

**Advanced Foundation Engineering**  
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**Lecture-13**  
**Shallow Foundation: Bearing Capacity VII**

So, last class, I was solving one particular problem on two-way eccentricity. And I did not mind the effective length and width of the foundation. Now today I will solve the remaining part of that problem.

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A rectangular footing of size 3m X 6m is founded at a depth of 1m in a homogeneous c- $\phi$  soil. The water table is at a great depth. The unit wt of soil 18 kN/m<sup>3</sup>. Determine net safe load. c= 100 kPa and  $\phi = 20^\circ$ . The load is vertical and  $e_L= 1.2m$  and  $e_B=0.3m$ . Use Meyerhof Equation and assume general shear failure and a factor of safety of 3.

$L = 6m, B = 3m, e_L = 1.2m, e_B = 0.3m$

$\frac{e_L}{L} = \frac{1.2}{6} = 0.2 < 0.5$      $0.2 < 0.5$  and  $0 < \frac{e_B}{B} < 0.167$   
 $\frac{e_B}{B} = \frac{0.3}{3} = 0.1$     Case II

$A' = \frac{1}{2} (L_1 + L_2) B$

$\frac{L_1}{L} = 0.88$      $\frac{e_L}{L} = 0.2$   
 $\frac{L_2}{L} = 0.12$      $\frac{e_B}{B} = 0.1$

$L_1 = 0.88 \times 6 = 5.28m$   
 $L_2 = 0.12 \times 6 = 0.72m$   
 $L' = 5.28m$

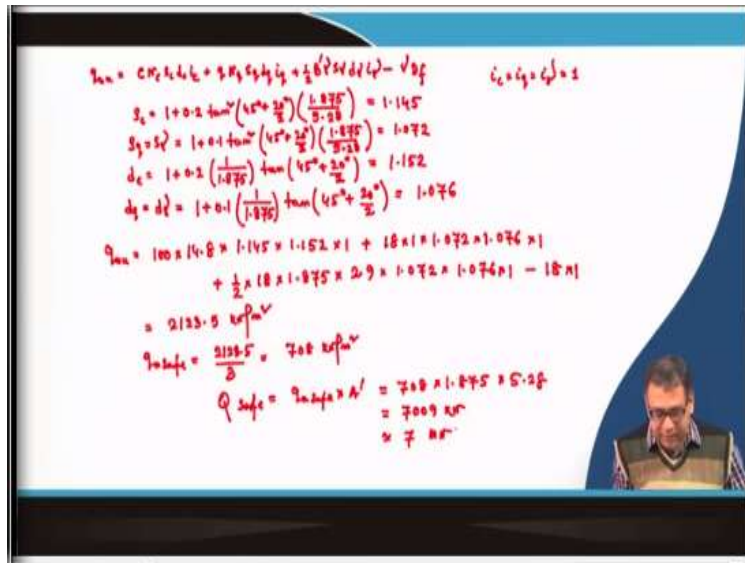
$A' = \frac{1}{2} (5.28 + 0.72) \times 3$   
 $= 9.9 m^2$   
 $B' = A'/L' = \frac{9.9}{5.28} = 1.875m$

Case II

Das, B. M., 1999

So, this was the problem that the foundation has width 3 m and length 6 m was placed at a depth of 1 m from the ground. And the  $e_L$  value is 1.2 m and  $e_B$  value is 0.3 m. And I have determined the effective length which is 5.28 m, and effective width which is 1.875 m original length was 6 m and original width was 3 m. And now we have to use Meyerhof's equation and then we assumed the general shear failure and the factor of safety is 3.

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So, now if we use Meyerhof's bearing capacity equation, so that equation for  $q_{mu}$  which is  $cN_c s_c d_c i_c + \gamma D_f N_q s_q d_q i_q + \frac{1}{2} B' \gamma N_\gamma s_\gamma d_\gamma i_\gamma - \gamma D_f$ . So, as there is the perfectly vertical load, so  $i_c, i_q, i_\gamma$  will be equal to 1. So, now let me determine those bearing capacity factors. So,  $s_c$  is  $1 + 0.2 \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) \times \frac{B'}{L'}$ .

And  $B'$  is 1.875 and  $L'$  is 5.28, so this is 5.28, yes, 5.28. So, now this value will be 1.145, similarly,  $s_q = s_\gamma = 1 + 0.1 \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) \left( \frac{B'}{L'} \right) = 1 + 0.1 \tan^2 \left( 45^\circ + \frac{20^\circ}{2} \right) \left( \frac{1.875}{5.28} \right) = 1.072$ .

Similar way I can calculate the  $d_c$  also this is  $1 + 0.2 \frac{D_f}{B'} \tan \left( 45^\circ + \frac{\phi}{2} \right) = 1 + 0.2 \times \frac{1}{1.875} \tan \left( 45^\circ + \frac{20^\circ}{2} \right)$ , so this value is 1.152. Now,  $d_q = d_\gamma = 1 + 0.1 \frac{D_f}{B'} \tan \left( 45^\circ + \frac{\phi}{2} \right) = 1.076$ .

So, net ultimate bearing capacity which is equal to  $100 \times 14.8 \times 1.145 \times 1.152 \times 1 + 18 \times 1 \times 1.072 \times 1.076 \times 1 + \frac{1}{2} \times \gamma B'$  is  $18 \times 1.875 \times 2.9 \times 1.072 \times 1.076 \times 1 - 18 \times 1$ . Then these two factors. So, now this value is coming out to be 2123.5 kN/m<sup>2</sup> and  $q_{n,safe} = \frac{2123.5}{3}$  which is 708 kN/m<sup>2</sup>.

Now the  $Q_{safe} = q_{n,safe} \times A' = 708 \times 1.875 \times 5.28$ , so that is around 7000 kN, which is roughly 7 mN. So, you can see that in previous problem I considered the one-way eccentricity and safe load was around 10,000 kN, but now here I have taking the two-way eccentricity. So, the

effective area has been further reduced and then the total safe load that the footing can carry is 7 mN or the 7000 kN, which is reduced compared to the first case.

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A rectangular footing of size 3m X 6m is founded at a depth of 1m in a homogeneous c- $\phi$  soil. The water table is at a great depth. The unit wt of soil 18 kN/m<sup>3</sup>, c = 100 kPa and  $\phi = 20^\circ$ . The load is vertical and  $e_L = 0.6$ m and  $e_B = 0.3$ m. Determine the effective area of the foundation.

Handwritten calculations:

$$L = 6\text{m}, B = 3\text{m}, e_L = 0.6\text{m}, e_B = 0.3\text{m}$$

$$\frac{e_L}{L} = \frac{0.6}{6} = 0.1, \frac{e_B}{B} = \frac{0.3}{3} = 0.1 \quad \frac{e_L}{L} < 0.167 \text{ and } \frac{e_B}{B} < 0.167$$

Case IV

$$A' = L_2 B + \frac{1}{2} (B + B_2) (L - L_2)$$

$$\frac{B_2}{B} = 0.24, \frac{L_2}{L} = 0.22$$

$$B_2 = 0.24 \times 3 = 0.72\text{m}, L_2 = 0.22 \times 6 = 1.32\text{m}$$

$$A' = 1.32 \times 3 + \frac{1}{2} (3 + 0.72) (6 - 1.32)$$

$$= 12.7\text{ m}^2$$

$$B' = \frac{A'}{L'} = \frac{12.7}{6} = 2.1\text{ m}$$

$$L' = L = 6\text{m}$$

Das, B. M., 1999

Case IV:

So, now the next problem that I will discuss or that I will explain that how I can use the case 4 to determine the effective area of the foundation. So, now this is the similar type of problem only difference is that here the  $e_L$  is taken as 0.6 m and  $e_B$  is taken as 0.3 m. So, now my  $L = 6$  m and  $B = 3$  m, now  $e_L = 0.6$  m and  $e_B = 0.3$  m. So, now  $\frac{e_L}{L} = \frac{0.6}{6}$  is 0.1 and  $\frac{e_B}{B}$  is also 0.1.

So, now in this case, both  $\frac{e_L}{L}$  and  $\frac{e_B}{B}$  are less than 0.167, so that means it is case 4. Now here we have to determine only the effective area but, why? I have already discussed that once you get the effective area or effective width and the length, how we can determine the ultimate bearing capacity or the safe load that can act on the foundation?

But remember that I have discussed only Meyerhof's method but you can use other methods also like Hansen's method, Vesic's method or even IS code method also. But in case of Terzaghi's this option is not there to incorporate this loading eccentricity. But remember that when you use the other methods, then you calculate the correction factors according to their suggestions.

That means some cases you have to use  $B$  and  $L$  and some cases you have to use  $B'$  and  $L'$ . Remember that in Meyerhof's theory all the factors are calculated using  $B'$  and  $L'$ . But in case of

say for example, Hansen's shape factor and the inclination factor you have to use effective area or  $B'$  and  $L'$  but for the depth factor you have to use  $B$  and  $L$ . Similarly for Vesic, all the factors you have to determine by using  $B$  and  $L$  not  $B'$  and  $L'$ . Except that inclination factor effective area, that area you have to use  $A'$ , effective area.

So, and the IS code method also all the factors are to be determined by using  $B'$  and  $L'$ . So, you just keep in mind that all based on their suggestions and I am giving as a note for all cases that when you have to consider the actual width or length, when you have to consider the effective width or the effective length. So, determine those factors according to that, so it is case 4. Now case 4, the effective area is given as  $B \times L_2 + \frac{1}{2}(B + B_2) \times (L - L_2)$ .

Now, the  $\frac{B_2}{B}$  here this is 0.1, this is also 0.1, so this is 0.1 is  $\frac{e_B}{B}$  and  $\frac{e_L}{L}$  also 0.1, so that means, this is 0.1. And if I use that upward curves, so that means this is the value which will give me the  $\frac{B_2}{B}$ . So, upward curve  $\frac{B_2}{B}$ , so that is this value is roughly 0.24 so this is the curve, so this is the curve that we have to use. So, this curve we have to use to calculate  $\frac{B_2}{B}$ . Now for the downward curve, so these are the values, so this is 0.02, 0.04, 0.06, 0.08, so this is 0.02 this line, this is 0.04, 0.06, 0.08, so this is the line.

So, this is the line which is coming from here and passing from here, so this is the line and which is roughly passing from here, so more or less the same value. So, here this downward curve will give me the  $\frac{L_2}{L}$ , so  $\frac{L_2}{L} = 0.22$  clear, so one is 0.24 another is 0.22. So, now my  $B_2 = 0.24 \times 3$  which is 0.72 m and the  $L_2 = 0.22 \times 6 = 1.32$  m. And now I can calculate the  $A'$  which is  $B \times L_2 + \frac{1}{2}(B + B_2) \times (L - L_2) = 3 \times 1.32 + \frac{1}{2}(3 + 0.72) \times (6 - 1.32) = 12.7 \text{ m}^2$ .

So, now the  $B'$  is given,  $L'$  is  $L$  which is 6 m in case 4, so  $B'$  will be  $\frac{A'}{L'}$ . So,  $A'$  is 12.7 and  $L'$  is 6, so  $B'$  will be 2.1 m. So, this way you will get the  $L'$  and the  $B'$ , now you can use these effective width and length to calculate the bearing capacity of the foundation. So, now I have discussed

the one-way eccentricity and which equation you should use for one-way eccentricity, and then I have discussed the two-way eccentricity also and then I have discussed four cases.

And then I will discuss how to calculate the effective length and width and the area for four cases and then how we can use those effective width and length to determine the bearing capacity? So, in the next effect, if the load is not vertical, it is inclined, then how I can determine the bearing capacity?

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### Foundation under Inclined Load

A rectangular footing of size 3m X 6m is founded at a depth of 1m in a c-φ soil. The water table is at a great depth. The unit wt of soil 18 kN/m<sup>3</sup>. 8000 kN load acts at an angle 15° to the vertical at the centre of the foundation. c=100 kPa and φ = 20°. Determine the factor of safety against bearing and sliding. Assume general shear failure. Take δ as 0.8φ and c<sub>v</sub> as 0.7c. Neglect the Passive Resistance.

**Meyerhof's Theory**

$$q_m = q_{ult} - \gamma D_f = c N_c s_c d_c l_c + \gamma D_f N_q s_q d_q l_q + 0.5 \gamma B N_{\gamma} s_{\gamma} d_{\gamma} l_{\gamma} - \gamma D_f$$

From table  $N_c = 14.8$ ,  $N_q = 6.4$ ,  $N_{\gamma} = 2.9$  for  $\phi = 20^\circ$

$$s_y = 1 + 0.2 \left( \frac{B}{L} \right) \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) = 1 + 0.2 \left( \frac{3}{6} \right) \tan^2 \left( 45 + \frac{20}{2} \right) = 1.20$$

$$s_x = s_y = 1 + 0.1 \left( \frac{B}{L} \right) \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) = 1 + 0.1 \left( \frac{3}{6} \right) \tan^2 \left( 45 + \frac{20}{2} \right) = 1.10$$

So, that means the foundation under inclined load, then how I can take care of that effect? So, first a simple problem I am discussing then I will go to some complicated problems, and then how to use different approaches in case of inclined load? Because for eccentric loading, all the approaches are same that means, you have to only know how you can determine  $B'$  and  $L'$ ?

Then based on the given suggestion by different methods or different theories, you put the  $B'$  or  $L'$  or  $B$  or  $L$  and you will get the bearing capacity. But for inclined loading, it is slightly complicated specifically for Hansen's theory. Because if you want to use Hansen's theory for inclined loading, then you have to take care of so many things, so that I will discuss in this part.

So, that foundation under inclined load the same problem more or less I am taking the same problem and only certain conditions will change. So, that means the rectangular footing of size 3

$m \times 6 m$  is placed at a depth of  $1 m$  in  $c-\phi$  soil. The water table is at a great depth unit weight it is again  $18 \text{ kN/m}^3$ . And at  $800 \text{ kN}$  load at an angle of  $15^\circ$  to the vertical at the center of the foundation.

That means there is no eccentricity and the load is acting at the center, but inclined at an angle of  $15^\circ$  with the vertical. Now there can be one case that it is acting at an angle, but with an eccentricity also. So, that means in that case both the conditions you have to consider but here I have not considered the eccentricity, I have consider only the inclination effect.

So, that means this is vertical, so load is acting at an angle of  $15^\circ$  with the vertical and load value is  $8000 \text{ kN}$ . And then we have to determine the factor of safety against bearing and sliding assumed the general shear failure take  $\delta = 0.85\phi$  and  $c_a = 0.7$  of  $c$ . Because for previous cases I have to check only the bearing capacity but here you have to check the sliding also.

Because when the load is acting there is a vertical component, there is horizontal component also. So, against the vertical component you have to check the bearing capacity. But against the horizontal component you have to check the sliding also because not only it will push the foundation in downward direction but the horizontal component also try to slide the foundation, so we have to take the sliding also.

That is why this  $\delta$  because this  $\delta$  is the friction angle between the foundation and the base soil. And this adhesion between the foundation and the base soil, remember that this is base soil. And why you have to consider or neglect the passive resistance? For example, suppose we have a foundation and this is my depth of foundation. Now there is an inclination of the load, so load is acting with an angle it is inclined.

So, it has two components, one is the vertical component and another one is the horizontal component. So, now that means this horizontal component will try to push the foundation or slide the foundation from right to left. So, that means the soil in this zone basically will give you a passive resistance, so from where I will get the resistance? So I will get this passive resistance from the base soil because your foundation is sliding.

So, in our retaining wall design also and this type of foundation design, we neglect generally passive resistance. Because if we do not consider the passive resistance and if my foundation is safe. Then if I consider the passive resistance obviously it will be oversafe. So, that is why you can neglect the passive resistance but if you want to accurately determine the factor of safety, then you have to consider the passive resistance but here the passive resistance is not considered. So, passive resistance means, if you consider the Rankine passive force, how to calculate?

That means you have to calculate this  $P_p$  which will be  $\frac{1}{2} \gamma D_f^2 K_p$ , where  $K_p = \frac{1 + \sin \phi}{1 - \sin \phi}$ . So, this is  $\gamma D_f \times K_p$  and  $D_f$  is the depth of foundation, so that amount of passive resistance the soil is giving. So, this is  $P_p$ , this is  $H$  and then you will get another shear resistance along this line, so that is the shear resistance you will get say  $T$ .

So, finally the net force or the shear resistance is that, so that means the factor of safety will be equal to this  $\frac{T + P_p}{H}$ , where,  $H$  is the horizontal component of the load and  $V$  is the vertical component of the load,  $R$  is the resultant here which is 8000 kN, clear. But in this case we have not considered the  $P_p$ , so in our case the factor of safety for sliding will be  $\frac{T}{H}$ , because we are neglecting this  $P_p$  part. So, now let us solve this question, so that means the we are calculating  $q_{nu}$  which is  $q_u - \gamma D_f$ .

So, this is the expression which I am using as it is a general shear failure. So, for  $N_c$  value corresponding to  $\phi = 20^\circ$ , is 14.8,  $N_q$  and  $N_\gamma$  also we will get from Meyerhof's bearing capacity factor table. Then I will calculate the shape factors  $s_c$ ,  $s_q$ ,  $s_\gamma$  by using these expressions because there is no eccentricity. So, you have to consider  $B$  and  $L$ .

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$$d_c = 1 + 0.2 \left( \frac{D_f}{B} \right) \tan \left( 45^\circ + \frac{\phi}{2} \right) = 1 + 0.2 \left( \frac{1}{3} \right) \tan \left( 45^\circ + \frac{20}{2} \right) = 1.095$$

$$d_q = d_r = 1 + 0.1 \left( \frac{D_f}{B} \right) \tan \left( 45^\circ + \frac{\phi}{2} \right) = 1 + 0.1 \left( \frac{1}{3} \right) \tan \left( 45^\circ + \frac{20}{2} \right) = 1.05$$

$$i_c = i_q = \left( 1 - \frac{\alpha}{90} \right)^2 = 0.69 \quad i_r = \left( 1 - \frac{\alpha}{\phi} \right)^2 = 0.063 \quad \alpha = 15^\circ$$

$$q_{ult} = q_{ult} - \gamma D_f = c N_c s_c d_c i_c + \gamma D_f N_q s_q d_q i_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma i_\gamma - \gamma D_f$$

$$q_{ult} = 100 \times 14.8 \times 1.20 \times 1.095 \times 0.69 + 18 \times 1 \times 6.4 \times 1.1 \times 1.05 \times 0.69 + 0.5 \times 18 \times 3 \times 2.9 \times 1.1 \times 1.05 \times 0.063 - 18 \times 1 = 1421.4 \text{ kN/m}^2$$

And then I will calculate the depth factor also, and now the inclination factor they are not 1. Now this inclination factor  $i_c, i_q$  this  $\alpha$  is  $15^\circ$  and this is  $\phi$ . Because  $\alpha$  is with vertical and as  $\alpha$  is given with vertical and so that means this is  $15^\circ$ , if you put the  $15^\circ$ , I will get these correction factors. Now if I put these values in the equation, finally I will get one value which is  $1421.4 \text{ kN/m}^2$ . So, net ultimate bearing capacity is  $1421.4 \text{ kN/m}^2$ .

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Factor safety against bearing =  $Aq_{ult}/V$   
 $= (3 \times 6 \times 1421.4) / 7727.4 = 3.31 > 2.5$

Factor safety against sliding =  $(V \tan \delta + A c_u) / H$   
 $= (7727.4 \tan (0.8 \times 20^\circ) + 3 \times 6 \times 0.7 \times 100) / 2070.6 = 1.68 > 1.5$

Note:  $\delta$  is taken as  $0.8\phi$  and  $c_u$  is  $0.7c$ . Passive Resistance is neglected.

$V = P \cos 15^\circ = 7727.4 \text{ kN}$   
 $H = P \sin 15^\circ = 2070.6 \text{ kN}$   
 $P = 8000 \text{ kN}$

So, now we have to calculate the factor of safety. So, as I mentioned against the vertical component we have to check the factor of safety against bearing. And against the horizontal component, we have to check the factor of safety against the sliding.



$T$  is the shear resistance between the foundation and the base soil, which is vertical component  $V \times \tan \delta +$  the effective area  $\times$  the adhesion. So, total vertical force  $\times \tan \delta +$  the effective area  $\times$  the adhesion and that is  $T$ , so factor of safety is basically  $\frac{T}{H}$  because we have neglected the passive resistance. So, this is the vertical component  $\times \tan \delta$  and then the area  $\times c_a$ .

And remember that if in your case it is eccentric loading, then you have to use  $A'$  not  $A$ . In case of eccentric loading, you have to use the  $A'$  but now for particular case that  $A' = A$  because there is no eccentric loading. So, effective area is equal to the actual area of the footing. So, now, what is the bearing capacity? So, net ultimate bearing capacity is this one.

Now if I multiply with the area in case of eccentric loading effective area, then I will get the total load carrying capacity of the soil, so that is total vertical load carrying capacity of the soil. Now if I divide it with the vertical component of the load, so vertical component how I can get? So, suppose this is my  $R$  and it has a horizontal component and the vertical component,  $V$  and this is  $H$ .

Now the horizontal component, so this angle is  $15^\circ$ , so the  $V$  will be  $P \times \cos 15^\circ$ . So, now here  $P$  is written, so you can write  $P$  also, if this is  $P$ , so  $P \times \cos 15^\circ$  and horizontal component will be  $P \times \sin 15^\circ$ , so this is the value and here  $P = 8000$  kN. So, now if I take the area  $\times$  the net ultimate bearing capacity I will get the total vertical load the foundation can take divided by the total load which is acting.

So, I will get a factor of safety  $3.31 > 2.55$ . Now let us check the sliding, so now as I mentioned  $\frac{T}{H}$ ,  $T$  is equal to this equation. So, this is vertical component  $\times \tan \delta$  and  $\delta$  is here taken 0.8 times of the friction angle. And the adhesion also taken 0.7 times of the cohesion value and because Hansen and Vesic recommended that you can take 0.621 times of the cohesion.

So, I have taken 0.7 times of the cohesion value. So, this is the area  $3 \times 6 \times 0.7$  times of the cohesion is 100 and divided by the horizontal force, so horizontal component is 2070.6, so 2070.6. So, it will give  $1.68 > 1.55$ . So, this way I can check the bearing capacity and the sliding

in case of inclined load but here one thing I want to mention that here directly your inclination is given.

And so, that means suppose if you have a footing like this. Now inclination can be in any direction I mean any two directions. That means the load can be inclined in this direction I mean it is inclined along this length direction and it can be inclined along the width direction. So, inclination can be in any direction, it can be in the width direction, it can be in the length direction.

But here if you use Meyerhof's bearing capacity equation, because we have used Meyerhof's bearing capacity equation, because it is not required to know your inclination is in which direction. So, that means the inclination if we know that what is the amount of inclination with respect to vertical axis, that is good enough to calculate the bearing capacity as per Meyerhof's bearing capacity theory.

So, that means here we know that it is acting at an angle of  $15^\circ$  with vertical axis, that is good enough. So, now these inclinations I mean the horizontal load can be parallel to width or can be parallel to length but it is not required to know to solve as per the bearing capacity equation proposed by Meyerhof, but if we use Hansen's or Vesic's bearing capacity equation.

Then we should know that which direction these horizontal component is acting whether it is parallel to  $B$  or parallel to  $L$  or in other way you can say that it is required to know, another way you can say that we can incorporate the direction of the horizontal loading also in Vesic and Hansen that we cannot do in Meyerhof but that is the requirement, so you can do.

So, but in Meyerhof's equation, it is good enough to know the inclination angle with respect to vertical but Hansen and Vesic we have to know that in which direction this horizontal load is acting whether it is width direction or the length direction. So, in the next class I will discuss that if you want to use Hansen and Vesic, then how you can incorporate the direction of the horizontal load in the bearing capacity equation? Thank you.