

Advanced Foundation Engineering
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Lecture-14
Shallow Foundation: Bearing Capacity VIII

So, last class I have discussed that how you can determine the bearing capacity for inclined load and I have use Meyerhof's bearing capacity equation. But today in this class I will discuss that how you can calculate the bearing capacity if you use Hansen's equation under inclined load.

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Foundation under Inclined Load (either H_B or H_L or both)

Hansen's Theory

Note: With a vertical load and a load H_L i.e $H_L > 0$ (horizontal load parallel to length) and either $H_B = 0$ or $H_B > 0$ (horizontal load parallel width), two sets of shape ($s_{c,B}$ and $s_{c,L}$), depth ($d_{c,B}$ and $d_{c,L}$) and inclination ($i_{c,B}$ and $i_{c,L}$) factors have to be computed. In case of i, L subscripts, use L/B or D/L ratio for shape and depth factors calculation, respectively. Bearing capacity has to be determined by two following equations (lowest of these two is the bearing capacity).

$$q_m = q_{ult} - \gamma D_f = cN_{c,B} s_{c,B} d_{c,B} i_{c,B} + \gamma D_f N_{q,B} s_{q,B} d_{q,B} i_{q,B} + 0.5 \gamma B N_{\gamma,B} s_{\gamma,B} d_{\gamma,B} i_{\gamma,B} - \gamma D_f \quad (1)$$

$$q_m = q_{ult} - \gamma D_f = cN_{c,L} s_{c,L} d_{c,L} i_{c,L} + \gamma D_f N_{q,L} s_{q,L} d_{q,L} i_{q,L} + 0.5 \gamma L N_{\gamma,L} s_{\gamma,L} d_{\gamma,L} i_{\gamma,L} - \gamma D_f \quad (2)$$

So, if the foundation is under inclined load, then either H_B or H_L , so what does it mean? That H_B means horizontal component of the inclined load which is parallel to B and H_L means the horizontal component of the inclined load which is parallel to length. So, sometimes in previous problem I have given an inclined load itself but sometimes the vertical load and the horizontal load can be given separately.

That means the footing is under vertical load as well as the horizontal load. So, you can get the resultant inclined load and then you can determine or find the inclination with respect to vertical and you can resolve it like the previous problem. But that means in two ways either the inclined load can be given directly then you have to determine the horizontal and vertical components of that load.

Another option is that vertical and horizontal components can be given separately. In such case you can get the resultant inclined load and the amount of inclination also. So, but as I mentioned if I want to use Hansen's theory under inclined load, then only inclination is not sufficient. We have to know that the vertical or the horizontal components of the load which are acting in which direction whether it is parallel to L or parallel to B or in both directions the horizontal load is acting.

That is also an option that in both the directions horizontal load can act along with the vertical load. So, in such case we can use Hansen's theory and Vesic's theory, but Vesic's theory is straightforward and not much modifications are required. But in Hansen's theory we have to do few modifications, as we proposed in previous lectures. So, I have already discussed about Hansen's theory in previous lectures.

But those you can use if loading is perfectly vertical but if the loading is inclined then you have to do few modifications, so what are those modifications? So, first note that with a vertical load and the horizontal load H_L ; that is $H_L > 0$ H_L means horizontal load parallel to length. And either $H_B = 0$ or $H_B > 0$, H_B means horizontal load parallel to width.

Then we have to calculate two sets of shape factors, inclination factors and the depth factors, what does it mean? That suppose if this is your foundation and you have a vertical load and horizontal load, H_L . For example, this is the width of the foundation and this is the length of the foundation. So, now if the horizontal load is parallel to length then H_L because horizontal load is acting.

So, I should then draw this is actually perpendicular to the plane, so this should be a dot or which is perpendicular to the plane, this horizontal load. So, that means this is H_L or your H_B can be 0 or $H_B > 0$. So, that means if your horizontal component of the inclined load or the horizontal load is parallel to L . In such case, we have to consider two sets of shape factor, depth factor and inclination factor to incorporate the inclined load effect in these factors or in bearing capacity equations.

So, and in such case when you use these two sets, we have to interchange the L and B also and if it there is eccentric loading, then L' and B' . But 2 sets we have to calculate only if H_L is present along or if H_B can be 0 or H_L is there along with H_B also. But if the H_L is 0 and you have only H_B then one set is sufficient, and that is your normal set, that I have discussed in the previous lectures, that means this equation number 1.

So, that we have to use only that if $H_L = 0$, then only one set is sufficient, that means if $H_L = 0$ then you calculate the shape factor, depth factor, inclination factor and all these factors as usual as you do, then that is sufficient. But if $H_L \neq 0$, then we have to use both equations 1 and 2. That means H_L is there, H_B can be 0 or there can be H_B also.

So, that means here $H_L = 0$ and $H_B \neq 0$; that mean this is the one case. Another case both H_L we have to use equation 1 and 2, if $H_L \neq 0$ but H_B can be 0 or $H_B \neq 0$ also. So, that means the 3 cases, one case that $H_L = 0$, $H_B \neq 0$, then only the equation 1 or the one set is sufficient. But if $H_L \neq 0$ $H_B = 0$, then 2 sets.

Again if $H_L = 0$ and $H_B = 0$ again 2 sets, that means whenever $H_L \neq 0$ whether H_B is present or not, then we have to take 2 sets of factors. And we have to calculate the bearing capacity by using these two equations. So, what is the difference between these two equations? As I mentioned you have to interchange L and B . So, that means in the first equation, the factors we have to calculate in the normal way as we do considering that is the B , all are in terms of B , that means $\frac{B}{L}$ and $\frac{D_f}{B}$, the way it is given.

And we have to use in the third term B , if it is eccentric loading then you have to go for B' . So, but if we have to go for both the sets, then one set in terms of B , another set in terms of L and that means L we have to interchange $\frac{B}{L}$ and $\frac{L}{B}$. That means now it is not $\frac{B}{L}$ it will be $\frac{L}{B}$ or $\frac{D_f}{L}$ and in the third term, we have to use the L not the B . And the lowest of these two will give us the bearing capacity, clear. Because this is important for Hansen's theory, but in Meyerhof's theory this type of recommendation is not there even not in IS code also, even not in Vesic also.

Because Vesic as you remember that in inclination factor calculation already that the direction of load, H parallel to L or parallel to B is already incorporated, so in Vesic also it is not required, but in Hansen it is required, so this is one modification, we have to do another modification.

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Shape, depth, inclination factor for the Hansen's bearing capacity equation: Bowles, 1997

Factors	Value
Shape	$s_x = 1 + \frac{N_q}{N_c} \left(\frac{B}{L} \right)$ for $\phi > 0$ $s_x = 1$ for strip footing
	$s_x = 0.2 \frac{B}{L}$ for $\phi = 0$
	$s_x = 1 + \sin(\phi) \left(\frac{B}{L} \right)$
	$s_x = (1 - 0.4 \frac{B}{L}) \geq 0.6$
Depth	$d_x = 0.4k$ For $\phi = 0$
	$d_x = 1 + 0.4k$ $k = \frac{D_f}{B}$ for $D_f/B \leq 1$ and $k = \tan^2(D_f/B)$ for $D_f/B > 1$, $\phi > 0$ <small>k in radian</small>
	$d_x = 1 + 2(\tan \phi)(1 - \sin \phi)^2 k$
	$d_x = 1$

So, that means these are the factors I am talking about. So, now when you calculate the s_{cB} because there is a s_{cB} , so that means s_{cB} then you use B . Now if you calculate say s_{cL} then the s_{cB} means, this equation if you use the s_{cL} then you have to use $1 + \frac{N_q}{N_c} \left(\frac{L}{B} \right)$ for s_{cL} . Similarly for $\frac{D_f}{B}$ also you have to replace by $\frac{D_f}{L}$ in calculation of d_{cL} or d_{qL} , like this.

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Factors	Value	H = horizontal component of inclined load, V = vertical component of inclined load
Inclination	$i_x = i_y = \frac{1 - i_q}{N_q - 1}$ For $\phi > 0^\circ$	$C_u =$ base adhesion, 0.6 to 1 times the base cohesion
	$i_x = 0.5 - \sqrt{1 - \frac{H}{A'c_u}}$ For $\phi = 0^\circ$	
	$i_y = \left(1 - \frac{0.5H}{V + A'c_u \cot \phi} \right)^2$	
	$i_z = \left(1 - \frac{0.7H}{V + A'c_u \cot \phi} \right)^2$	
	$i_z = \left(1 - \frac{(0.7 - \eta^2/450^\circ)H}{V + A'c_u \cot \phi} \right)^2$ If foundation has tilted base	

Note: Some researches suggested to use exponent value of 2 to 3 for i_q and 3 to 4 for i_x .

Bowles, 1997

Note: In case of eccentric loading, one can use Hanson's Equation with B' and L' to compute the shape and inclination (effective are $A'=B'L'$) factors, but B and L to compute the depth factors and B' (or L') in the term $0.57B' N_q$.

And then these factors again you have to calculate whose effective area whether A or B does not matter, it is area. Again these other things are same as I discussed and for eccentric loading you remember where you have to use L' and B' , where you have to use L and B .

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Hansen's Theory (edited)

Note: For Hansen theory, it is suggested to use the **inclination factors** to compute the shape factors as follows:

- Calculate the inclination factors are per the inclination factor table.
- Compute the shape factor considering inclination factors as:

Note: in case of eccentric loading, one can use B' and L' to compute the shape factors.

$$s_{c,B} = 0.2 \frac{B i_{c,B}}{L} \quad s_{c,L} = 0.2 \frac{L i_{c,L}}{B} \quad (\text{For } \phi=0)$$

$$s_{e,B} = 1 + \frac{N_c}{N_q} \left(\frac{B i_{e,B}}{L} \right) \quad s_{e,L} = 1 + \frac{N_c}{N_q} \left(\frac{L i_{e,L}}{B} \right) \quad (\text{For } \phi>0)$$

$$s_{c,B} = 1 + \sin(\phi) \left(\frac{B i_{c,B}}{L} \right) \quad s_{c,L} = 1 + \sin(\phi) \left(\frac{L i_{c,L}}{B} \right) \quad \text{and} \quad s_{e,B} = (1 - 0.4 \frac{B i_{e,B}}{L i_{e,L}}) \geq 0.6 \quad s_{e,L} = (1 - 0.4 \frac{L i_{e,L}}{B i_{e,B}}) \geq 0.6$$

• Bearing capacity has to be determined by two following equations (lowest of these two is the bearing capacity).

$$q_{un} = q_{ult} - \gamma D_f = c N_c s_{c,B} d_{c,B} i_{c,B} + \gamma D_f N_q s_{q,B} d_{q,B} i_{q,B} + 0.5 \gamma B N_s s_{s,B} d_{s,B} i_{s,B} - \gamma D_f$$

$$q_{un} = q_{ult} - \gamma D_f = c N_c s_{c,L} d_{c,L} i_{c,L} + \gamma D_f N_q s_{q,L} d_{q,L} i_{q,L} + 0.5 \gamma L N_s s_{s,L} d_{s,L} i_{s,L} - \gamma D_f$$

Now the second modification to incorporate this inclination effect that for Hansen's theory it is suggested to use the inclination factor to compute the shape factor as follows. That if the load is inclined, so Hansen's bearing capacity equation, we calculate the shape factor and then you modify the shape factor because original shape factors what is given? The shape factor is $s_c = 1 + \frac{N_q}{N_c} \left(\frac{B}{L} \right)$.

So, now that you have to multiply by $i_{c,B}$, that means first you have to calculate these inclination factors, then that inclination factor we have to multiply with these respective terms in shape factor. So, initially shape factor if you see this table, so that means this table you can use if loading is perfectly vertical, you cannot use this table to calculate the shape factor if loading is inclined or if a horizontal load is present as per Hansen.

In such case we have to modify that, first we have to calculate the inclination factors, then we have to modify these equations. Because you can see that if it is $s_{c,B}$ this $i_{c,B}$ we have to multiply with B . If it is $s_{c,L}$ then this will be L , this will be B , then we have to multiply with $i_{c,L}$, $i_{c,L}$ means whether there is a horizontal load, H_L or H_B , that you have to consider.

If it is H_L then you have to consider H_L when calculate the $i_{c,L}$, if this is $i_{c,B}$, then you consider H_B , if $H_B = 0$, then the $i_{c,B}$ will be just 0.5. So, now $i_{c,L}$ means these will be so for example I can write that for say $i_{q,B}$, so that mean this will be your H_B . And if it is $i_{q,L}$ then it is be L , that means the horizontal load parallel to L . Now sometimes the H_B can be 0, H_L can be 0 also.

So, in such case you put 0 and calculate the H_B and H_L all those things. But this as I mentioned these H_B , H_L and B that part you have to use only if $H_L \neq 0$. So, that means there is the option when H_L cannot be 0 and when you calculate $i_{q,L}$ or $i_{q,B}$, only option is that H_B can be 0 not has to be 0, it can be 0 it may not be 0 also it maybe 0 may not be 0.


But H_L will always be there, then only I have to go with these two sets otherwise only one set is sufficient and if H_L is 0 then only one set is sufficient. If $H_L \neq 0$ then only you have to go for two sets, that is $i_{q,B}$ or $i_{q,L}$, in that case, H_B may be 0 may not be 0. So, this is the case, we have to modify, that means we have to interchange this L , that means this is B $i_{c,B}$ and similarly L B and then for $s_{\gamma,B}$, this is B by L $i_{\gamma,B}$, $i_{L,B}$ $i_{\gamma,L}$.

And also interchange that $i_{\gamma,L}$, B $i_{\gamma,B}$, and these things we have to modify the shape factors. And again these two bearing capacity equations you have to use and lowest of these two will give us the bearing capacity value and this note is also given. And one more thing that I want to mention that suppose if your H_L is 0 but only H_B is present then also you have to do this modification.

So, that means you have to calculate the inclination factor and then you have to multiply these inclination factors with the shape factors. But in such case this B $s_{c,B}$ or $s_{c,L}$ these two sets are not required, we have to go for only one set, that is $s_{c,B}$ but this modification is must. That means if the loading is inclined or the horizontal load is parallel to L or B but this modification to calculate the shape factor you have to do. But if $H_L \neq 0$, then we have to go for two sets and if $H_L = 0$ you have to go for one set.


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Shape, depth, inclination factor for the Vesic's bearing capacity equation:

Factors	Value	Bowles, 1997
Shape	$s_r = 1 + \frac{N_q}{N_c} \left(\frac{B}{L} \right)$	<p>Note: Use B and L not B' and L' to compute shape and depth factors in case of eccentric loading.</p>
	$s_c = 1$ for strip footing	
	$s_q = 1 + \tan(\phi) \left(\frac{B}{L} \right)$	
	$s_r = \left(1 - 0.4 \frac{B}{L} \right) \geq 0.6$	
Depth	$d_r = 1 + 0.4k \cdot k = \frac{D_r}{B}$ for $D_r/B \leq 1$ and $k = \tan^{-1}(D_r/B)$ for $D_r/B > 1$, k in radian	
	$d_q = 1 + 2(\tan \phi)(1 - \sin \phi)^2 k$	
	$d_r = 1$	

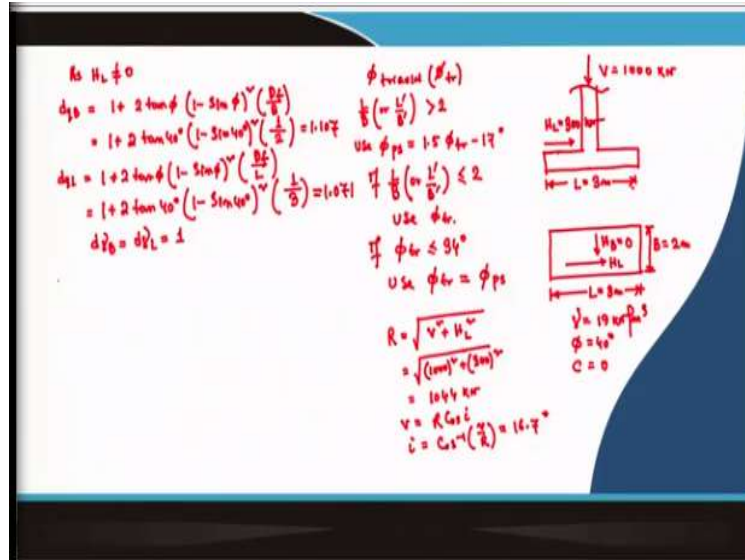
So, now this is Hansen, as per Vesic you can see but remember that these modifications you have to go only with shape factor not with depth factor, it is shape factor only not with depth factor. So, and for Vesic, this is Vesic's table here no such modifications are required, the only one way you have to do it whether it is parallel to L or parallel to B .

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Factors	Value	Bowles, 1997
Inclination	$i_r = i_q = \frac{1 - i_c}{N_q - 1}$ For $\varphi > 0^\circ$	<p>Note: Use effective are ($A'=B'L'$) to compute inclination factors in case of eccentric loading. Use B' in the term $0.5\gamma B'N_c$ (even if $H_1=H_2$)</p>
	$i_r = 1 - \frac{mH}{A_c N_c}$ For $\varphi = 0^\circ$	
	$i_q = \left(1 - \frac{H}{V + A_c c \cot \phi} \right)^m$	
	$i_r = \left(1 - \frac{H}{V + A_c c \cot \phi} \right)^{m+1}$	
$m = m_q = \frac{2 + B/L}{1 + B/L}$ When H parallel to B, $m = m_r = \frac{2 + L/B}{1 + L/B}$ When H parallel to L If you have both H_q and H_r use $m = \sqrt{m_q^2 + m_r^2}$ Note: Use B and L not B' and L' to compute m		

And because that parallel to L or parallel to B effect is already incorporated to calculate the inclination factor, so no such modifications are required in case of Vesic, so you can do as it is. But for Hansen these modifications are suggested.

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So, now let us do one particular problem and then I can more clearly explain that how these modifications can be incorporated in Hansen's equation. So, the problem is that this is the footing, so this is in length direction. So, the length of the footing is 3 m. So, there is a vertical component of the load which is 1000 kN and that is a horizontal component of the load H_L which is say 300 kN.

And which is parallel to L because this is the L direction. So, that means if I go for this plan of the footing. So, this is the length direction L which is 3 m and this is the width direction B , which is 2 m. So, the horizontal load which is parallel to L and parallel to B , $H_B = 0$. That means no component of load parallel to B is acting.

But that does not matter as $H_L \neq 0$, and then you have to go for 2 sets of equation as per Hansen because we are using Hansen's bearing capacity, that is what the equation. But in such case as I mentioned H_B may be 0 may not be 0, in this case H_B is 0, but there is a case where both H_L and H_B are present. So, that means this is the case and then my soil property, unit weight of the soil is 19 kN/m^3 , $\phi = 40^\circ$, $c = 0$, water table is not considered.

And before we solve this question one more thing I want to mention that this is a rectangular footing. So, now if sometimes you will find that we will test the soil sample. And then you will

get a ϕ value and that I want to use that ϕ value for this case or if my loading is strip footing but I cannot use directly that ϕ value for strip footing condition.

Because strip footing is just plane-strain condition and when you will determine the triaxial ϕ that is not under plane-strain condition. So, now if ϕ_{triaxial} value is given and if you want to use them under plane-strain condition where the length of foundation is higher than the width of the foundation, then we have to modify the ϕ value.

So, in your case if directly ϕ is given, then you can use that ϕ either it is a strip footing or the rectangular footing or square footing. But if in the question, it is given ϕ_{triaxial} , then you check whether that ϕ value you can directly use or you have to do some modification to use it for the strip footing case or when the length of the foundation is much higher than the width of the foundation, so that I am giving.

Suppose, if your ϕ_{triaxial} is given, which is called as ϕ_r . Now if your $\frac{L}{B}$ or if it is eccentric loading $\frac{L'}{B'} > 2$. Then you use $\phi_{\text{plan-strain}}$ or modified ϕ which is $1.5 \times \phi_{\text{triaxial}} - 17^\circ$. So, that means if length is 2 times more than the width then if ϕ_{triaxial} is given, then you convert it by using this equation.

Now if your $\frac{L}{B}$ or $\frac{L'}{B'} \leq 2$, then you use ϕ_{triaxial} . Now if any case that $\phi_{\text{triaxial}} \leq 34^\circ$ then you use whether it the length is more than 2 times the width, you use $\phi_{\text{triaxial}} = \phi_{\text{plan-strain}}$. That means, if it is greater than 34° and if length is more than 2 times the width, then you can convert the ϕ_{triaxial} with this equation.

If it is less than 34° or equal to 34° then whatever the case use that 34° . And if length is less than equal to 2 times the width and but $\phi_{\text{triaxial}} > 34^\circ$ then you use that ϕ_{triaxial} . So, but if only ϕ is given for such case, for these problems only ϕ is given because it is specifically not mentioned that how this ϕ is been determined. Then whatever is the condition, whatever the $\frac{L}{B}$ ratio use the ϕ as equal to whatever ϕ is given.

So, and then for one more condition as I mentioned, in the previous problem I have given the inclined load directly and the inclination. But in this problem as in Vesic's equation it needs to note whether the horizontal load is parallel to L or B . So, it is given like this, you can convert it to inclined load also by using this equation or just take the resultant that is $\sqrt{V^2 + H_L^2} = \sqrt{1000^2 + 300^2} = 1044 \text{ kN}$.

Now you know that $V = R \cos i$ or, $V = R \cos \alpha$ so that $i = \cos^{-1} \left(\frac{V}{R} \right) = 16.7^\circ$. So, the angle with vertical is 16.7° and the resultant load or inclined load is 1044 kN. But as we are using Hansen's equation, so we will use this way. So, now we have to calculate the two sets of the bearing capacity factors because your $H_L \neq 0$. As $H_L \neq 0$, then we have to go for two sets of bearing capacity or the correction factors.

And so, as for depth correction factors no modifications are suggested as per Hansen's theory, so first we will calculate the depth factors but depth factors also you have to go for two sets but only that modification to incorporate the inclination factor is suggested only with shape factor not with depth factor. But depth factor also we have to go for two sets, if $H_L \neq 0$.

So, now my $d_{q,B}$ is equal to why I am not going for d_c because as $c = 0$, so first time I am not calculating. So, $d_{q,B} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left(\frac{D_f}{B} \right)$. So, that is $1 + 2 \tan 40^\circ (1 - \sin 40^\circ)^2 \times \left(\frac{1}{2} \right)$, so this is 1.107. Now, I will go for $d_{q,L}$, so that is $d_{q,B} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left(\frac{D_f}{L} \right)$.

Now this is equal to $1 + 2 \tan 40^\circ (1 - \sin 40^\circ)^2 \times \left(\frac{1}{3} \right)$, so that value is 1.071, and as per Hansen the $d_{\gamma,B}$ and $d_{\gamma,L}$ all both are 1. So, now I have to go for $i_{q,B}$ and $i_{q,\gamma}$, $i_{\gamma,B}$, $i_{\gamma,L}$. First we have to calculate the inclination factor then we have to modify the shape factor and then we have to use them.

So, in the next class I will solve the remaining part of this problem because I have determined the depth factors only. Because where only we have to go for two sets as $H_L = 0$, but that modification you have to go for the shape factors, so that I will do in the next class, thank you.