

**Advanced Foundation Engineering**  
**Prof. Kousik Deb**  
**Department of Civil Engineering**  
**Indian Institute of Technology-Kharagpur**

**Lecture-15**  
**Shallow Foundation: Bearing Capacity IX**

So, last class I was solving one problem where foundation was subjected to both vertical and the horizontal loads and horizontal load was parallel to length. So, as per Hansen's bearing capacity theory, we have to do some modifications and that I have done only for the depth correction factors. And then we have to calculate the shape correction factors and the inclination factor. So, that I will do in this class, so here.

**(Refer Slide Time: 01:09)**

The whiteboard contains the following handwritten notes:

As  $H_L \neq 0$

$d_{qB} = 1 + 2 \tan \phi \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)^2 \left( \frac{D_f}{B} \right)$   
 $= 1 + 2 \tan 40^\circ \left( \frac{1 - \sin 40^\circ}{1 + \sin 40^\circ} \right)^2 \left( \frac{1}{2} \right) = 1.107$

$d_{qL} = 1 + 2 \tan \phi \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)^2 \left( \frac{D_f}{L} \right)$   
 $= 1 + 2 \tan 40^\circ \left( \frac{1 - \sin 40^\circ}{1 + \sin 40^\circ} \right)^2 \left( \frac{1}{3} \right) = 1.071$

$d_{\gamma B} = d_{\gamma L} = 1$

$i_{qB} = \left[ 1 - \frac{0.5 H_0}{V + A' C_u C_1 \phi} \right]^5 = 1$  (As  $H_0 = 0$ )  
 $i_{qL} = \left[ 1 - \frac{0.5 H_0}{V + A' C_u C_1 \phi} \right]^5 = 1$  (As  $H_0 = 0$ )  
 $i_{\gamma B} = \left[ 1 - \frac{0.5 H_L}{V + A' C_u C_1 \phi} \right]^5 = \left[ 1 - \frac{0.5 H_L}{V} \right]^5$   
 $= \left( 1 - \frac{0.5 \times 300}{1000} \right)^5 = 0.444$   
 $i_{\gamma L} = \left[ 1 - \frac{0.5 H_L}{V + A' C_u C_1 \phi} \right]^5 = \left( 1 - \frac{0.5 \times 300}{1000} \right)^5 = 0.31$

$\phi$  increased ( $\phi' = 40^\circ$ )  
 $\frac{1}{2} \left( \frac{V}{B} \right) > 2$   
 Use  $\phi_{ps} = 1.5 \phi - 17^\circ$   
 $\frac{1}{2} \left( \frac{V}{B} \right) \leq 2$   
 Use  $\phi_r$   
 $\frac{1}{2} \phi_r \leq 34^\circ$   
 Use  $\phi_r = \phi_{ps}$

$R = \sqrt{\frac{V + H_L}{V}}$   
 $= \sqrt{\frac{1000 + 300}{1000}} = 1.141$   
 $V = R C_u i_{\gamma} \left( \frac{V}{B} \right) = 16.7^\circ$

Diagram: Foundation with  $V = 1000 \text{ kN}$ ,  $H = 300 \text{ kN}$ ,  $L = 3 \text{ m}$ ,  $B = 2 \text{ m}$ .  $H_0 = 0$ ,  $H_L = 300$ .  $\phi = 40^\circ$ ,  $c = 0$ .

So, this is the problem that load vertical load of 1000 kN was acting, horizontal load was 300 kN and horizontal load was parallel to the length of the foundation. And it is parallel to  $B$  the horizontal load is 0 but that can be both also. So, and then I talked about how to convert if the  $\phi$  value is measured in triaxial test and then you have to use it for under plane-strain condition.

And then we determine because as your  $H_L \neq 0$  then you have to go for two sets of factors and equations. So that I did for depth correction factors that  $d_{q,B}$  and  $d_{q,L}$  ok. And now  $d_{\gamma,B}$  and  $d_{\gamma,L}$  is one as per Hansen's bearing capacity equation. So, now I will go for the inclination factor first, because that inclination factor you have to multiply with the shape factor.

So, with certain term in the shape factor, so now here  $i_{q,B} = \left(1 - \frac{0.5H_B}{V+A'c_a \cot \phi}\right)^5$ , remember that if there are horizontal and vertical loads which are inclined and eccentric then we have to use that  $B'$  and  $L'$  as per the suggestions given in different theories.

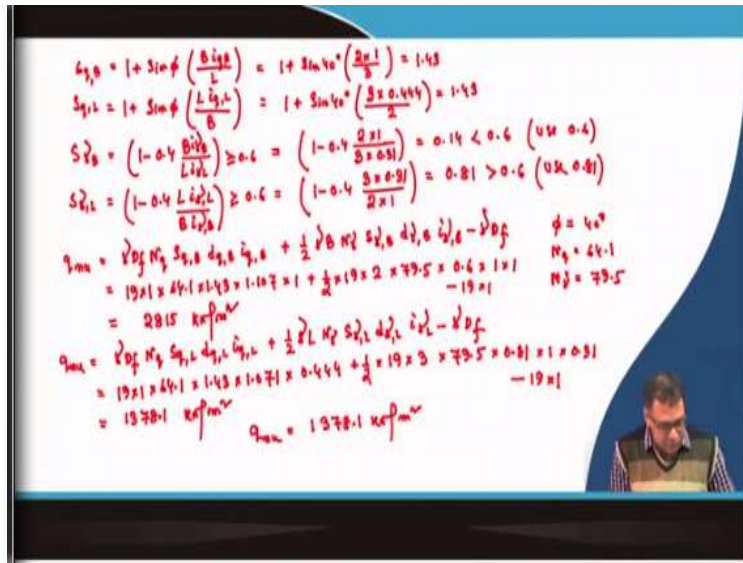
Even Hansen also suggested that you have to use  $B', L'$  that means for inclination factor and the shape factor you have to use  $B'$  and  $L'$ . In the third term you have to use either  $B'$  or  $L'$  and for depth factor you have to use  $\frac{L}{B}$  and  $B$ , remember that. So, this is your value and now this is equal to 1 as  $H_B = 0$ .

Similarly,  $i_{\gamma,B} = \left(1 - \frac{0.7H_B}{V+A'c_a \cot \phi}\right)^5$ . As I mentioned if nothing is mentioned in the question I will use this value as  $\phi$ . But, I have discussed in previous class that some researchers have suggested to use the lower value. But if it is mentioned and the range is given you use this value then only in the question or in the example problem I will use that.

Otherwise I will go with the original one which is  $\phi$ . So, this is equal to  $\phi$  as  $H_B = 0$ . And now  $i_{q,L} = \left(1 - \frac{0.5H_L}{V+A'c_a \cot \phi}\right)^5$ . So, now as the  $c_a$  part that mean the  $c$  is 0, so this part will be 0. So, now the equation will be  $i_{q,L} = \left(1 - \frac{0.5H_L}{V}\right)^5$  because  $c_a = 0$  as cohesion is 0.

So,  $i_{q,L} = \left(1 - \frac{0.5H_L}{V}\right)^5 = \left(1 - \frac{0.5 \times 300}{1000}\right)^5 = 0.444$ . Similarly,  $i_{\gamma,L} = \left(1 - \frac{0.7H_L}{V+A'c_a \cot \phi}\right)^5$ . Here again that your  $c = 0$ , so I will neglect that, so I will write  $i_{q,L} = \left(1 - \frac{0.7H_L}{V}\right)^5 = \left(1 - \frac{0.7 \times 300}{1000}\right)^5 = 0.31$ . So, I have calculated these factors.

**(Refer Slide Time: 06:22)**



So, now I will go to the modification in the shape factor. So, now I will calculate the  $s_{q,B}$ , so this is  $1 + \sin \phi \left( \frac{B i_{q,B}}{L} \right)$  and if loading is not inclined then  $i_{q,B}$  will be 1, so that will go back to our original table equation that I have given. So, but that type of modification is not required for other theories, it is only required for Hansen's theory.

So, now this is equal to  $1 + \sin \phi$  is  $\sin 40^\circ$  and  $B$  is 2 and  $i_{q,B} = 1$  because your  $i_{q,B} = 1$ ,  $i_{\gamma,B} = 1$ ,  $L = 3$ , so this is 1.43. Then  $s_{q,L} = 1 + \sin \phi \left( \frac{L i_{q,L}}{B} \right)$ , so this is  $1 + \sin 40^\circ$ ,  $L$  is 3 m, now  $i_{q,L}$  is 0.444, so divided by 2, so that value is also 1.43. Now,  $s_{\gamma,B} = 1 - 0.4 \frac{B i_{\gamma,B}}{L i_{\gamma,L}}$  and that should be greater than equal to 0.6.

So, now I can write  $1 - 0.4$ ,  $B$  is 2,  $i_{\gamma,B}$  is 1, then  $L$  is 3,  $i_{\gamma,L}$  is 0.31 because this is 0.31. So, this value is 0.14 as it is less than 0.6, so use 0.6 because if it is greater than equal to 0.6, then use 0.6. Then  $s_{\gamma,L} = 1 - 0.4 \frac{L i_{\gamma,L}}{B i_{\gamma,B}} \geq 0.6$ , so I can write  $1 - 0.4$   $L$  is 3,  $i_{\gamma,L}$  is 0.31 divided by  $2 \times 1$ , so that is equal to 0.81 which is greater than 0.6, so use 0.81.

So, now we have calculated two sets of factors or depth factors, shape factors and inclination factors. So, now I will use the two sets of equations, so that is net ultimate bearing capacity, so this equation will be first part is 0 then  $\gamma D_f N_q$  then  $s_{q,B} d_{q,B} i_{q,B} + \frac{1}{2} \gamma B N_q s_{\gamma,B} d_{\gamma,B} i_{\gamma,B}$ . Again all the

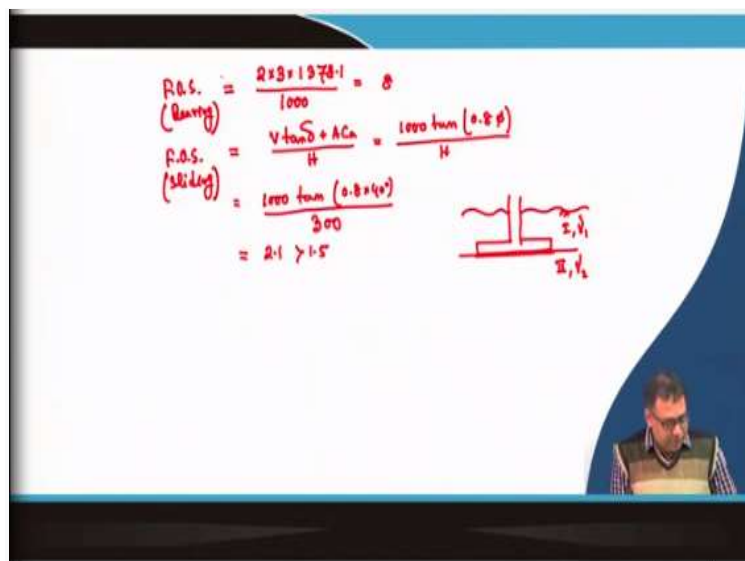
cases I am mentioning that if there is compressibility effect that you have to incorporate compressibility factor.

But the data is not given that is why we are not incorporating that, so that is  $-\gamma D_f$ . So, now I put that this  $\gamma$  is 19,  $D_f$  is 1. So, that depth of foundation is  $D_f = 1$  m. So, this is 1 and for  $\phi = 40^\circ$   $N_q$  is 64.1 and  $N_\gamma$  is 79.5, that I will get from the table of Hansen's bearing capacity equation 64.1 and 79.5.

So, this is  $N_q$  is 64.1, then  $s_{q,B}$  is 1.43 then  $d_{q,B}$  is how much?  $d_{q,B}$  is 1.107, so this is 1.107, then  $i_{q,B} = 1$ , then  $+\frac{1}{2} \times 19 \times B$  is  $2 \text{ m} \times 79.5$ , then  $s_{\gamma,B} = 0.6$ . Then  $d_{\gamma,B}$  is 1 and then  $i_{\gamma,B}$  is also 1 and then  $-19 \times 1$ , so this value is coming out to be  $2815 \text{ kN/m}^2$ , this is one set of equation. Now we will go for the next set of equation and here we have to replace  $B$  by  $L$ , so this will be  $\gamma D_f N_q s_{q,L} d_{q,L} i_{q,L} + \frac{1}{2} \gamma L N_\gamma s_{\gamma,L} d_{\gamma,L} i_{\gamma,L} - \gamma D_f$ .

So, I will put this value  $19 \times 1 \times 64.1 \times s_{q,L}$  which is 1.43 then  $d_{q,L}$  is 1.071. Then  $i_{q,L}$  is 0.444, then  $+\frac{1}{2} \times 19 L$  is  $3 \text{ m} \times 79.5 \times d_{\gamma,L} \times s_{\gamma,L}$  is 0.81. Then  $d_{\gamma,L} = 1$  and  $i_{\gamma,L}$  is 0.31, so this is  $1 \times 0.31 - 19 \times 1$ , so this is coming out to be  $1378.1 \text{ kN/m}^2$ . So, the minimum of these two will give us the  $q_{nu}$  or net ultimate bearing capacity, so  $q_{nu} = 1378.1 \text{ kN/m}^2$ .

**(Refer Slide Time: 15:30)**



Now we have to calculate the factor of safety, so for the factor of safety for bearing that is equal to that total load the footing can carry, that is  $2 \times 3$ , area into the net ultimate bearing capacity, that is 1378.1 divided the total load that is acting on the footing and that value is 8 which is very high. But let us see factor of safety for sliding is equal to that your  $T$  that is  $\frac{V \tan \delta + AC_a}{H}$ , now  $AC_a$  is 0,  $V$  is 1000  $\tan \delta$  and  $\delta$  is taken 0.8 times of  $\phi$  and this is  $H$ .

And one thing I want to mention that sometimes it may happen in that this is the foundation base and this is the ground surface. Now soil in this zone and soil in this zone can be different also. So, now when you calculate that  $\delta$  and the  $c_a$  do not take the soil properties of layer 1, you take the soil properties of the layer 2 because it is the base interaction, so where the foundation is resting.

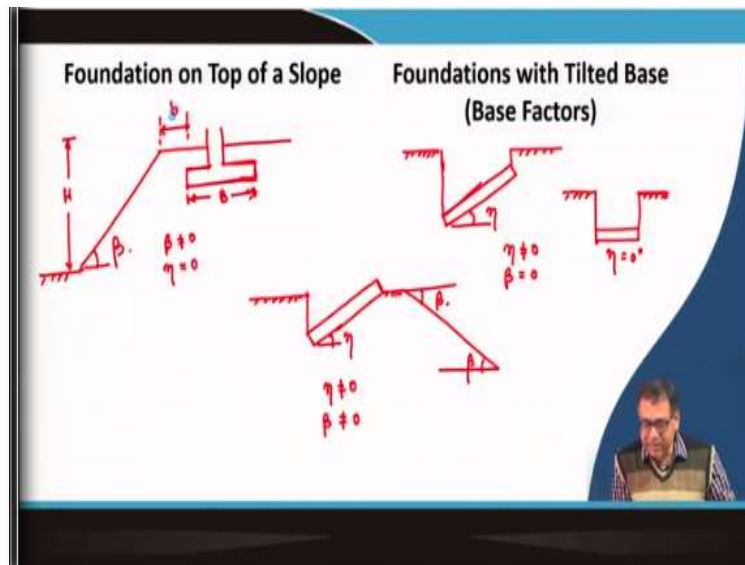
So, here the properties that you have to consider for  $\delta$  and the  $c_a$ , similarly for again the second term because that is bearing capacity for second term  $\gamma$  will be  $\gamma_1$  and the third term  $\gamma$  will be  $\gamma_2$ . Because that is below the base of the foundation, so remember that. So, now here it is  $1000 \times \tan(0.8 \times 40^\circ)$  and the  $H$  is 300 kN, so which is  $2.1 > 1.5$ , it is fine.

So, bearing capacity factor is very high but as per the sliding it is more or less, but still you can reduce some factor of safety. So, you can modify your design and then you can make the design economical also. But here only the bearing capacity or the factor safety is asked to be determined, so that is why we have determined the factor safety.

So, but here it is very high whereas for the sliding it is but still you can modify the design because it is over safe, you can say. So, this is the example to calculate the bearing capacity for inclined load as per Vesic's bearing capacity equation. Remember that if your  $H_L = 0$  but  $H_B \neq 0$  then these two sets are not required, only one set you have to do that is  $d_{q,B}$ .

But this modification you have to do, this modification  $1 + \sin \phi B i_{q,B}$  that modification you have to do. But only that  $L$  part no need to do, only the  $B$  part is sufficient and only the first equation you have to use, not the second equation. So, these two sets are only required if  $H_L \neq 0$ .

(Refer Slide Time: 19:53)



Now the next effect that I will discuss that if the foundation is with tilted base. That means till now we have discussed the foundation where the base is perfectly horizontal but there may be a possibility that foundation base can be tilted also. Suppose you have a foundation like this, so this is your foundation which is tilted at an angle  $\eta$  with the horizontal. And that can be like this also, this is your ground surface.

And here the ground surface can be at the same level, but their foundation is tilted it is not our normal foundation like this where the tilt is 0, this is our foundation where tilt is 0. But here the foundation is tilted with an angle, then how we can determine the bearing capacity? Another case is that foundation may be in the sloping ground, then how we can calculate bearing capacities?

That our foundation is in the sloping ground, for example, this is your foundation and this is a slope. And your foundation is here on the top of a slope, it may be on the surface of the slope or it may be at a depth below the surface of the slope. So, this is your  $B$ , so this is the slope with an angle of  $\beta$  and this is the height of the slope, clear. Now how we can calculate the bearing capacity in such cases?

That the foundation on top of a slope or foundation with tilted base, now it can be both also that foundation on the slope plus it is tilted also. For example, this is the foundation which is tilted

one and then it is on the slope also. So, that means it is on the slope also, with an angle of  $\beta$  say, but at the same time it is tilted, so that different combinations can be possible.

That it has both  $\eta$  and  $\beta$ , so this case your  $\beta \neq 0$ , but your  $\eta = 0$ . This case  $\eta \neq 0$  but  $\beta = 0$ , this case  $\eta \neq 0$  and  $\beta \neq 0$ . So, that means foundation is on the slope at the same time it is tilted. So, then how we can calculate the bearing capacity all those things that we have to discuss. So, now if the foundation is on the slope and foundation with a tilted base, this tilted base.

(Refer Slide Time: 24:56)

**Effect of Ground Factors (base on slope) and Base Factors (tilted base)  
Hansen's or Vesic's bearing capacity Theory**

$$q_n = cN_c s_c d_c i_c g_c b_c + qN_q s_q d_q i_q g_q b_q + 0.5\gamma BN_r s_r d_r i_r g_r b_r$$

$g_i$  is the ground factor (base on slope) and  $b_i$  is the base factor (tilted base)

Hansen		Vesic	
Factors	Value	Factors	Value
Ground	$g_c = \frac{\beta^2}{147^\circ}$ for $\phi = 0$	Ground	$g_c = \frac{\beta}{5.14}$ for $\phi = 0$ $\beta$ in radians
	$g_c = 1 - \frac{\beta^2}{147^\circ}$ for $\phi > 0$		$g_c = i_c - \frac{1-i_c}{5.14 \tan \phi}$ for $\phi > 0$
	$g_c = g_r = (1 - 0.5 \tan \beta)^2$		$g_r = g_c = (1 - \tan \beta)^2$
Base	$b_c = \frac{\eta^2}{147^\circ}$ for $\phi = 0$ $b_c = 1 - \frac{\eta^2}{147^\circ}$ for $\phi > 0$	Base	$b_c = g_c$ (for $\phi = 0$ )
	$b_c = \exp(-2\eta \tan \phi)$ $\eta$ in radians		$b_c = 1 - \frac{2\beta}{5.14 \tan \phi}$ for $\phi > 0$
	$b_c = \exp(-2.7\eta \tan \phi)$		$b_c = b_r = (1 - \eta \tan \phi)^2$ $\eta$ in radians

Vesic and Hansen have suggested the factors. Now as I mentioned that originally we know that there are three correction factors. That means shape factor, depth factor, inclination factor but I have discussed about one more factor that is the compressibility factor. Now here two more factors are introduced by Vesic and Hansen. That is  $b$ ,  $g$  where  $g$  is the ground factor that means base on slope if the foundation is on slope that is the ground factor.

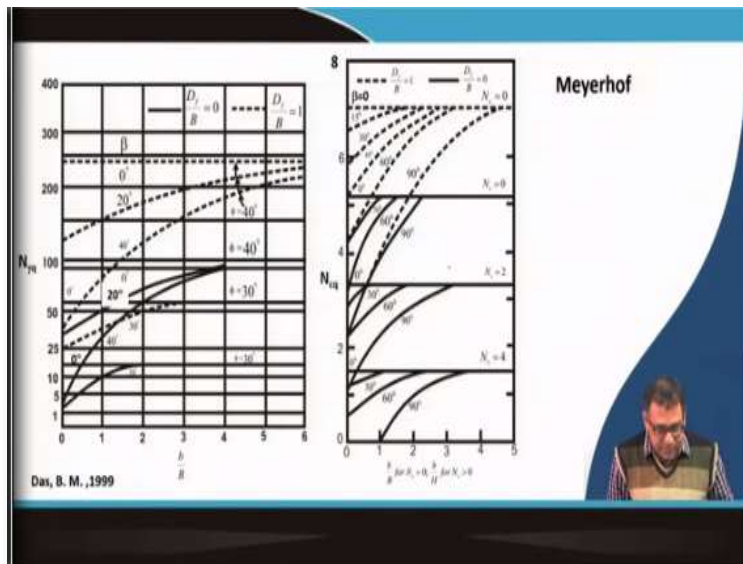
And  $b$  is the base factor if it has a tilted base, so that means the  $s_c d_c i_c g_c b_c$ . So, that means this  $g$  is the ground factor if foundation is on the slope and  $b$  is the base factor if it has a tilted base, so these two factors are also introduced. In addition to that if we have the properties then we can introduce the  $c_c$  also, that means there were total six factors shape factor, depth factor, inclination factor, ground factor, base factor and the compressibility factor.

So, these six factors can be used, so this base factor and shape factor, these two factors are proposed in Hansen's and Vesic's bearing capacity theories, so these two theories. So, now what is the  $g_c$ , how I can get the ground factor and the base factor? So, this is the  $g_c$  expression,  $\beta$  is the slope angle and this is  $147^\circ$  if  $\phi = 0^\circ$  and  $g_c = 1 - \frac{\beta}{147^\circ}$  if  $\phi \geq 0^\circ$ .

And  $g_q$  and  $g_\gamma$  can be determined by using this equation for any  $\phi$  value. Similarly  $b_c$  also can be determined by using this equation for  $\phi = 0^\circ$  and  $b_c$  by this equation if  $\phi > 0^\circ$ , and  $b_q$  by this equation but remember that here  $\eta$  is in radian and  $b_\gamma$  also can be determined by using this equation where  $\eta$  is also in radian.

Similarly as per Vesic this is the table by which we can determine the ground factors and the base factors. And remember that all this  $\beta$  and  $\eta$  are in radians in these equations, that means actually it is in degree. So, when you put them in this equation you have to convert them to radian by which we can determine all these factors as suggested by Hansen and Vesic.

**(Refer Slide Time: 28:06)**



And then one thing that I want to mention that here Meyerhof also suggested that how we can determine the bearing capacity when foundation is resting on a slope? And Meyerhof's suggestion, so this is the normal these two are the suggestions by Hansen and Vesic where it is in



the slope and the base is tilted. Meyerhof also suggested and he has incorporated the effect of this footing distance or distance of edge of the footing from the slope edge, so that is  $b$ .

But the effect of that distance of edge of the foundation from the slope edge is not in the factors those suggested by Hansen and Vesic. Where this  $b$  value effect is not there because whether you place the foundation at any distance but you will get more or less same factors. Because those factors are either functions of  $\beta$  or  $\eta$ , you can see these are all either functions of  $\beta$  or  $\eta$ .

So, that depth is not given but Meyerhof has suggested when foundation is resting on a slope with the distance  $H$  from the slope edge. But Meyerhof has not given again this in tilted base factors, so that means if you go for tilted base case, then you have to go only for either Vesic or Hansen. But if you want to go for slope if foundation is on the slope then you can use either Vesic or Hansen or Meyerhof.

So, in the next class I will solve one particular problem and then I will discuss all these effects. That how you can determine the bearing capacity when the foundation is resting on a slope by using Meyerhof's approach then Hansen's and Vesic's approach? And if the foundation has a tilted base then how you can determine the bearing capacity by using Meyerhof's and Vesic's approach? So, these things I will discuss in next class, thank you.