

Advanced Foundation Engineering
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Lecture-17
Shallow Foundation: Bearing Capacity XI

So, last class I have discussed about Meyerhof's bearing capacity theory if the foundation is on the slope or with a certain depth within the slope. So, and then I started one example problem where the foundation base was tilted. So, I will finish that problem first.

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Handwritten calculations for Hansen's bearing capacity theory:

Given: $\phi = 20^\circ$, $N_c = 14.8$, $N_q = 6.4$, $N_\gamma = 2.9$

Shape factors:

$$d_q = 1 + 0.4 \left(\frac{B_f}{B} \right) = 1 + 0.4 \left(\frac{0.3}{2} \right) = 1.06$$

$$d_s = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left(\frac{B_f}{B} \right)$$

$$= 1 + 2 \tan 20^\circ (1 - \sin 20^\circ)^2 \left(\frac{0.3}{2} \right) = 1.05$$

Correction factors:

$$i_{qs} = \left(1 - \frac{0.5 H_b}{V + A C_u \tan \phi} \right)^5$$

$$= \left(1 - \frac{0.5 \times 300}{1000 + 2 \times 3 \times 0.7 \times 100 \times C_u \tan 20^\circ} \right)^5 = 0.7$$

$$i_{qg} = \left[1 - \frac{(0.7 - \eta/40) H_b}{V + A C_u \tan \phi} \right]^5 = \left[1 - \frac{(0.7 - 10/40) 300}{1000 + 2 \times 3 \times 0.7 \times 100 \times C_u \tan 20^\circ} \right]^5 = 0.61$$

Final bearing capacity: $i_{qsq} = 1$

Diagram details: Foundation width $B = 2\text{ m}$, length $L = 3\text{ m}$, area $A = 6\text{ m}^2$. Applied forces: $V = 1000\text{ kN}$, $H_b = 300\text{ kN}$. Soil properties: $\gamma = 19\text{ kN/m}^3$, $c = 100\text{ kN/m}^2$, $\phi = 20^\circ$. Failure mode: General shear failure.

So, this was the problem that there is a foundation with the tilted base with the angle $\eta = 10^\circ$ and it is not on the sloping ground, so $\beta = 0^\circ$. So, one vertical force $V = 1000\text{ kN}$ which is acting at the center of the foundation or that value and one horizontal force $H_B = 300\text{ kN}$ which is parallel to the width is acting but $H_L = 0$ and the ϕ value is given as 20° , cohesion is 100 kN/m^2 , length of the foundation is 3 m and general shear failure is considered and depth of foundation is 0.3 m .

So, as I mentioned as the $H_L = 0$ and the horizontal force is parallel to width. So, we have to take one set of factors and the bearing capacity equation for Hansen's bearing capacity theory, so first I will solve it with Hansen then I will solve it with Vesic. So, Hansen's bearing capacity theory, so if I go for Hansen, for $\phi = 20^\circ$ from the table $N_c = 14.8$, $N_q = 6.4$ and $N_\gamma = 2.9$.

So, your $\frac{D_f}{B}$ which is $\frac{0.3}{0.2} = 0.1$, so which is ≤ 1 . So, this information will be required for depth factor calculation. Now first we will calculate that d_c and I am going for d_c because here now we do not need to go for the d_{cB} or d_{cL} , because we have to determine only one set of factors because $H_L = 0$, that is why I am writing this in previous problem I have given d_{cB} and d_{cL} .

Because, we have to calculate two sets of factors and the bearing capacity equation, that is why, here only d_c because only once it actually this is d_{cB} . So, this value is as it is $\frac{D_f}{B} \leq 1$. So, this will be $1 + 0.4 \frac{D_f}{B}$. So, this will be $1 + 0.4$ your D_f is 0.3 and B is 2, so this is 1.06. Similarly, $d_q = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left(\frac{D_f}{B} \right)$, so this is $1 + 2 \tan 20^\circ (1 - \sin 20^\circ)^2$, D_f is 0.3, B is 2, so that value is equal to 1.05 and $d_\gamma = 1$.

So, these are the values, now we have to calculate the inclination factor because we have to modify the shape factor by using the inclination factors. So, first we will calculate the inclination factor. So, that is your i_{qB} here I am using B either you can use B or not because you can use i_q also because it is actually i_{qB} . So, here also you can use the d_{cB} or d_{qB} or $d_{\gamma B}$ because these are B .

So, this is $1 - 0.5H_B$ and $V + A$ or A' both are same, because it is not an eccentric loading, so to the power 5. As I mentioned if nothing is mentioned in the problem I will use the 5. So, now I have to take that values which is recommended that for δ I am taking 0.85 and c_a it is 0.621, so I am taking $0.7c$. These things will be mentioned in the question in your case, but here I am assuming during design and you can assume it within the given range.

But in the assignment problem or the example problem it will be given specifically, so this is point C. So, finally I will get $1 - 0.5H_B = 300$ then $V = 1000$ then area is 2×3 , cohesion is 100 kN/m^2 c_a is 0.7×100 because c value is 100, and then $\cot \phi$ i.e. $\cot 20^\circ$ to the power 5, so this is coming out to be 0.7. Similarly that $i_{\gamma B}$ will be calculated and then as I mentioned in the table if you look at that table of Hansen's bearing capacity.

So, I have given that value of i_γ for two conditions, one is the foundation base is not tilted another one if the foundation base is tilted. So, now in this case your previous question I use the case where the foundation base is not tilted. So, I use the equation as for that, but here the foundation base is tilted, so I have to use the dot equation whether we can use for foundation base tilted and that is point this is a third bracket, it is within bracket 0.7.

This $i_{\gamma B} = \left[1 - \frac{(0.7 - \frac{\eta}{450}) H_B}{V + A c_a \cot \phi} \right]^5$. So, now if I now put this value, you will get this equation it is given in the table. So 1 minus this is $0.7 - \eta$ is 10° or sorry I should write 10 and this is 450, then H_B is 300 then divided by V is $1000 + 2 \times 3 \times 0.7 \times 100 \times \cot 20^\circ$ to the power 5. So this value is coming out as 0.61.

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Now I will calculate the i_{cB} which is related to i_{qB} that is why I have calculated i_{qB} first. Now, $i_{cB} = i_{qB} - \frac{1-i_{qB}}{N_q-1}$. So, i_{qB} is 0.7 and N_q is 6.4. So, $i_{cB} = 0.7 - \frac{1-0.7}{6.4-1}$. So this value is 0.644. Now we have to modify our shape factor by using these inclination factors though you have to use one set but we have to modify the shape factors as per Hansen.

So, $s_{cB} = 1 + \frac{N_q}{N_c} \left(\frac{Bi_{cB}}{L} \right)$. So, now this is $1 + \frac{6.4}{14.8} \left(\frac{2 \times 0.644}{3} \right)$, so this is 1.186. Similarly, $s_{qB} = 1 + \sin \phi \left(\frac{Bi_{qB}}{L} \right)$, so these B is in suffix remember that, so $1 + \sin 20^\circ \left(\frac{2 \times 0.7}{3} \right)$, so this is 1.16. Similarly, $s_{\gamma B}$ which is $1 - 0.4 \frac{Bi_{\gamma B}}{Li_{\gamma L}}$, that should be ≥ 0.6 .

Now how $i_{\gamma L}$ I can calculate? So, $i_{\gamma L}$ obviously will be 1. So, in this part in place of H_B you have to write H_L and $H_L = 0$, so that whole term will be 0, so it will be only 1^5 means it is 1, so $i_{\gamma L}$ will be 1.

So, now you put these values, so this is $1 - 0.4 B$ is 2, then $i_{\gamma B}$ is 0.61 then divided by $L = 3$, $i_{\gamma L} = 1$, so that value is 0.837. So, the depth factor, inclination factor and the shape factors are calculated. Now our foundation is tilted, so we have to incorporate that factor.

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**Effect of Ground Factors (base on slope) and Base Factors (tilted base)
Hansen's or Vesic's bearing capacity Theory**

$$q_u = cN_c s_c d_c i_c g_c b_c + qN_q s_q d_q i_q g_q b_q + 0.5\gamma BN_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$$

g_i is the ground factor (base on slope) and b_i is the base factor (tilted base)

Hansen		Vesic	
Factors	Value	Factors	Value
Ground	$g_c = \frac{\beta^\circ}{147^\circ}$ for $\phi = 0$	Ground	$g_c = \frac{\beta}{5.14}$ for $\phi = 0$ β in radians
	$g_c = 1 - \frac{\beta^\circ}{147^\circ}$ for $\phi > 0$		$g_c = i_c - \frac{1-i_c}{5.14 \tan \phi}$ for $\phi > 0$
	$g_q = g_\gamma = (1 - 0.5 \tan \beta^\circ)^2$		$g_q = g_\gamma = (1 - \tan \beta^\circ)^2$
Base	$b_c = \frac{\eta^\circ}{147^\circ}$ for $\phi = 0$ $b_\gamma = 1 - \frac{\eta^\circ}{147^\circ}$ for $\phi > 0$	Base	$b_c = g_c$ (for $\phi = 0$)
	$b_q = \exp(-2\eta \tan \phi)$ η in radians		$b_\gamma = 1 - \frac{2\beta}{5.14 \tan \phi}$ for $\phi > 0$
	$b_\gamma = \exp(-2.7\eta \tan \phi)$		$b_q = b_\gamma = (1 - \eta \tan \phi^\circ)^2$ η in radians

So, here Hansen because this is for the ground factor (base on slope), so g_q , g_γ all will be 1, because it is not a sloping ground but we have to calculate the b_c , b_q and b_γ by using these expressions. So, remember Vesic also given, so you will use these expressions I will solve the Vesic's bearing capacity equation, so remember that. So, now I will calculate b_c where, $b_c = 1 - \frac{\eta^\circ}{147^\circ}$, remember that here $\eta = 10^\circ$.

So, $1 - \frac{10^\circ}{147^\circ}$, so this is 0.932 and $b_q = \exp(-2\eta \tan \phi)$, but here η is in radian. So, what I will do? That $\eta = 10^\circ$ then I have to multiply by $\frac{\pi}{180^\circ}$ to convert it to radians and then it is $\tan 20^\circ$. So, just careful which one is degree, which one is radian is mentioned in the table specifically, so use according to that. So, this is equal to 0.881.

Now, $b_\gamma, b_q = \exp(-2.7\eta \tan \phi)$ again η is in radian here $-2.7 \times 10^\circ \times \frac{\pi}{180^\circ} \times \tan 20^\circ$, so this is equal to 0.842 and as I mentioned that $g_{qc} = g_q = g_\gamma = 1$, because it is not in a sloping ground.

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$$q_{ult} = 100 \times 14.8 \times 1.186 \times 1.06 \times 0.644 \times 1 \times 0.932 + 6.4 \times 19 \times 0.3 \times 1.16 \times 1.05 \times 0.7 \times 1 \times 0.881$$

$$+ \frac{1}{2} \times 19 \times 2 \times 2.9 \times 0.897 \times 1 \times 0.61 \times 1 \times 0.842 - 19 \times 0.3$$

$$= 1162 \text{ kN/m}^2$$

ii) Velic. $\phi = 20^\circ, N_c = 14.8, N_1 = 6.4, N_2 = 5.4$
 $s_c = 1 + \frac{N_2}{N_c} \left(\frac{B}{L}\right) = 1 + \frac{6.4}{14.8} \left(\frac{2}{8}\right) = 1.288$
 $s_\gamma = 1 + \tan \phi \left(\frac{B}{L}\right) = 1 + \left(\frac{2}{8}\right) \tan 20^\circ = 1.243$
 $s_d = (1 - 0.4 \frac{D_f}{L}) \geq 0.6 = (1 - 0.4 \times \frac{2}{8}) = 0.73$
 $d_c = 1 + 0.4 \left(\frac{D_f}{B}\right) = 1 + 0.4 \left(\frac{2.3}{2}\right) = 1.06$
 $d_\gamma = 1 + 2 \tan \phi (1 - \sin \phi) \left(\frac{D_f}{B}\right) = 1 + 2 \tan 20^\circ (1 - \sin 20^\circ) \left(\frac{2.3}{2}\right) = 1.047$
 $i_f = 1$

So, finally if I put these values in the equation, so net ultimate bearing capacity. So, that equation means all the factors we have to consider except the compressibility factor we are not considering because those data are not available. So, that c value is 100 kN, N_c is 14.8, now s_c is 1.186, d_c is your 1.06 then i_c is 0.644 then your ground factor g_c is 1 and the base factor or the this tilted base factor b_c is 0.932.

Similarly for this part it will be 6.1 that is the factor $\times \gamma D_f$, γ is 19, γ value is given. So, γ you can consider as 19 kN/m³. So, γ is 19 and then your depth of foundation is 0.3 then the all the factors. So, I am just putting all the values as you know which factor is what? So, that means, it is $1.16 \times 1.05 \times 0.7 \times 1 \times 0.881$ then the third factor which is $\frac{1}{2} \times 19$ width, $B = 2$ m.

So, width is 2 m, then bearing capacity factor N_γ is 2.9, then all the factors 0.837 then 1, then 0.61, then 1×0.842 and then the γD_f , γ is 19.3 because we are calculating net, so finally this value is 1162 kN/m², so this is as per Hansen. So, now I will calculate the bearing capacity factor as per Vesic. So, initially one as per Hansen this is your case 1 and the case 2 as per Vesic.

So, from Vesic's table corresponding to $\phi = 20^\circ$ we will get $N_c = 14.8$ then $N_q = 6.4$ and $N_\gamma = 5.4$. So, now I will calculate the s_c and remember that in the previous problem we have taken only one set of equation and the factors for Hansen. Because here your $H_L = 0$, but in Vesic whether $H_L = 0$ or not it does not matter, you have to take always one set of factors and equation.

And no modifications are required, you can use directly those values which are given in the table and you can put. Because those parallel to L or B effects already incorporated in the factors. So, $s_c = 1 + \frac{N_q}{N_c} \left(\frac{B}{L}\right)$ and that is $1 + \frac{6.4}{14.8} \left(\frac{2}{3}\right)$, so this will be 1.288. Now $s_q = 1 + \tan \phi \left(\frac{B}{L}\right)$, so this is $1 + \tan 20^\circ \times \left(\frac{2}{3}\right)$ that is 1.243. Similarly $s_\gamma = 1 - 0.4 \left(\frac{B}{L}\right)$ that should be ≥ 0.6 , so it is $1 - 0.4 \left(\frac{2}{3}\right)$, which is $0.73 > 0.6$ fine.

So, previous one let me check which is $0.837 > 0.6$ it is fine, so this is 0.3, so 0.3 you have to use. Similarly d_c I will get $1 + 0.4 \frac{D_f}{B}$. So, this is $1 + 0.4 \times \frac{0.3}{2}$ actually this value will come. So, this is the same value that we will get from this here 1.06, so this is $0.4 \times \frac{0.3}{2}$ i.e. 1.06, so these values will come your 1.06. So, 1.06 and $d_q = 1 + 2 \tan \phi (1 - \sin \phi)^2 \frac{D_f}{B}$, so this is $1 + 2 \tan 20^\circ (1 - \sin 20^\circ)^2 \times \frac{0.3}{2} = 1.047$, which is roughly 1.05, that is the value I have considered in previous case, so 1.05 also you can consider, so that is because it is 1.047 and then $d_\gamma = 1$.

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$$i_c = i_q - \frac{1-i_q}{N_q-1}$$

$$i_q = \left(1 - \frac{H}{V + A'c_a \cot \phi}\right)^m$$

$$m = m_b = \frac{2 + \left(\frac{B}{L}\right)}{1 + \left(\frac{B}{L}\right)} = \frac{2 + \left(\frac{2}{3}\right)}{1 + \left(\frac{2}{3}\right)} = 1.6$$

$$= \left(1 - \frac{300}{1000 + 2 \times 3 + 0.7 \times 100 \times \cot 20^\circ}\right)^{1.6} = 0.787$$

$$i_q = \left(1 - \frac{H}{V + A'c_a \cot \phi}\right)^{m+1} = \left(1 - \frac{300}{1000 + 2 \times 3 + 0.7 \times 100 \times \cot 20^\circ}\right)^{2.6}$$

$$= 0.677$$

$$i_c = i_q - \frac{1-i_q}{N_q-1} = 0.677 - \frac{1-0.677}{10-1} = 0.677 - \frac{0.323}{9} = 0.677 - 0.0359 = 0.641$$

Now I will calculate i_c and $i_c = i_q - \frac{1-i_q}{N_q-1}$, so you have to calculate i_q first. So, $i_q = \left[1 - \frac{H}{V + A'c_a \cot \phi}\right]^m$. Now your H is parallel to B , so if you look at that table where given if H is parallel to B , how you can calculate m ? If H is parallel to L then how you can calculate m ? But if both H_B and H_L are present then how to calculate m ?

So, as it is parallel to B , so your $m = m_b = \frac{2 + \frac{B}{L}}{1 + \frac{B}{L}}$, so this is $\frac{2 + \frac{2}{3}}{1 + \frac{2}{3}}$, so this is 1.6. Again I am saying that if this is a eccentricity you use the recommendation to calculate this factor where you have to use the B or L or where you have to use the L' or B' those are already been explained. So, now I can put these values because here H_B in Vesic.

According to Vesic H_B and H_L are not required to be present as mentioned because it is only one set and that is H but m you have to calculate whether it is H_B or H_L based on that. So, now this is your H is 300 and then V is $1000 + 2 \times 3$ the same 0.7×100 I am taking, then $\cot 20^\circ$, so that now $m = 1.6$, so this will be 1.6, so this is 0.787. Now i_q we will get

$$\left[1 - \frac{H}{V + A'c_a \cot \phi}\right]^{m+1}$$

because here A' and A are same.

So, that is $\left[1 - \frac{300}{1000+2 \times 3 \times 0.7 \times 100 \times \cot 20^\circ}\right]^{1.6+1}$, so this is 0.677. So, now I will use these ground factors are all 1 again because but base factor I have to use by using these tables or these expressions, so base factor I will use now. So, now the base factor which is given that my g_c , g_q and g_γ all are 1 now $b_c = 1 - \frac{2\beta}{5.14 \tan \phi}$

Now it will be 1 as $\beta = 0$, now $b_q = b_\gamma = (1 - \eta \tan \phi)^2$. So, η is in radian, so this is $\left(1 - 10^\circ \times \frac{\pi}{180^\circ} \times \tan 20^\circ\right)^2$, so this is 0.877.

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The image shows a handwritten calculation for the net ultimate bearing capacity q_{nu} . The calculation is as follows:

$$q_{nu} = 100 \times 14.8 \times 1.288 \times 1.06 \times 0.748 \times 1 \times 1 + 19 \times 0.3 \times 6.4 \times 1.243 \times 1.047 \times 0.787 \times 1 \times 0.877$$

$$+ \frac{1}{2} \times 19 \times 2 \times 5.4 \times 0.73 \times 1 \times 0.677 \times 1 \times 0.877 - 19 \times 0.3$$

$$= 1780 \text{ kN/m}^2 - 19 \times 0.3 = 1775 \text{ kN/m}^2$$

So, now if I put these values I will get net ultimate bearing capacity, q_{nu} and the values that is $100 \times 14.8 \times 1.288 \times$ your s_q is 1.28, s_c is 1.288, d_c is 1.06. So, this is $1.06 \times 0.748 \times 1 \times 1$ both the factors are 1 here + γ is 19 D_f is $0.3 \times 6.4 \times 1.243 \times 1.047 \times 0.787 \times 1 \times 0.877$. Then $+\frac{1}{2} \times 19$ is the unit weight 2 is the width $\times 5.4$, because here it is 5.4 for Vesic it is 5.4.

Now 5.4 is the bearing capacity factor then your s_c is 0.73 because s_γ is 0.73. So, s_γ is 0.73, then the d_γ which is 1, then the inclination factor is not here. So we will put sorry inclination factor is here and that is 0.677, inclination factor is 0.677 and then the ground factor is 1 and base factor is 0.877 - 19×0.3 as it is net. So, it will be roughly around $1780 \text{ kN/m}^2 - 19 \times 0.3$.

So, this will be roughly 1775 kN/m². So, as per Hansen we got the value 1162 and as per Vesic we are getting 1775, so these are the values that we are getting. So, now we have solved this problem by using two approaches, one is Hansen and one is Vesic.

And hope we understand what is the difference between these two approaches, if there is a inclined load and what is the difference in Hansen's approach also, if $H_L = 0$. Because previous example 1, now relative to the inclined loading as per Hansen's theory, I use $H_L \neq 0$, but here I have used $H_L = 0$. So, both the cases I have discussed, so hope you understand the differences.

And then differences between Hansen's theory and Vesic's theory, but remember that in both the problems for inclined loading I have used either H_L or H_B I have not used both at a time, but it is also possible, to use both at a time if these are present. So, you have to use it both at a time then definitely for Hansen's theory you have to consider two sets of equation and the factors as $H_L \neq 0$.

But in that case remember that $H_B \neq 0$ there will be H_B value. So, when you calculate the inclination factor in both the problems either we put $H_L = 0$ or $H_B = 0$ but and the inclination factor corresponding to those cases are coming 1. But here, if both horizontal loads are present, then it will not come as 1, according to that it will you have to calculate, clear.

So, these are the problems, so next class I will discuss one example problem when the foundation is on the sloping ground, I have discussed the theories and now in the next class I will solve one example problem, thank you.