

Advanced Foundation Engineering
Prof. Kousik Deb
Department of Civil Engineering
Indian Institute of Technology-Kharagpur

Lecture-18
Shallow Foundation: Bearing Capacity XII

So, in this lecture first I will discuss one example problem. When foundation is resting on slope and then I will discuss about another case when the foundation is resting on layered soil.

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The slide contains the following handwritten calculations and a diagram:

Diagram: A foundation of width $b = 2\text{ m}$ is shown on a slope of height $H = 6\text{ m}$. The slope angle is $\beta = 20^\circ$. The foundation is on the surface of the slope, so $\eta = 0^\circ$. Soil properties are given as $c = 0$, $\phi = 40^\circ$, $\gamma = 19\text{ kN/m}^3$, and $D_f = 0$.

I Meyerhof

$$q_u = \frac{1}{2} \gamma B N_q \left(\frac{b}{B} \right)^2 \left(\frac{D_f}{B} \right)^2$$

$$= \frac{1}{2} \times 19 \times 2 \times 40 \left(\frac{2}{2} \right)^2 \left(\frac{0}{2} \right)^2, N_q = 40$$

$$= 760\text{ kN/m}^2$$

II Hansen

$$q_u = \frac{1}{2} \gamma B N_q (s_1)(s_2)(i)(c)(d)(b)$$

$$\phi = 40^\circ, N_q = 79.5$$

$$= \frac{1}{2} \times 19 \times 2 \times 79.5 \times 0.966$$

$$= 559\text{ kN/m}^2$$

III Vesic

$$q_u = \frac{1}{2} \gamma B N_q (s_1)$$

$$= \frac{1}{2} \times 19 \times 2 \times 109.4$$

$$= 842\text{ kN/m}^2$$

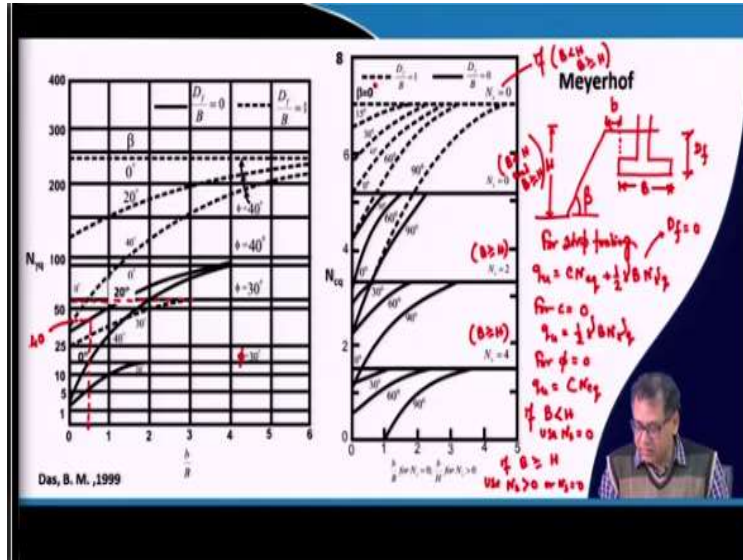
Additional notes on the slide include: $c = 0, \phi = 40^\circ, \beta = 20^\circ, \gamma = 19\text{ kN/m}^3, D_f = 0$. For Hansen, $s_1 = 1, c_1 = 1, d_1 = 1, b_1 = 1, i_1 = 1, j_1 = 1$. For Vesic, $s_1 = (1 - 0.5 \tan \phi)^2 = (1 - 0.5 \tan 40^\circ)^2 = 0.966$ and $s_2 = (1 - \tan \beta)^2 = (1 - \tan 20^\circ)^2 = 0.405$.

So, let me discuss first the example problem when the foundation is resting on a slope, that your foundation which is on the slope. So, here it is on the surface of the slope, so β value is given as 20° but here η is 0° because it is not the inclined base. But the foundation is on the surface of the slope and the width of the foundation is 2 m and the distance of the foundation from the slope is 1 m and height of the slope H is 6 m.

So, what are the properties given $c = 0$, ϕ is 40° and β is 20° , unit weight is 19 kN/m^3 and $D_f = 0$. But remember that D_f can be certain value also and I have discussed how you can use those charts by using D_f . So, now first I will use Meyerhof's equation and then I will go for Hansen and Vesic. So, Meyerhof the N_q for $c = 0$ ultimate bearing capacity, $q_u = \frac{1}{2} \gamma B N_q$.

Now, $\frac{b}{B} = \frac{1}{2} = 0.5$, now $\frac{D_f}{B} = 0$.

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So, now we have to go for the chart, so that chart as your $c = 0$ and only ϕ and ϕ is 40° . So, $\frac{b}{B} = 0.5$, so 0.5 means it will be somewhere here. So, this is 0.5 and your $\phi = 40^\circ$ and $\frac{D_f}{B} = 0$ that means firm line, so $\phi = 40^\circ$ means firm line. And so and your β value is 20° , so that means this is the line. So, this is the line, so if I extend this line, so it will here and remember that in the chart in few cases these are normal scale and few cases these are in log scale.

So, when you are taking the values from the chart you keep in mind it is not in normal scale it is sometimes in log scale also. So, log scale you know that how the values are represented, so based on that you have to pick the value, so similar to here it is a log scale. You can see that 100 to 200 difference is more compared 100, 200 to 300 then 300 to 400 and so on. So, here, so this is the value, so it is 40.

So, that means your $N_{q\phi}$ is 40, if it is a cohesive slope then you have to use these charts remember that. And as I mentioned that if your width is greater than or less than or equal to height of the slope, then how you can use this chart, so these I have already discussed. So, now $D_f = 0$, so corresponding your $N_{q\phi} = 40$. So, q_u will be $\frac{1}{2} \times 19 \times 2 \times 40$, so it is 760 kN/m^2 .

And remember that this is the strip footing and the one reason that why this equation is developed for strip footing also and no such recommendations are given that if there is a rectangular footing what to do? Because in most of the cases as the slope length will be very high compared to the width, so it is expected that the foundation that will be placed also continue along the slope.

So, that is why it is also a strip footing kind of foundation where the length is much, much longer compared to the width of the foundation that is why it is originally developed for the strip footing. But as I mentioned that if it is a rectangular footing or square footing and you have other conditions or loading conditions like inclined load or soil compressibility and all other factors, if it is present.

Then you have to use those factors here with some caution because originally it is developed for the strip footing. So, now if I use Hansen's theory, your q_u as it is on the surface, as $c = 0$, the first term will be 0, only the second term will present. And then you know that equation γB , so here I can write N_γ , then s_γ , then d_γ , i_γ , then c_γ say compressibility then g_γ then b_γ , so all the six factors I have written.

But here, so let me write one by one what will be the s_γ , because it is strip footing so, s_γ will be 1. Then d_γ because it is on the surface, so d_γ will also be 1. Then your i_γ it is not inclined loading, so i_γ will be also 1, then c_c , compressibility those values are not available. So, we assume it is the rigid one when the ϕ is 40° , so definitely it will be a general shear failure.

So, c_γ will be 1, then it is not tilting base. So, your b_γ will be 1, so only you have to calculate your b_γ . So, that g_γ as per Hansen which is the ground factor for the sloping ground. So, that is $(1 - 0.5 \tan \beta)^5$. So, you can write $(1 - 0.5 \tan 20^\circ)^5$, so this is 0.366. So, I can write this is half, so all these values are 1, so I am not writing them.

So, in $19 \times 2 \times 1$ for $\phi = 0$, now this case of $\phi = 40^\circ$ as per Hansen, $N_\gamma = 79.5$ that I will put here, that is 79.5 and it is 0.366. So, it is 553 kN/m². So, now I will go for the third approach which is

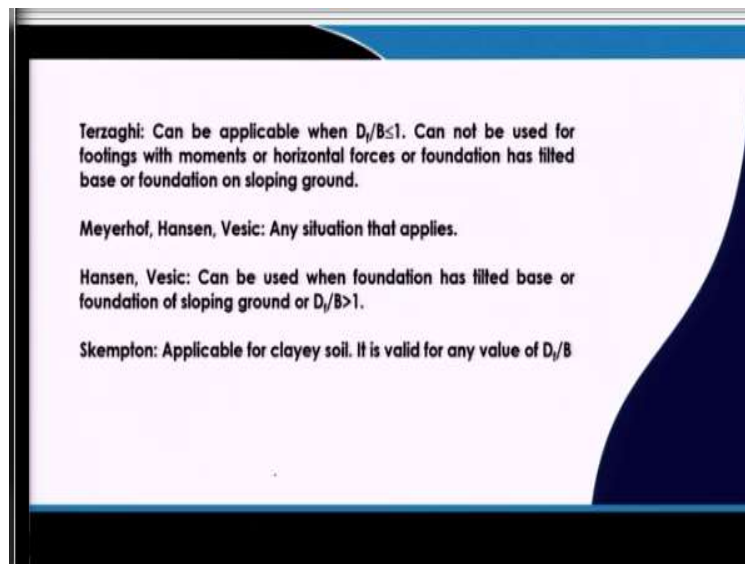
Vesic. And again if we have in the sloping ground here you can put those recommendations if it is eccentric loading, if there is compressibility, shape factor you can provide.

Remember that if it is eccentric loading again you have to go for Hansen whether it is parallel to L or B all these issues that I have discussed we have to apply here. If your footing is that kind and loading is that kind, because here I have taken the same place to the strip footing, vertical load, no horizontal load so just to show you how we can incorporate that sloping ground effects.

So, if your different problem as per that you have to apply those factors and you will get the value. So, as per Vesic that a similar expression this is I am not writing the coefficient those are 1 because I have already explained. So, this is $\frac{1}{2} \gamma B N_\gamma$ and g_γ will be there. Now for $\phi = 40^\circ$ as per Vesic the N_γ is 109.4 and g_γ is $(1 - \tan \beta)^2$ as per Vesic, so it is $(1 - \tan 20^\circ)^5$, so it is 0.405.

So, this is $\frac{1}{2} \gamma \times 2 \times 109.4 \times 0.405$, so this is 842 kN/m². So, this way you can calculate the bearing capacity by using different approaches like Meyerhof, Hansen and Vesic.

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So, now after comparison of all the methods, when you use which approach? That is the question, I have discussed several approaches I have discussed Terzaghi's, Skempton's, Meyerhof's then Hansen's, Vesic's and IS code approach, so all approaches I have discussed.

Now we have to use which approach, you know that Terzaghi can be applicable when $\frac{D_f}{B} \leq 1$, but Hansen, Vesic and Skempton.

So, these approaches you can use even for $\frac{D_f}{B} \geq 1$ also. So, even Meyerhof's approach also you can use. So, that means Terzaghi can be applicable when $\frac{D_f}{B} \leq 1$ and it can be used for footing with no moment, no horizontal force, the foundation is not tilted and foundation is not on the sloping ground. So, that means in simple case you can use Terzaghi's bearing capacity equation.

You cannot use for this condition if there is a moment you cannot use if there is a horizontal load, you cannot use that if the foundation is tilted, you cannot use if foundation is on the sloping ground. Even if the foundation is placed at a depth $> B$ width of the foundation, then also you cannot use Terzaghi's bearing capacity equation. In general Meyerhof, Hansen and Vesic these three equations can be used more or less for all purposes, and that is why these three equations are very commonly used or famous.

Because they can be used for more or less all the cases except Meyerhof's equation you cannot use if there is a tilting base condition because that is not recommended. Otherwise Meyerhof has suggested for sloping ground and all the other cases also, inclined footing, if there is a moment and then you can use that. So, that means Meyerhof, Vesic and Hansen you can use for all the conditions, even you can use for these Vesic and Hansen's equations if $\frac{D_f}{B} \geq 1$ also.

And you can use for tilted base, foundation on sloping ground as I mentioned. Skempton's equation you can use for clay soil and it is valid for any $\frac{D_f}{B}$. So, that means you cannot say which equation I should use always, no there is no such condition which one you use always because it is depending upon your requirement. Suppose but in general Meyerhof, Hansen, Vesic can be applicable for more or less all the cases.

But Skempton is only applicable for clay soil and in my next design problems I will prefer to use Skempton if the soil is clay soil. So and then if your foundation base is within the width you can

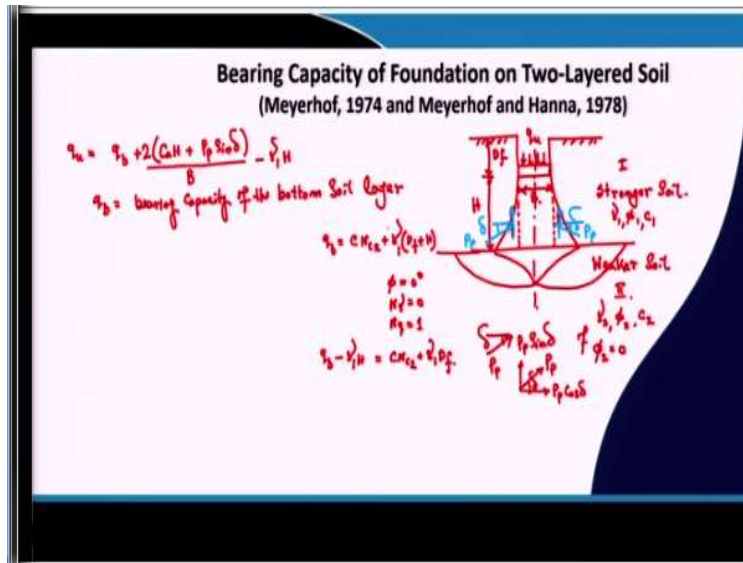
go for Terzaghi under simple condition. But if it is different loading condition or the moment or other condition is there you cannot use Terzaghi. So, depending upon your requirement, you have to select which one you will use.

And as I mentioned that Meyerhof, Hansen and Vesic all these equations can be applicable, now the question is when I will use these equations. That also depends on availability of loading and the other conditions. For example, in Meyerhof only the loading direction only the loading angle with vertical axis is sufficient to calculate but in case of Hansen and Vesic it is not ok.

In Hansen's and Vesic's approach, you should know whether the horizontal loading, H is parallel to B or parallel to L . Because those informations you should know to use Hansen's theory. Suppose you do not have those informations available, you have the information only the angle is given, so then you go for Meyerhof. But if you have that information that means you know the horizontal load is parallel to L or parallel to B then you can go for the any equations Hansen and Vesic.

So, that is why depending upon your requirement and availability of the values and the conditions, you have to choose the appropriate bearing capacity equation. But in my recommendation you always use two equations, any of the two methods always and then consider the minimum one or the lowest one as your bearing capacity value, so always it is better to use two equations.

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So, now the next topic that I will start is the bearing capacity of foundation on layered soil. So, in our foundation engineering problem generally in the foundation engineering course, we prefer to use the weighted average value for your layered soil case. And you can consider those properties for that particular one layer and then I will get the bearing capacity of the soil.

But now if the soil is layered and there are different cases, so basically in this case, I will discuss the two-layered soil case only. That means either the sand is resting on clay or clay is resting on clay or the sand. So, that means, I will discuss the three cases or even four cases you can say. That mean the sand, then sand clay and clay and there is another case I will discuss later on.

So, first I will provide a layered soil case where this is my ground surface, and you have a foundation here, this is the foundation and this is the ground surface. So, this is the ground surface and you have a foundation where your ultimate bearing capacity is q_u and the width of the foundation is B , suppose this is the width of the foundation and this is the centerline of the foundation.

Now we have two layers, this is my layer 1, and this is layer 2. And in this particular case, first I will discuss always your first layer is stronger and the second layer is weaker. In the next case I will discuss the second layer is stronger and the first layer is weaker. But the first case I will

discuss that always this first layer is the stronger layer, stronger soil and this one is that weaker soil, so what will happen?

So, as your lower layer is weaker, now if this is your depth of foundation D_f and this H is the thickness of the stronger layer below foundation. Now if the H value is very high, so that means your foundation failure zone will be on the stronger layer, so there will be no effect of the weaker layer. So you have to calculate the bearing capacity as if the foundation is resting on that stronger layer.

And you will get the values as per the equations or the procedure that I have discussed by using different approaches. So, that means you will get if the H value is very high, that means the influence zone is not going to the weaker layer. But if H value is not high and your influence zone is going to the second layer which is weaker layer, then, your actual failure will occur in the second layer.

So, now the failure surface, if I draw the failure surface, so this is the probable shape of the failure surface as proposed by these theories, proposed originally by Meyerhof in 1972, and then it is further modified by Meyerhof and Hanna in 1978. So, it is proposed that this is a possible failure surface. So, this is in the first layer, stronger layer, remember that if this H value is not very large, if H value is very small, so that the second layer will be influenced.

And this is the failure surface, so this is the probable failure surface in this case. So, as if the stronger soil layer is basically going down as foundation is going down and it is punching this eagerly. So, there will be a punching type of shear failure in the stronger layer because your weaker layer will fail which is very, very low strength layer, so that mean total system will try to sink.

So, that is why this type of failure surface is proposed, so now here the stronger layer this is unit weight, ϕ value and the c value, this is ϕ_1 , c_1 and γ_1 and for the weaker layer it is γ_2 , ϕ_2 where, γ , ϕ , c are the unit weight, friction angle and cohesion, respectively. So, now how it can be

calculated? Now it is assumed there will be strip, so this actual failure surface is assumed as a straight failure surface now, for the calculation purpose.

So, that means now your failure surfaces are perfectly straight, now the dotted line, say this is the dotted line but the firm one is the actual failure surface. But we assume that or it is assumed by these researchers that you consider this straight line. So, now when this foundation will move along this dotted line because now this foundation will move along the dotted line and the soil will also move along the dotted lines.

So, this portion of the soil within the dotted lines will deform or move and this is the movement along the dotted lines. So, what will happen if there is cohesion? So there will be a shear resistance that will act in upward direction. So, there is a shear resistance, so that will act in upward direction, so that shear resistance we have to calculate.

Now again when these soil will try to move as you see the original straight line. So, the soil is moving in outer direction, obviously the soil will move in the outer direction. So, that means there will be a passive resistance that the soil outside these failure zones gives because your soil is trying to push in the outer side, two sides of the foundation ok. So, soil is trying to push in the two sides of the foundation.

So, those soils will give you a passive resistance, ok, so that passive resistance is say P_p , and that will act at an angle of δ , now I am considering δ later on I will change it. So, this is P_p and that will act in both directions. So, at an angle of δ to the vertical to the surface and this dotted line you make a vertical line and with that vertical line it is making an angle δ because that is the case and it is common for your earthquake theories.

So, if you read the earthquake theories, you know that this will act at an angle of δ . So, what is δ ? δ is the friction angle between the soil and any other object. But here this angle basically along that dotted line, so it is soil versus soil, because later on I will modify that but now it is considered as a δ . Now how I will calculate the q_u ? So, $q_u = q_b + \frac{2(c_u H + P_p \sin \delta)}{B} - \gamma_t H$

H is the thickness of this soil below the foundation, so your cohesion will act along the H . So, that means $c_a \times H$ will give you the resistance due to cohesion or shear stress due to cohesion. And due to passive resistance also there will be value because it has two components. So, now this is your δ angle, your P_p is acting, so it has two components, so one is your if this is the P_p which is acting.

So, there will be one vertical component, there will be one horizontal component. So, this angle is δ means this angle is also δ , so this is your $P_p \cos \delta$ and this is your $P_p \sin \delta$. So, that $P_p \sin \delta$ will act in the vertical direction, so that $P_p \sin \delta$ this cohesion part and the bearing capacity that this base of the foundation is giving will be equal to the total load that is coming on the foundation.

Or the total load carrying capacity of the foundation that is q_u . So, this is c_a and then this $P_p \sin \delta$ term and this will act on both the sides. So, right side as well as the left side, so that is why I have to multiply it 2. So, this is the $c_a \times H$ and this is the P_p , so we have to express them in per width of the foundation. And then $-\gamma_1 H$, now why this $-\gamma_1 H$? I am just explaining that.

So, now your q_b is the bearing capacity of the base soil. So, that means from where this foundation is getting resistance. That as I mentioned the failure will be on the weaker soil, so that means this weaker soil will give you the resistance and that will get these weaker soil q_b , I will get by using the normal bearing capacity equation. Because now the failure if you look at this failure surface, because in weaker soil this is the failure surface.

So, now that can be your failure surface depending upon the type of soil, that can be your local shear failure also but the failure will be on the second surface. So, that means that way I can get the resistance by the bearing capacity of the second layer. So, that is the bearing capacity of the second layer that you will get from the second weaker soil layer and that I can use in our normal bearing capacity equations, I will explain that how I can calculate the q_b .

So, I will get the q_b , secondly our bearing capacity plus the resistance this soil is getting because of its movement along the dotted lines. So, that one cohesion part and another is the friction part.

So, if the cohesion is present then $c_a \times H$, H is the total thickness, then the passive resistance that soil is getting. That is the vertical component $\times 2$, because it is acting on two sides, then that will be equal to the q_b .

Basically I am taking all the vertical components. That means $q_u = q_b +$ this resistance. Now, one thing that why this $\gamma_1 H$ is used? So, $\gamma_1 H$ is used because it is assumed as weaker soil, if you assume that weaker soil is c soil. That means, in weaker soil you find $\phi_2 = 0$, you assume for the time being that my weaker soil is the cohesive soil, so $\phi_2 = 0$.

So, now how I will calculate the q_b ? That q_b will be given by which equation? Suppose if $\phi = 0$, q_b expression will be what? cN_{c2} . Then it is the strip footing originally let us first discuss for strip footing. So, for the strip footing the $s_c = 1$, and inclination is not given, $i_c = 1$. And then because these depth factors are already taken care of because along this depth all these resistances are considered.

So, that depth factor also you need to apply. So, it will be $cN_{c2} + \gamma(D_f + H)$ and for $\phi = 0$ your $N_\gamma = 0$ and $N_q = 1$, so it is 1. Because in the second weaker soil layer what is the thickness of the soil or the structure? Because second layer is due to the structures that is q . So, second layer is basically the second term is basically q , so $N_q = 1$ and $q = \gamma(D_f + H)$, so this is the q_b part.

Now in this way I can calculate q_b , but if you see that $\gamma(D_f + H)$, so now this H part we are basically taking twice, how I am saying that. Because when I am calculating the bearing capacity I am taking $\gamma(D_f + H)$. So, that the $\gamma D_f + \gamma H$, that structure contribution I am taking. As well as the same time when I am taking the resistance along the soil this part, here also I am taking the H, D_f part I am not taking any time.

So, that I can calculate within the q_b , but the thickness of this part I have already been taken during the resistance calculating around this dotted line because it is $c_a \times H$. So, the H part contribution is already been taken. Now if I take again during the q_b calculation. So, we are

basically taking twice this contribution in the surcharge to calculate the q_b and as resistance calculation along the dotted line.

So, it is recommended that the contribution we should take only one time. So, that is why this γ_1 because this is also γ_1 because $\gamma_1(D_f + H)$ because this is in the first plane. So, that is why it is recommended that you just subtract γ_1 each, because we have taken already this H term into this resistance calculation. So, it is recommended to neglect the H term when you calculate the bearing capacity for the bottom layer.

And that is why this $\gamma_1 H$ basically we are neglecting, now if you take that $q_b - \gamma_1 H$, so that will be basically $cN_{c2} + \gamma_1 D_f$. So, we are basically neglecting this H contribution when you calculate the q_b . So, that is the reason why I am putting $\gamma_1 H$. Now, one thing that if during q_b calculation if you neglect that value and you calculate only that $cN_{c2} + \gamma_1 D_f$, then no need to take $-\gamma_1 H$.

Because either you calculate q_b considering D_f and H , then you have to subtract that H contribution. But during calculation, if you take q_b is only the D_f contribution, then no need to subtract it. So, either you subtract it considering the total for doing q_b calculation or you just do not need to subtract if you do not consider the H contribution during your q_b contribution. So, one way, do not subtract twice, that means once you are subtracting during q_b contribution calculation.

And then again you are subtracting by using this equation, no, either you subtract in this equation. In that case you have to consider H during the q_b calculation or you consider only the D_f contribution, do not consider the H contribution during q_b calculation, then no need to substitute this value, clear. So, in the next class, I will complete the full derivation, then I will solve a few example problems, thank you.