surface of each layer. So, this first and second term were 0, then the third term, this will be $\frac{\frac{1}{2}\gamma_2 B N_{\gamma_2}}{\frac{1}{2}\gamma_1 B N_{\gamma_1}}$. Now what is γ_2 , what will be γ_1 ? Because the γ_2 is below the water table. So, definitely it will be γ' .

So, now we put on the surface. So, we have to put it below the water table. So, I can write because $\frac{1}{2}B$ will cancel out, so there will be γ_2 and N_{γ_2} and there will be γ_1 and N_{γ_1} . So, now $\gamma_2 = \gamma' = 19 - 10$, because it is below the water table and second layer N_{γ_2} is 6.8.

And then γ_1 again I will take γ' because again like the case 2 water table position is not changed. So, I will take $\bar{\gamma} = 14$. So, I can write $\gamma' = 14 \text{ kN/m}^3$. So, I can directly write this value as 14. So, this is $14 \times$ so this is 93.7. So, in previous problem you can see it is 93.7.

So, this value is roughly coming out to be 0.044 okay. So, now we will go back to that chart. So, 0.044 where is the 0.044? This is 0.044 means roughly 0.05. So, this is 0.0. So, this is 0.2, this is 0.1. So, this will be your 0.04 which is close to here. So, this value is here and that will be roughly 2. So, now I will calculate the q_u value.

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So, q_u value will be $420 + \gamma'$ is 10, H^2 is 1, then $1 + \frac{2}{3}$, then $\left(1 + 2 \times \frac{1}{1} \times \frac{18}{10}\right) \times \frac{2 \tan^2 \alpha}{2}$, then -10×1 . So, this value is coming out to be around, this value and one thing that during my calculation I have taken this is 5. So, there will be some higher value that I am giving, you just calculate this and you will get the actual value.

So, these values you can calculate and then you will get the actual answer, because I have done the calculation by taking $K_s = 2$ because I pick the value of K_s from the chart because you can see the value corresponding to 0.04 which is close to 0. So, it will be around 2. So, case value will be 2. So, you just calculate considering 2.

Just do the calculation you will get the value of ultimate bearing capacity, q_u . Now, one thing I want to mention that during the derivation I told you that I am taking this basic equation where I am giving this $\gamma_1 H$ or here it will be $\gamma' H$ basically because it is below the water table. Now, this as per my understanding this $\gamma' H$ we are subtracting.

Why we are subtracting? Because we have taken the effect of H in the q_b calculation, you can see when I calculate q_b I have taken this H because $H = 1$, this is nothing but that $\gamma' H$ which I am just subtracting here. Now, this is if I subtract directly $\gamma' H$, if it is a clay soil then there is absolutely no issue. Because if this is a clay soil, then this N_q is 1, then shape factor is also 1, you can see that if it is a clay soil for case 1.

So, you can see that N_q is 1, your shape factor is 1. So, this total will be $\gamma \times (D_f + H)$, but there is no other factor. In such case subtracting $\gamma \times H$ is absolutely fine, that is no issue, because you are considering the H effect during the q_b calculation, that you are subtracting in the main equation. So, it is balance, because this H effect is already been considered during the soil friction calculation.

So, it is better to do not consider this twice to neglect that. So, that is no issue. But if you do the same thing for the sandy soils, then I think it is not the right approach, you have to do some modification. What is that? Why do we have to do the modification? Because in q_b calculation now the shape factor and bearing capacity factor are not 1, both are not 1, they have basically some value okay.

And now if I only subtract $\gamma' H$ basically I am not subtracting the total contribution of H , that is considered during the q_b calculation. So, I am subtracting less, because those multiplication factors are not used during the subtraction. Because this equation is initially developed if bottom layer is clay. So, my suggestion is that when you do this type of calculation, so, instead of using $\gamma' H$, you just use during the subtraction, that $\gamma' H \times N_{\gamma 2} \times s_{\gamma 2}$ or in another way, you just consider this first part because this is the same part that I am subtracting.

As I mentioned, you can do in two ways, either you do whatever way I have discussed otherwise, during the q_b calculation directly do not consider this part, do not consider this second term. So, you do not consider this part, this part means this part, do not consider this part. If you do not consider this part, then no need to subtract anything.

Then you have to end the equation here. And do not consider this part, you have to consider only up to this part, do not consider this one, if you do it in this way. Otherwise, if you consider the q_b part this part the second term during q_b calculation then you have to subtract only $\gamma' H$ for clay soil.

I think that is no problem, but sandy soil as per my suggestion, you calculate the $N_{\gamma 2}$ and $s_{\gamma 2}$ incorporate $N_{\gamma 2}$ and $s_{\gamma 2}$ within this calculation also. So, now, if you incorporate that then your

q_b will be again I am just writing that you do the calculation later on, q_b is $\frac{1}{2} \times 10 \times 1 \times \left(1 + \frac{2}{3}\right) \left(1 + 2 \times \frac{1}{1} \times \frac{18}{10}\right) \times \frac{2 \tan 40^\circ}{2}$. Now - 10.

Now, $N_{q\gamma}$ is 10.7, $N_{\gamma 2}$ is 10.7 and $s_{\gamma 2}$ is 1.164. So, I have to multiply these things also 10.7×1.164 . So, that will be your answer. So, few researchers have suggested that you use the first one, that means if I write this equation A and equation B, a few cases are suggested that you use equation A.

Whenever the soil is clay or sand, we just multiply directly $\gamma_1 H$, but few are suggested that you go for second option, that is B. So, I will recommend that you go for the second option. So, that is one thing that I want to mention, but this calculation I have not just done to do the calculation, I have given the values, you will get the value. So, now these three cases are completed.

Now quickly I will give you the bearing capacity for other types of foundation because I have discussed those rectangular foundation, now square foundation it is very simple, in the rectangular foundation that the equation I have given or the calculation that I have done in that case you put just $B = L$ equal to the value. So, that means the rectangular case will be converted to the square case.

So, square case is very simple that I am not discussing, you have to put only the $B = L$ equal to the value. Now, I am doing for the strip footing case and the circular footing case. Now, among these three cases that I have discussed, I am taking case 2 for the calculation of other footings. Because that means your top layer is strongest sand and the bottom layer is weaker clay.

And water table is also at the base of the foundation and dimension of the foundation will change now, in the strip footing, the B value is 2 m because it is a strip footing. So, L is not required. So, it is a strip footing case. So, this is for case 2, I have taken first one I will do for strip footing. So, that means I have taken $B = 2$ m.

So, if I take $B = 2$ m now, it is the case 2 the q_b will be what? q_b will be 40×5.14 . And in the previous case when I solved the case 2 problem, I have taken shape factor as $1 + 0.2 \times \frac{2}{3}$. So, that shape factor is not required. So, that means here it will be 1 because it is a strip footing, then $+ 18 \times 1 + (20 - 10) \times 1$. So, that value is 233.6 kN/m^2 .

Now, q_t also I can calculate. So, q_t value is $18 \times 1 \times 64.1$, then the factor is 1 because q_t value for case 2, this is the q_t value $618 \times 1 \times 64.1 \times$ that shape factor. Now shape factor is 1. So, both the shape factors are 1. So, I am just putting that shape factor as 1. So, this shape factor is 1 then $+\frac{1}{2} \times (20 - 10) \times$ it is $2 \times 93.7 \times 1$. So, that is 2091 kN/m^2 .

Now again whether the strip footing or the square footing or rectangular footing because $\frac{q_2}{q_1}$ determination we are going for the strip footing all the cases. So, because previous case also I do not mind that. So, in this case also $\frac{q_t}{q_1}$ will not change. So, this is 0.16 and K_s value is roughly 3.5. So, now, if I put these values I will get the q_u that is $233.6 +$ this is 10×1 .

Then the shape factor is $1 \times \left(1 + 2 \times \frac{D_f}{H} \times \frac{\gamma_{\text{bulk}}}{\gamma'}\right) \times \frac{3.5 \times \tan 40^\circ}{2}$, again - 10×1 . So, that is equal to 291 kN/m^2 . So, in case of rectangular footing the value was 363. So, definitely first footing it will reduce. So, it will be 291 kN/m^2 . And again I want to mention that this A case and B case. So, these modifications you have to do if the bottom layer is sand only, but otherwise it is not required. So now next one I will go for the circular footing.

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• Circular footing ($D = 6 = 2m$)
 $q_b = 1.2 c_2 N_c(s) + \gamma'_1 (D_f + H)$
 $= 1.2 \times 40 + 514 + [18 \times 1 + (20 - 10) \times 1] = 275 \text{ kN/m}^2$
 $q_t = \gamma'_1 N_q(s) + 0.3 \gamma'_1 D N_q(s)$
 $= 18 \times 1 \times 64.1 + 0.3 \times 18 \times 1 \times 64.1 = 1716 \text{ kN/m}^2$
 $q_u = q_b + 2 \gamma'_1 H \left[1 + 2 \left(\frac{D_f}{H} \right) \frac{K_s \tan \phi}{\gamma'} - \gamma'_1 H \right] \quad s = 1$
 $= 275 + 2 \times 18 \times 1 \left[1 + 2 \left(\frac{1.5}{10} \right) \left(\frac{1.5}{10} \right) \frac{3.5 \tan 40^\circ}{2} - 10 \times 1 \right]$
 $= 400 \text{ kN/m}^2$

So, for the circular footing I will go the same way calculation with your diameter or width is equal to 2 m, circular footing with diameter 2 m. So, now in this case I have given the equations. So, I can calculate the q_b which is given by $1.2c_2N_{c2} + \gamma_1(D_f + H)$ because this is the general type of equation and then I am writing here 1.2, c_2 is 40, then N_{c2} is 5.14 then +. This is $18 \times 1 + (20 - 10) \times 1$. So, that is 275.

So, that value is coming out to be 275 kN/m². Now, q_t value also I have given the equation for the circular footing. So, q_t value that equation is $\gamma D_f N_{q1}$ + now, the change is 0.3 because for strip footing it is 0.5. So, now it is $0.3\gamma_1 B N_{q1}$. So, now I can write γ_{bulk} because it will be within the depth of the foundation D_f is 1, N_{q1} is 64.1 + 0.3.

Then this value is below the base of the foundation. So, γ will be 20 - 10, I should write D here. So, that D value is equal to 2 m \times 93.7. So, this value is 1716 kN/m². So, now I will calculate the q_u which is q_b +. Now, this equation is given as $2\gamma H^2 \left(1 + 2\frac{D_f}{H}\right) \frac{SK_s \tan \phi}{B} - \gamma_1 H$.

This is the equation which was given in one of the previous classes. So, for the circular footing the equation for q_{ultimate} is given, but this case is when the top layer is sand. So, we have to do the modification and top layer is sand and this is also for top layer sand, but here the water table is there. So, we have to do the modification. So, that modification is the same as this will be $q_b + 2\gamma' H^2 \left(1 + 2\frac{D_f}{H} \times \frac{\gamma_{\text{bulk}}}{\gamma'}\right) \frac{SK_s \tan \phi}{B} - \gamma_1 H$.

So, as I mentioned that your $S = 1$ but that is a very conservative case. So, for conservative case you can consider $S = 1$. So, if I consider that conservative case then it will be equal to q_b is $275 + 2 \times 10 \times 1 \times \left(1 + 2 \times \frac{1}{1} \times \frac{18}{10}\right) \frac{SK_s \tan \phi}{B} - \gamma_1 H$, I have taken $S = 1$ and K_s again it will be 3.5.

Because I have already mentioned for different cases of footing also, we have to consider $\frac{q_t}{q_1}$ for strip footing. So, this is one conservative solution, then this is 3.5, then $\frac{\tan 40^\circ}{2}$, I should write this is also B or D . So, this is 2. So, then - 10 \times 1. So, this value is 400 kN/m². So, that means, these are the three cases that I have discussed for the strip footing, circular footing.

Then for rectangular footing, $B = L$, you can convert it to square footing also. So, these are the bearing capacity equations for layered soil and that can be used to determine the bearing capacity if there is a two-layered case. So, just one part is remaining for this bearing capacity calculation and that is the fourth case.

Because I have discussed three cases and then during my discussion also I have told that there is a fourth case. So, that is a separate case which is not the same as case 1, 2, 3. In the fourth case, the top layer is the weaker layer and the bottom layer is the stronger layer. So, the fourth case I will discuss in the next class and then I will start the next module, the next part that is the settlement of the foundation, thank you.