

Advanced Foundation Engineering
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Lecture-10
Shallow Foundation: Bearing Capacity IV

So, last class I have discussed about the bearing capacity determination, then if the soil has the local shear failure intermediate case. Now today I will discuss how this effect can be incorporated in another way by considering the effect of soil compressibility.

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Effect of Soil Compressibility (Vesic, 1973)

$$q_u = cN_c s_c d_c i_c c_c + q_0 N_q s_q d_q i_q c_q + 0.5 \gamma B N_{\gamma} s_{\gamma} d_{\gamma} i_{\gamma} c_{\gamma}$$

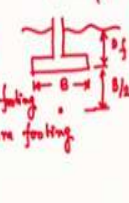
- Calculate the Rigidity Index, I_r at a depth of $\frac{B}{2}$ from the base of the foundation:

$$I_r = \frac{G_s}{c + q' \tan \phi}$$

G_s = Shear Modulus
 q' = Effective overburden pressure at a depth of $(2z + \frac{B}{2})$
- Calculate 'Critical Rigidity Index'

$$I_r(c) = \frac{1}{2} \left\{ \exp \left[(3.30 - 0.45 \frac{B}{L}) C_{\phi} \left(45^{\circ} - \frac{\phi}{2} \right) \right] \right\}$$

if $\frac{B}{L} = 0 \rightarrow$ Strip footing
if $\frac{B}{L} = 1 \rightarrow$ Square footing
- If $I_r \geq I_r(c)$
 $c_c = c_q = c_{\gamma} = 1$



So, that means as I discuss that the local shear failure can be incorporated by compressibility effect also in the soil, because this is due to the compressibility of the soil. So, that means this is our general bearing capacity equation that means the shape factor, depth factor and inclination factor are given. In addition to there is another factor introduced that is called compressibility factor.

So, this is c_c , c_q and c_{γ} , so now how I will calculate these values, so Vesic suggested this compressibility effect of the soil in 1973. So, where another factor is incorporated, so we know generally basically we have 3 factors that is the shape factor, depth factor, inclination factor, so another factor is introduced which is called the compressibility factor which is the c_c .

And later on I will introduce two more factors, one is the tilt base factor and another is the sloping ground factor. So, basically it is not three factors basically it is six factors, total we can incorporate in a bearing capacity equation. So, in our normal foundation engineering course, we generally deal with these three factors. But actually we can incorporate three more factors one is compressibility factor another is the tilting base factor, another is the sloping ground factor.

So, those things I will discuss later on but now I will discuss about the compressibility effect. So, I will discuss the steps, first I have to calculate the rigidity index. So, step 1 that we have to calculate the rigidity index I_r ok at a depth of $\frac{B}{2}$ where B is the width of the footing from the base of the foundation. So, we have to calculate the rigidity index I_r at a depth of $\frac{B}{2}$ from the base of the foundation.

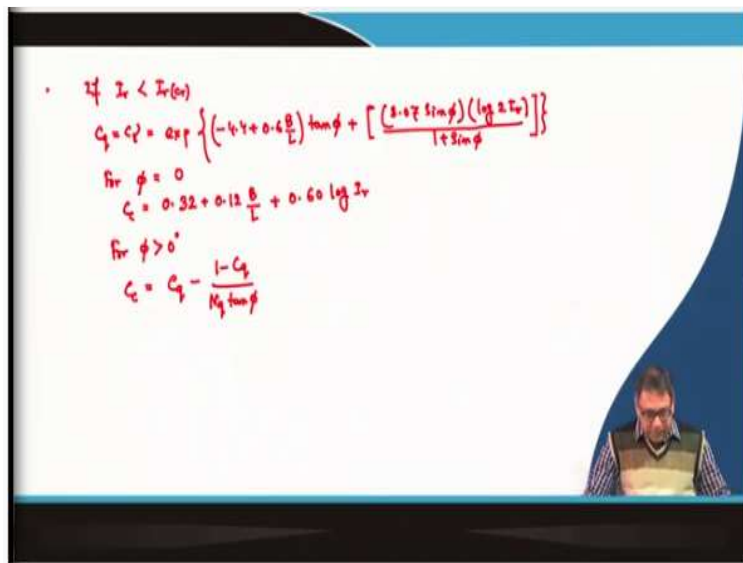
So, how will I calculate the rigidity index I_r , so that I_r can be calculated by $\frac{G}{c+q' \tan \phi}$ ok. So, where G is the shear modulus and q' is effective overburden pressure at a depth of $D_f + \frac{B}{2}$. So, that means suppose we have a footing whose width is B and this is the depth of the foundation. So, we have to calculate this rigidity index at a depth of $D_f + \frac{B}{2}$, ok, so at the depth of $D_f + \frac{B}{2}$.

Then next step is to calculate critical rigidity index ok, so calculate critical rigidity index which is $I_{r(cr)}$ is equal to $\frac{1}{2} \left\{ \exp \left[\left(3.30 - 0.45 \frac{B}{L} \right) \cot \left(45^\circ - \frac{\phi}{2} \right) \right] \right\}$. Now this case that if $\frac{B}{L} = 0$, that means length is much larger than the B then it is called the strip footing, this is true for all the cases actually. Strip footing and if $\frac{B}{L} = 1$ then it is called square footing ok.

So, now this is the case if $\frac{B}{L} = 0$ that means this part will be 0 only $3.3 \cot \left(45^\circ - \frac{\phi}{2} \right)$ within the exponential part. Now if on the next condition that if I_r , that means the rigidity index is greater than equal to $I_{r(cr)}$. So, that means that this is a critical value of the rigidity index, that means soil rigidity index is more than the critical rigidity index that means the soil is very stiff.

So, that means no compressibility effect or corrections are not required, so that means similar to the general shear failure. So, that means if $I_r \geq I_{r(cr)}$, that means it is similar to a general shear failure, ok, where this correction will be definitely 1, because the soil is very rigid. So, that means it is the general shear failure type, so all the corrections will be 1. So, that means in such case the c_c , c_q and c_γ all will be 1. So, that means it is similar to the general shear failure case.

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But if it is not, now if your $I_r < I_{r(cr)}$, then we have to apply the corrections. Because now that may be the local shear failure or in between that, so we have to apply the correction factors. So, that correction factor we can calculate using the expression, $c_q = c_\gamma = \exp\left\{\left(-4.4 + 0.6 \frac{B}{L}\right) \tan \phi + \left[\frac{(3.07 \sin \phi)(\log I_r)}{1 + \sin \phi}\right]\right\}$ ok. This is the equation to calculate c_q and c_γ .

Now for $\phi = 0^\circ$, $c_c = 0.32 + 0.12 \frac{B}{L} + 0.60 \log I_r$. If, $\phi > 0^\circ$ then $c_c = c_q - \frac{1 - c_q}{N_q \tan \phi}$. So, these are the corrections factors we have to apply to incorporate the compressibility of the soil. Now I will solve one example problem and then we will see how we can incorporate that effect, ok, into the bearing capacity.

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A rectangular footing of size 3m X 6m is founded at a depth of 1m in a homogeneous c-φ soil. The water table is at a great depth. The unit wt of soil 18 kN/m³. Determine net ultimate bearing capacity, c = 50 kPa and φ = 20°. Modulus of Elasticity of the soil is 6MPa and Poisson's Ratio=0.35. Use Vesic's Equation. The load is vertical and acting at the centre of the foundation.

$$q_{nu} = cN_c s_c d_c c_c + q_0 N_q s_q d_q c_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma c_\gamma - \gamma D_f$$

$I_r = \frac{G}{c \gamma' \tan \phi} = \frac{2222.22}{50 + 15 \tan 20^\circ} = 33.48$
 $G = \frac{E}{2(1+\mu)} = \frac{6 \times 10^6}{2(1+0.35)} = 2222.22 \text{ kN/m}^2$

$I_r(\phi) = \frac{1}{2} \left\{ \exp \left[\left(3.3 - 0.45 \frac{B}{L} \right) \tan \left(45^\circ - \frac{\phi}{2} \right) \right] \right\}$
 $I_r(\phi) = \frac{1}{2} \left\{ \exp \left[\left(3.3 - 0.45 \times \frac{3}{6} \right) \tan \left(45^\circ - \frac{20^\circ}{2} \right) \right] \right\}$
 $I_r(\phi) = 40.38$

$I_r < I_r(\phi)$
 $c_q = c \left\{ (-4.4 + 0.4 \frac{B}{L}) \tan \phi + \left[\frac{(3.07 \sin \phi) (1 + 2 \frac{B}{L})}{1 + 2 \frac{B}{L}} \right] \right\}$
 $c_q = 50 \left\{ (-4.4 + 0.4 \times \frac{3}{6}) \tan 20^\circ + \left[\frac{(3.07 \sin 20^\circ) (1 + 2 \times \frac{3}{6})}{1 + 2 \frac{3}{6}} \right] \right\}$
 $c_q = 0.94$

$D_f + \frac{B}{2} = 1 + \frac{3}{2} = 1 + 1.5 = 2.5 \text{ m}$
 $q' = 2.5 \times 18 = 45 \text{ kN/m}^2$

So, that example problem is given for similar rectangular footing of size 3 m × 6 m placed at a depth of 1 m in a homogeneous c-φ soil, the water table is at a great depth, the unit weight of the soil is 18 kN/m³. Determine the net ultimate bearing capacity when c = 50 kPa, φ = 20°, modulus of elasticity of the soil is 6 MPa and Poisson's ratio is 0.35.

Use Vesic's equation, the load is vertical and acting at the center of the foundation. So, that means no eccentricity, no inclined load is acting at the center of the foundation, it is vertical also. So, we have to use Vesic's bearing capacity equation. So, first as the modulus of elasticity is given, so we can incorporate the compressibility effect because in previous case in Terzaghi's theory the example that I have solved.

Where based on the φ value we have to judge whether we have to apply the local shear failure or the general shear failure. But here if we have the modulus of elasticity or shear modulus or Poisson's ratio, then automatically we can understand whether this compressibility correction you have to apply or not, by calculating the critical rigidity index and rigidity index.

So, let us see whether in this case the compressibility correction is required or not, ok. So, this is the equation that I have discussed, so first I have to calculate the I_r which is the rigidity index, so

that value is given $\frac{G}{c+q' \tan \phi}$, ok. So, now here E is given, so from the E I can get the shear modulus, so that is $G = \frac{E}{2(1+\mu)}$, so E is given 6 MPa, that means $\frac{6 \times 10^3 \text{ kPa}}{2(1+0.35)}$.

So, G value is coming out to be 2222.22 kN/m², ok. So, G value is this one, so I can put that G value divided by c is 50 kPa + the effective overburden pressure. Because here we have to calculate the effective overburden pressure at a depth of $D_f + \frac{B}{2}$, ok. So, that D_f is 1 m, depth of foundation is given 1 m, B is 3 m, so 3 divided by 2, so 1 + 1.5, so 2.5.

So, we have to calculate that q' at a depth of 2.5 m from the ground as the water table is not present. So, remember that it is effective overburden pressure as in this case water table is not present, so that is why directly I can calculate the total pressure. Because here total pressure and effective pressure both are same. But if water table is present then you have to calculate the effective overburden pressure, remember that.

So, that means here q' will be $2.5 \times$ unit weight is 18, 2.5×18 which is 45 kN/m², so that is why $45 \times \tan 20^\circ$, ok. So, that means here this value is coming out to be 33.48. Now I will calculate $I_{r(\text{cr})}$ which is $\frac{1}{2} \left\{ \exp \left[\left(3.30 - 0.45 \frac{B}{L} \right) \cot \left(45^\circ - \frac{\phi}{2} \right) \right] \right\}$, ok. So, I will put this value half 3.3 - 0.45, B is 3 m, L is 6 m then $\cot \left(45^\circ - \frac{20^\circ}{2} \right)$, so this is coming out to be 40.38.

So, you can see that the critical rigidity index is 40.4 around and rigidity index is 34, so which is very close to the critical rigidity index but it is less than critical rigidity index. So, we have to apply the corrections, these values I mean correction factors are not 1 ok. But by looking at these values you can understand that these values will be close to 1 because the difference between I_r and $I_{r(\text{cr})}$ are very small.

So, that is why the rigidity of the soil is very close to the critical value, so that is why it will be because if the $I_r = I_{r(\text{cr})}$ then the correction factors will be 1. So, as these values are very, very close to each other, so it is expected the critical I mean thus corrections factors will be very close to 1, ok. Because if the difference is more than this value will be less than 1.

But here it will be less than 1 but very close to 1, let us see. So, now I can see that my $I_r < I_{r(cr)}$, so I have to calculate these factors. So, these factors, $c_q = c_\gamma = \exp \left\{ \left(-4.4 + 0.6 \frac{B}{L} \right) \tan \phi + \left[\frac{(3.07 \sin \phi)(\log I_r)}{1 + \sin \phi} \right] \right\}$, ok. So, I can write that is $\exp \left\{ \left(-4.4 + 0.6 \times \frac{3}{6} \right) \tan 20^\circ + \left[\frac{(3.07 \sin 20^\circ)(\log \times 33.48)}{1 + \sin 20^\circ} \right] \right\}$, ok. So, this is the value, so finally it is coming out to be 0.94, as I mentioned the value will be close to 1, so it is coming as 0.94.

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Similarly I can calculate the other factors that is c_c , so that is $c_q = \frac{1 - c_q}{N_q \tan \phi}$. Now it is Vesic's equation you have to consider, now for $\phi = 20^\circ$ my $N_c = 14.8$, $N_q = 6.4$ and $N_\gamma = 5.4$. Because here we have to consider the general shear values directly from that table, because the compressibility effect itself will take care of the local shear failure or intermediate failure, so that correction will be itself taken care of.

So, that means here this is the value, so now c_c is $0.94 - \frac{1 - 0.94}{6.4 \times \tan 20^\circ}$. So, this value is also very close to 1, which is 0.914. So, now we have calculated the compressibility factors, now we have to calculate the shape factor, depth factor also inclination factor will be 1 all the cases. So, I will because as per Vesic, so s_c is $1 + \frac{N_q}{N_c} \left(\frac{B}{L} \right)$ which is $1 + \frac{6.4}{14.8} \left(\frac{3}{6} \right)$ which is 1.216, ok.

Now $s_q = 1 + \tan \phi \left(\frac{B}{L}\right)$ which is $1 + \tan 20^\circ \left(\frac{3}{6}\right)$ which is 1.182. Similarly, $s_\gamma = 1 - 0.4 \left(\frac{B}{L}\right) \geq 0.6$, so it is coming as $1 - 0.4 \left(\frac{3}{6}\right) = 0.8$. So, which is greater than 0.6, so we have to consider the 0.8. Now similarly d_c we can get $1 + 0.4K$, now for $\frac{D_f}{B} \leq 1$, my K value is $\frac{D_f}{B}$.

And our case this is D_f is 1 and this is 3 which is less than 1, so this will be $\frac{D_f}{B}$. So, this will be $1 + 0.4 \times \frac{1}{3}$, so that will be 1.133, ok. So, now similarly I can calculate the d_q which is $1 + 2 \tan \phi (1 - \sin \phi)^2 K$, so K is also $\frac{D_f}{B}$ here, so that will be $1 + 2 \tan 20^\circ (1 - \sin 20^\circ)^2 \times \frac{D_f}{B}$. So, finally I can put $1 + 2 \tan 20^\circ (1 - \sin 20^\circ)^2 \times \frac{1}{3}$, so this is 1.105, ok.

And we know that $d_\gamma = 1$ for all the cases. So, finally net ultimate, q_{nu} will be $50 \times 14.8 \times 1.216$. Because that is the s_c is 1.1 into d_c is 1.133, then inclination factor is 1 because all i_c, i_q, i_γ will be 1. So, 1 but the compressibility factor c_c is 0.914 and then $+ 18 \times 1$, then the 6.4×1.182 is $s_q \times d_q$ is 1.105 and that is 0.94.

The compressibility factor then the inclination factor is 1 then $+\frac{1}{2}$ unit weight is 18, B value is 3, bearing capacity factor is 5.4, then all the factors this is s_q is 0.8, sorry s_γ is 0.8, then d_γ is 1, then inclination factor is 1, then the compressibility factor is 0.94, ok. So, the value and then it is net ultimate, so that will be $- \gamma$ is 18 and D_f is 1, so this value is 1164.92, ok.

So, this way we can incorporate the compressibility effect of the foundation. So, if you neglect the compressibility effect as you can see the compressibility factors are close to 1. So, this value will be more or less same there are the values will be slightly higher, if I do not consider the compressibility effect. But in this case this value is close to 1 but maybe if the elastic modulus or the rigidity of the soil is less then I will get a significant amount of reduction of the bearing capacity value, if I consider the compressibility effect.

So, that depends on the type of soil, the properties of the soil. But again when you calculate these factors for eccentric loading, remember that. Then in other theories this recommendation is not

given, so in my opinion you consider B' and L' in place of B and L ok. So, in this class I have discussed the effect of compressibility and in the next class I will discuss the other different effects in this course. So, first one I have discussed the effect of soil compressibility, so next class I will discuss the other effects like the loading eccentricity and then the inclination of the load and the other effects, thank you.