Soil Mechanics/Geotechnical Engineering I
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Lecture - 06
Three-phase diagram

Good morning, welcome you again in the second with lecture. Last week we have learnt to identify the soil type, based on grain size and all, and you have learned also to classify the soil. And if you can classify the soil, we will be able to tell the qualitatively, the different characteristics of the soil. And now we will try to go one by one, to quantify them, and for that first lesson, we will try to discuss in this session; that is three phase diagram and wed volume relationship.

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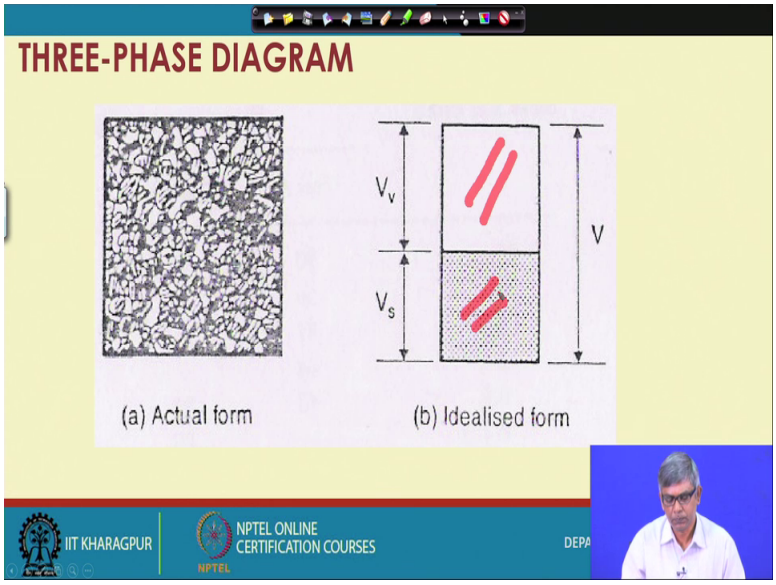

If typically if I take a mass of soil, and microscopically if you see. You will see something like this inside the grain, and there is a voids and voids can be filled up with air or water, and there will be third party solids. And if you can ideally separate them, void you separate volume, and solid you separate volume, and then if you can imagine this is the total volume of voids, and this is actually total volume of solid.

This is total volume of solid and this is a total volume the entire volume is total. Suppose in this mass, and then if you separate all voids, then this is the volume for voids, and this is the volume for your solid, and then this is the one then. Sorry.

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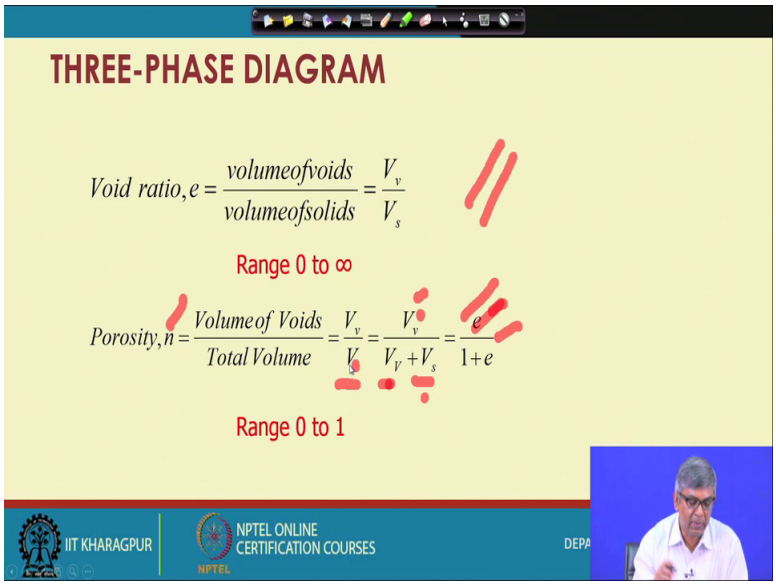

You can see that we can define a term that is void ratio, which is denoted as e, and that the definition is volume of voids divide by volume of solids, it is equal to Vv by Vs. And this Vv by Vs, what could be the range of void ratios or values. You can see, you can imagine that the solid portion is very close to 0 in a mass, then the ratio become close to 0, and you can imagine also that void portion is very close to 0 or entire mass is void less, than your, sorry solid less, and entire volume is the by void, voids only. In that case void by 0 that becomes infinity

So, theoretically this range can be 0 to infinity, and then similarly we can define another term; that is porosity which is also denoted as n, which is equal to volume of voids divided by total volume, and this value volume of voids by total value which is easy Vv by v. And this again that is a porosity, and void ratio there is a definite relationship we can see here Vv is kept as it is, and v is can be v can be denoted as Vv plus Vs. And now if I divide by Vs, both from here and here, then Vv by Vs becomes e and Vv by Vv become 1, and Vv by Vs equal to e, and Vs by Vv sorry.

So, Vv by Vs become, Vv by Vs become e, so this one, and Vs by Vs equal to 1 and Vv by Vs equal to e so, that n to e relationship, n equal to e by 1 plus e. So, and what is it is, what is the range of value of this. Its value actually you can imagine Vv by V; that means, if it is void is close to 0, then this ratio, the over total volume becomes 0, and it

become void become close to total volume, that voids if solid is very less, then that ratio close to 1.

So; that means, porosity void ratio range between 0 to infinity any value; that means, it will not be surprised, if you see any soil has void ratio of more than one; that is no surprise, actually it can happen many soil can have more than one void ratio. Whereas, porosity, the value of porosity it cannot be more than one, it should be ranging between 0 and 1.

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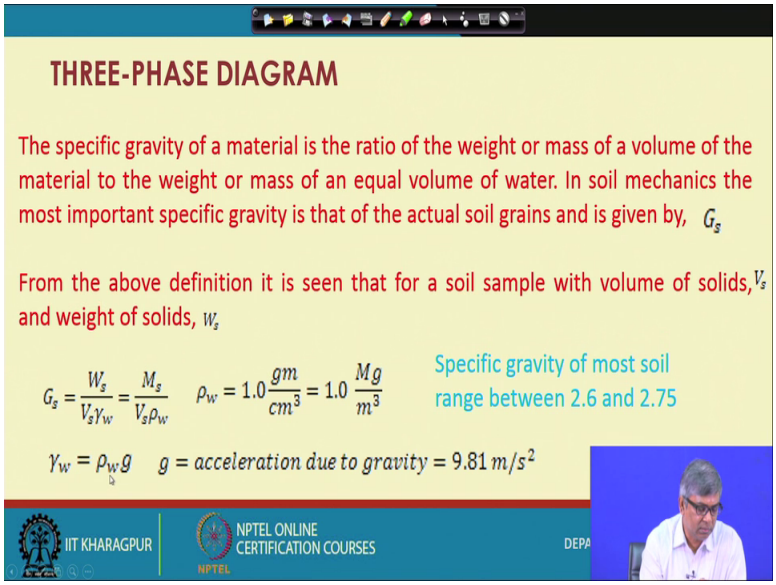

And the specific gravity of then another term is that is very important soil mechanic that is specific gravity of solids. We can know the mass, but soil grains, what is the specific gravity. The specific gravity of a material is the ratio of the wrist weight or mass of a volume of the material, to the weight or mass of an equal value of water. In soil mechanics, the most important specific gravity is that of the actual soil grain, and is given by Gs. And from the above definition it is seen that, for a soil sample with volume of solids Vs, and weight of solids equal to Ws. Your Gs will become Ws by Vs gamma w

So; that means, this is the one, Ws is the weight of solid and equal value of water weight. So, Vs that is Vs into gamma w, this solid value weight actually occupied by the solid only, because void does not have any weight, so that volume of solid into water; that means, equal volume of water, this ratio is the by definition is specific gravity, and that is also can be in terms of mass if you write, mass by Vs into rho s.

And rho w is 1 gram per centimeter cube, and your 1 mega gram per meter cube, either way it can be. So, this is known value this water density unit weight or mass density for water is known, and specific gravity of most soil, this is the definition we have given, but specific gravity of the most of the soil generally ranging between 2.6 to 2.75, and most of the time for solving any numerical problem, if specific gravity is not given. So, automatically one can assume, the specific gravity as 2.6 to 7 in between any value over 2.65 or 2.7.

And; obviously, there is a gamma, and rho, there is a relationship. Gamma will be rho times g; that is where g is the acceleration due to gravity, the value is 9.81 meter per second square.

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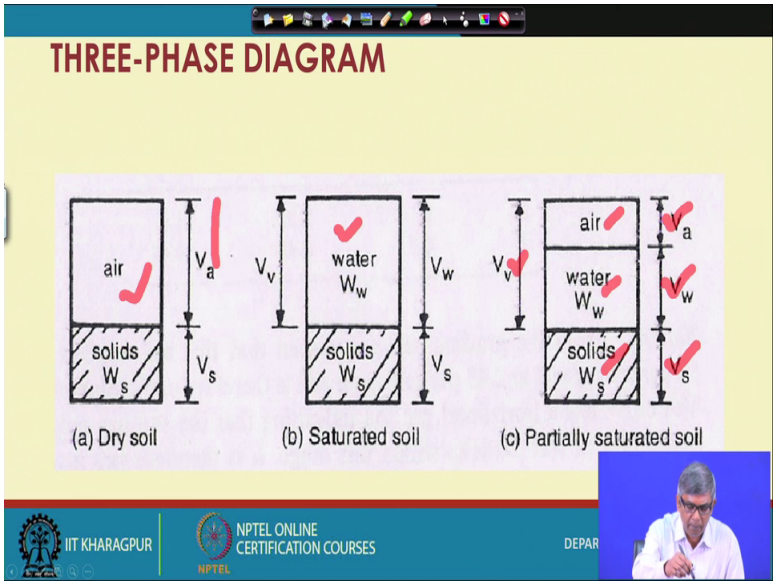

And now I have shown in the previous diagram only two phase diagram, because only I have considered voids only, and we have considered solid, but if I consider void is completely dry, then this is the diagram we can think of. This is the diagram we can think of; that is dry condition only air, and only solid and if the all the voids filled up with water; that means, saturated then I can think of, this is the diagram; that means, this is the volume of water and this is the value of solid.

So, these are actually two phase diagram which I have discussed previous slide. Now this one actually, in most soil will be there in a three phase form; that means, it will have partly occupied by air here, partly occupied by water, and partly occupied by solid, and

they are given the symbol like this. The volume of void total volume of void is shown here, out of total void value of air given here, volume of water is given here, and volume of solid is given here, and this is actually total volume V.

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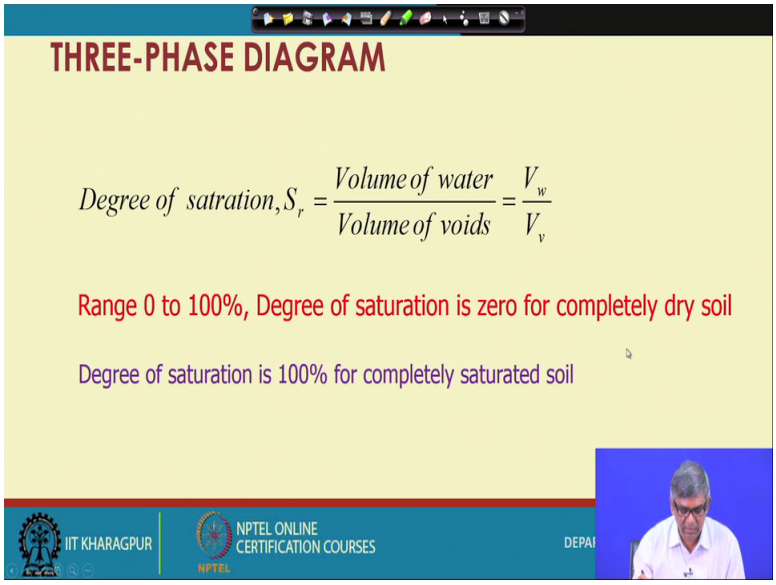

And now based of these three phase diagram, we can define a few more term; that is degree of saturation. Total volume of voids at they known, but now what percentage of volume is occupied or filled up by water, that actually we denote by using a term; that is degree of saturation, that is Sr which is equal to volume of water by volume of voids. So, volume of water I have denoted previous slides by Vv, and we have void I have denoted Vw, and total voids I have denoted as Vv.

So, this ratio is degree of saturation. Now this what will be the range of degree of saturation. Degree of saturation can be from 0 to 100 percent, mainly because it can be completely dry, in that case Vw will be 0, and Vw by Vv that become 0, and if it is completely saturated; that means, Vw become is Vv. So, that ratio become 1, 1 means 100 percent. And degree of saturation is 100 percent for completely saturated soil and.

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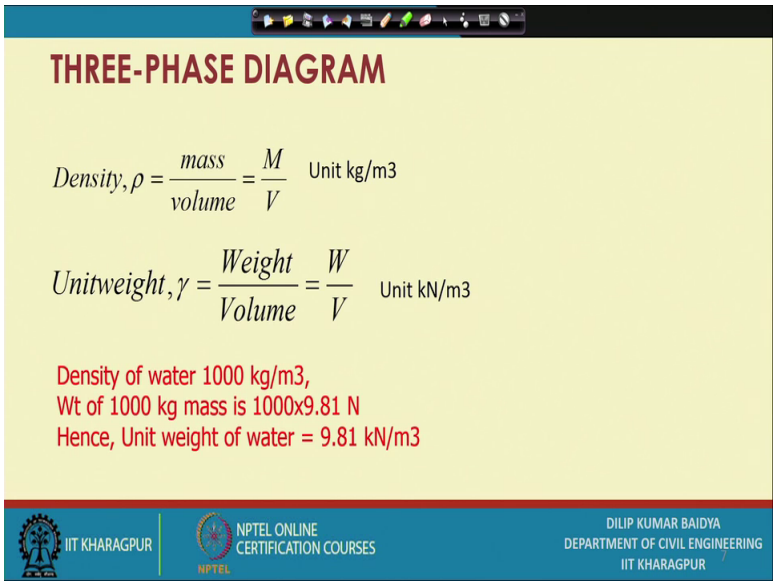

Now, the density is, definition of density. Density is mass by volume, and the unit will be kg per meter cube, and un[it] unit weight gamma actually, it is weight by volume; that is w by v, and unit weight will be kilo newton per meter cube

And these are the things actually very carefully one has to keep in mind, that mass and unit weight is just a multiplication of, multiplied by g only. And density of water, it is known generally to be assumed or it is known, for any calculation this is 100. It is a 100 kg per meter cube, and weight of 100 kg mass is a 109.1 newton. Hence unit weight of water is 9.81 kilo newton per meter cube.

So, most of the calculation, or civil engineering calculation, instead using 9.81 kilo newton per meter cube as unit weight of water, sometime we take 10, but if you want to do actual calculation, then it is better to use 9.81, but for simplification, if you want to do then we can use unit weight of water as 10, and. So, these are the things to be remembered, that unit weight of soil most soil, what will be the value actually. If it is expressed in kilo newton per meter cube, most soil will have unit weight between 13 14 to 20 21 22 kilo newton meter cube, an average value of unit weight of soil. Mostly it is 16 17 18 like that, kilo newton per meter cube.

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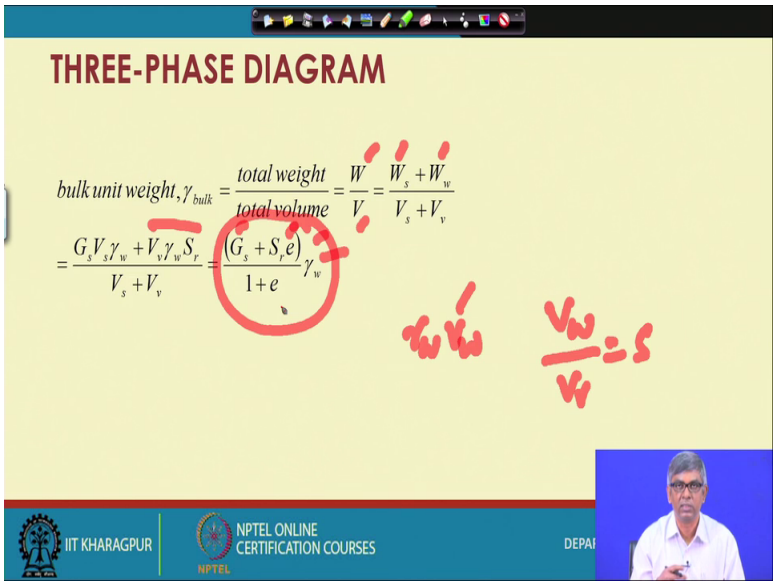

Now, bulk unit weight of water actually we bulk unit weight as I have mentioned before that it is total weight divided by total volume, total bulk weight by total volume, that can get bulk unit weight, but we want to express we have learnt now few term, that is e then n, then we have learnt Sr. So, we will try to express the soil bulk unit weight in terms of those, and if you want to try that first attempt is here.

You can see W by V, and that W can be since, if it is a, if you consider also three phase diagram, then weight of air will be ignored, then your total weight will become summation of Ws plus Ww. And total volume can be expressed a Vs plus Vv. Now this Ws and Ww, Ws can be expressed by Gs Vs gamma W, Gs Vs gamma W, is the Ws by definition of specific gravity, Gs gamma w is the weight and volume Vs. So, this become the total weight of the solid and. In fact, it is weight of w is nothing, but your gamma w into Vw. But if I again use the degree of, introduce degree of saturation, degree of saturation exactly Vs by Vv, this is S.

So, now I can substitute Vw this Vw. I can substitute actually S into Vv. So, that is the thing is done. So, this one Ww is expressed in terms of Vv gamma w into Sr. And now if I divide it by Vs, then you can see, this become one, this actually Vv by Vs become e. These are actually become Gs. Here actually Vv by Vs, then it becomes e, e and Sr into c. So, if I take gamma out from here, then it becomes Gs plus Sr into e divide 1 plus e into gamma w.

So, that mean bulk if you know that these are the quantity like specific gravity of the soil if you know, and if you know the degree of saturation. And then void ratio, and then you can find out by this from this expression, we can find out the bulk unit weight of the soil. Sometime bulk unit weight also you can find out, but you want to find out some of the parameter like either void ratio is known, or degree of saturation is known; one of them is not known, then we can also find out from this using this equation.

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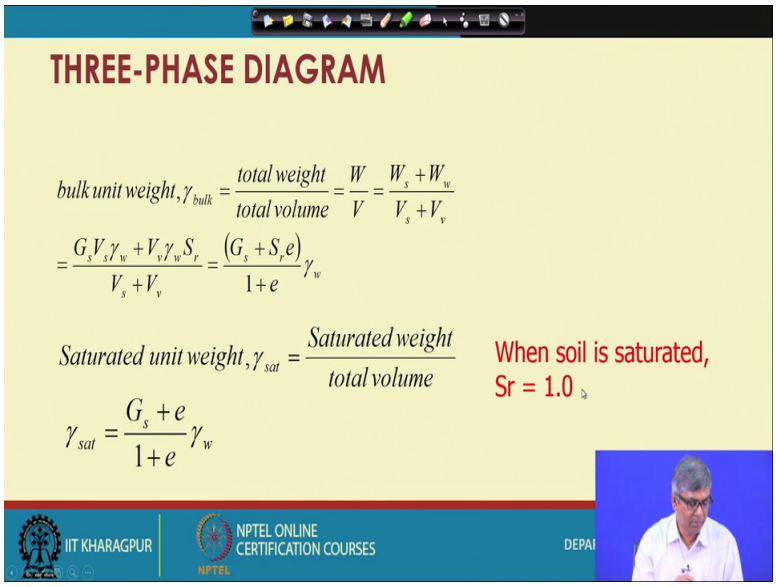

And unsaturated unit weight will be equal to gamma set; that is saturated weight by total volume, and when it is saturated Sr become 1. When it is fully saturated; that is 100 percent, then Sr is 1. I have already told in the previous slide. So, in that case in that case your this equation can be modified, if I put Sr equal to 1 here, then your equation become gamma sat equal to G plus e by 1 plus e into gamma w. So, this is another equation to be remembered, gamma bulk when it is partly saturated, gamma bulk when it is a bulk sorry, and gamma saturated when it is fully saturated.

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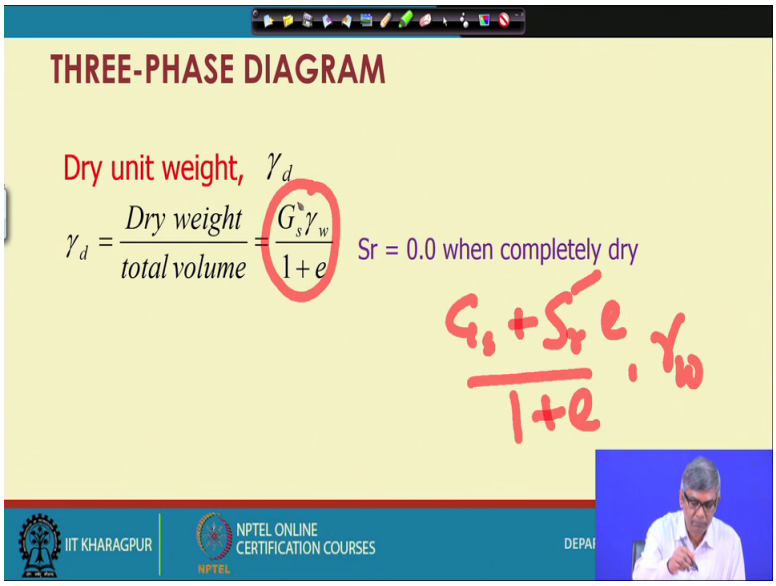

So, two different equation are shown which to be remembered. Next is dry unit weight, dry unit weight, gamma d will be dry weight by total volume, and this is, you can put Sr. Dry unit weight actually; that means, Sr becomes 0 dry means completely dry then in that case sr becomes 0 and if I put in the previous equation. Previous equation was Gs plus Sr into e by 1 plus e into gamma w. In this if Sr if I put equal to 0. In that case this term become 0, and then we have Gs by 1 plus into gamma w; that is the thing given here.

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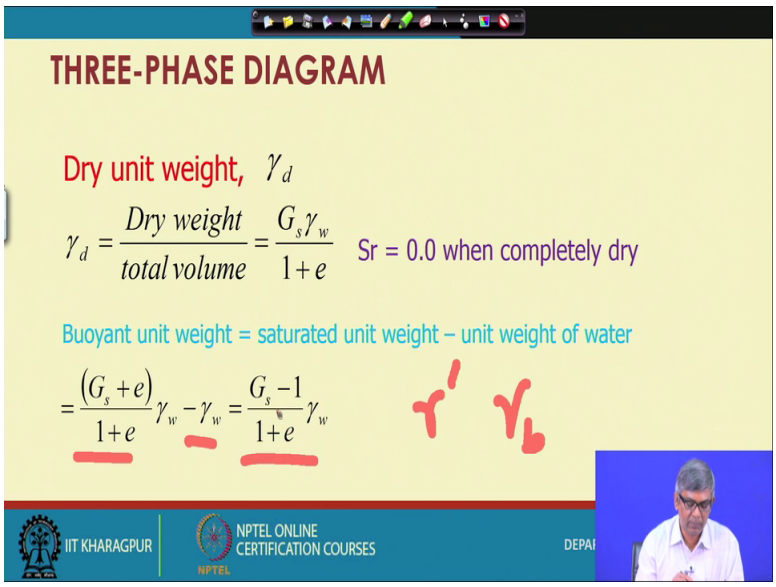

Buoyant unit weight, actually that is buoyant unit, when is what, what ii is underwater that unit weight will be different from that normal unit weight. So, that is actually by definition it is saturated in the weight, saturated unit weight minus unit weight of water. And if I do that, then you can see saturated unit weight we have derived, this is the equation we have derived, and unit weight of water is this one, and if you subtract this, then buoyant unit weight, it comes g minus 1 by 1 plus e into gamma w.

So, this is the one, another relationship weight volumes relationship, one has to remember. So, gamma, buoyant unit weight sometime, it is gamma dash u, sometime gamma buoyant it will be symbol will be used, like that different ways this can be expressed, but their expression is Gs minus 1 by 1 plus e into gamma w. So, all those things if I know the specific gravity and void ratio. I can find out the buoyant unit weight, or some other way if this is known, then we can find out the buoyant ratio also, whenever this is informed this expression.

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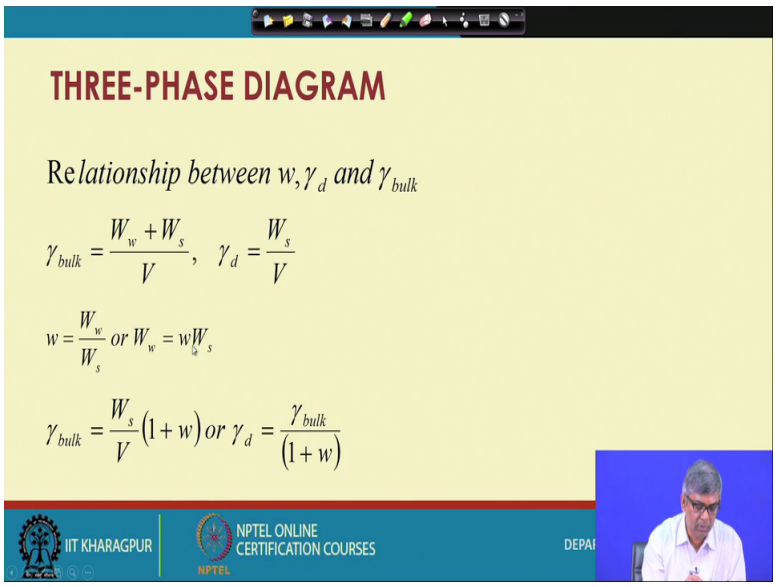

Relationship between water content, then gamma d, the dry unit weight and gamma bulk, you can see gamma bulk equal to Ww plus Ws by V, and gamma d is actually Ws by V and using these, and your water content is definition Ww by Ws, or Ws equal to Ww weight of water, will be equal to water content into weight of solid and if you substitute these Ww equal to weight of water. Sorry, weight of water, if you substitute weight of water, these by water quantity into Ws in this expression, then you can see this Ws Ws

can be taken out and v will be taken out, and this will be 1 plus w it is coming, and then gamma d will become Ws by V is nothing, but dry

So, this Ws by V is nothing, but gamma d. So, gamma d will be equal to this gamma d. These become gamma d. So, this gamma d will be equal to gamma bulk divided by 1 plus w. So, this is the formula; that means, if I know, the bulk unit weight of the soil, then if and water content. Then if I want to find out what is the value of dry unit weight, than simply bulk unit weight divided by 1 plus water content. This is the thing if I do then I will get the dry unit weight. Or in the other way if I want to, if I know the dry unit weight and water content, if I want to find out bulk unit weight, then gamma d multiplied by 1 plus water content is the bulk unit weight.

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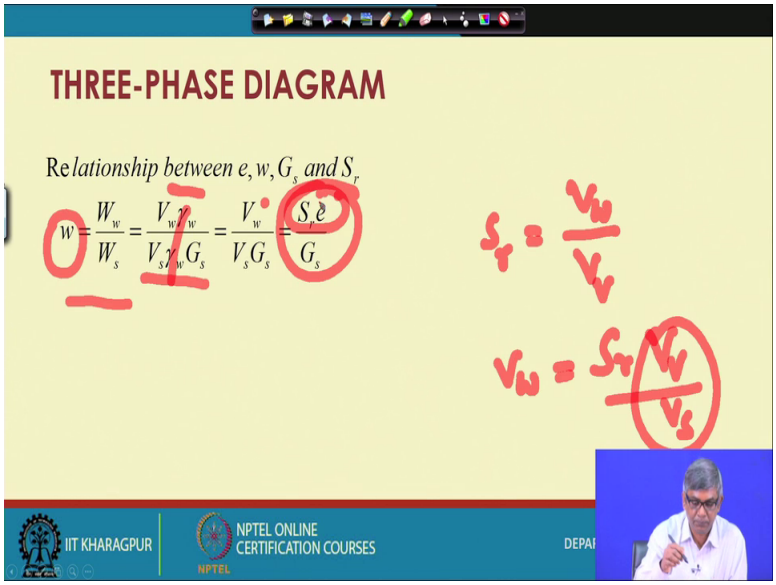

So, relationship between void ratio, water content, specific gravity and degree of saturation. You can see that by definition, your water content is nothing, but Ww by Ws. And Ww can be written as Vw into gamma w and Ws can be written as Vs gamma w Gs. And gamma w get cancelled from here, then it will become Vw divided by Vs Gs, and Vw can be written into. Again Vw can be written in terms of Sr and void ratio. Your Sr actually equal to volume of water by volume of void, and then Vw can be written as Sr into Vv. So, this one can be expressed in the Sr into Vv, and if I divide it by Vs both from here and here, then here actually it become Sr by Vv by Vs. So, Sr Vv by if this is e. So, that becomes Sr into e, and Vs by Vs become 1, so Sr into e by Gs

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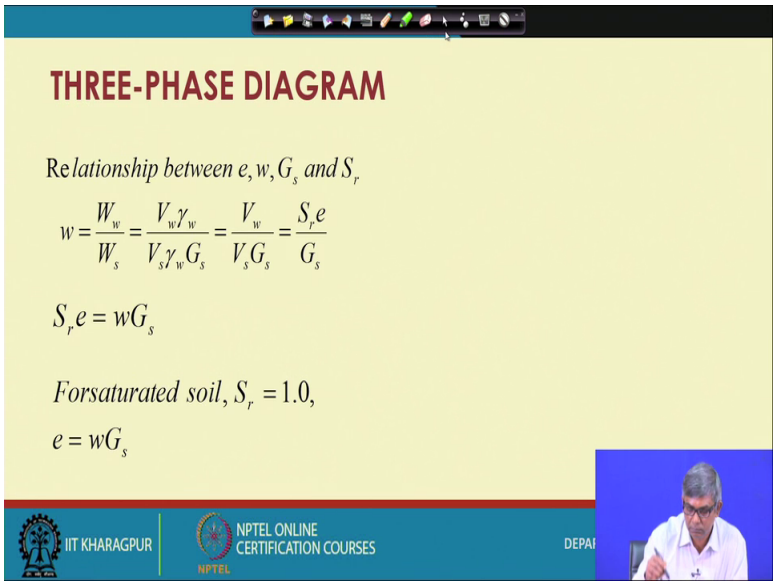

So, ultimately water content will be equal to S into e divided by Gs and. So, you can see the relationship finally, sorry, relationship finally, Sr into e equal to w into Gs. And for saturated soil, if; that means, Sr equal to 1, then in that case your, for saturated soil your e become e equal to water content into specific gravity. And if it is partially saturated, then degree of saturation multiplied by void ratio will be equal to water content to specific gravity is the relationship. This is another relation, this is another. So, this is the specific. From these actually you can derive by putting Sr equal to 1.

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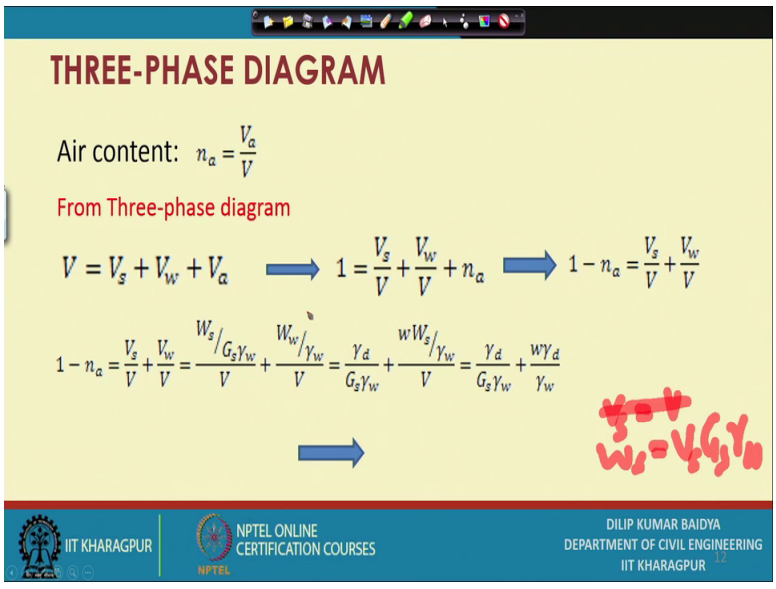

Now, air content; air content actually is defined as na, and which is by definition Va by total volume; that means, with respect to total volume, how much volume of air is present. So, you can, as we have done that three phase diagram, the total v can be expressed into Vs plus Vw plus Va, and this one if I divide it by Va if I divide it by V everywhere, and you can see this side this become 1 Vs by V Vw by V and this will become Va by V. So, this is n na.

and from here I can take it to here; that means, Vs by 1 minus n equal to Vs by V plus Vw by V. And from there we can further modify 1 minus na. You can see now 1 minus na Vs by V already there, and Vw by V and this is Vs by V, Vs actually can be written Ws by actually Vs equal to b. Sorry Ws equal to Vs Gs gamma w. So, if I want to express Vs, then it will be Ws by Gs gamma w.

So, Ws by Gs gamma w I have written G. Similarly if Ww is nothing, but sorry Vw is nothing, but Ww by gamma w. So, actually Vw into gamma w is the Ww. So, Ww weight of water divided by gamma w is Vw. So, this we have converted, and then I can see here Ws by V is a gamma d, and Gs gamma w, Ws by V is gamma d is taken here and Gs gamma w is here. And here actually again Ww I can express by w into Ws, water content into total solid by gamma w, and if we further simplify then you can gamma d by Gs gamma w plus water content into gamma d by gamma w.

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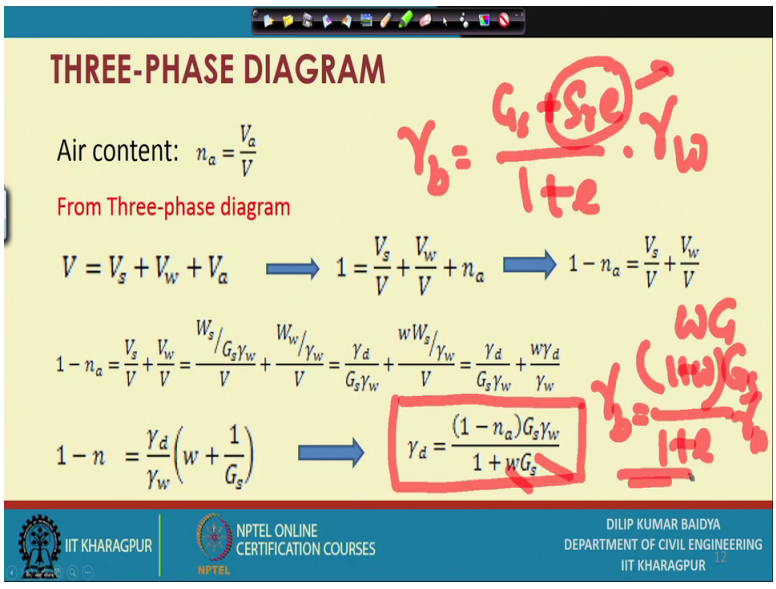

So, from these if I further, if I go, then 1 minus, if this is a, a is missing, gamma d by gamma w, gamma d by gamma w. If I take common w plus 1 by Gs, and further if I go. So, this is the relationship for your gamma d Vs gamma d equal to 1 minus na; that means, air content, 1 minus n is air content, Gs that is specific gravity gamma w. The unit weight of water 1 plus water content of the soil, Gs is the specific gravity. So, this is expression for a gamma dry. So, this is the one number of quantities in soil mechanics we have to calculate time to time. Sometime void ratio is given, you to find out water content. sometime water content is given you have to find out void ratio. Sometime both are given you have to find out your bulk unit weight or dry unit weight or submerge unit weight

So, we have given one equation actually gamma bulk equal to Gs plus Sr into e by 1 plus e into gamma w. This here actually, once you have derived Sr into e equal to w into g, Sr into e equal to w into g. So, if I substitute these, then another equation for bulk unit weight will become 1 plus w into Gs divided by 1 plus e into this gamma w. So, this equation we have not derived, but we have derived this equation. We have derived relationship between degree of saturation, void ratio, water content and specific gravity, and in the; if that relationship, if I substitute here then you will get another equation for solving. So, this is by and large different weight volume relationship, we have used in various calculation in soil mechanics.

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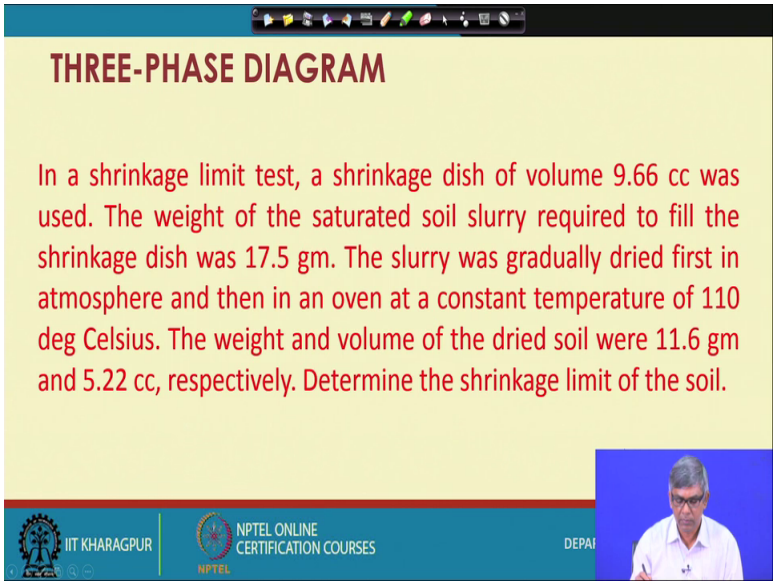

Now, I will I think we will go to the next topic, maybe with some application of those whatever you have learnt so far, from the first class, some application we will do with several problems, and we will take it in the next lecture.

Thank you.