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NPTEL
National Programme
on
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Probability Methods in Civil Engineering

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Lecture – 08

Topic

CDF and Descriptors of Random Variables

Hello welcome to the third lecture of our module 3 and in this lecture, we will cover mainly that descriptors of random variables.

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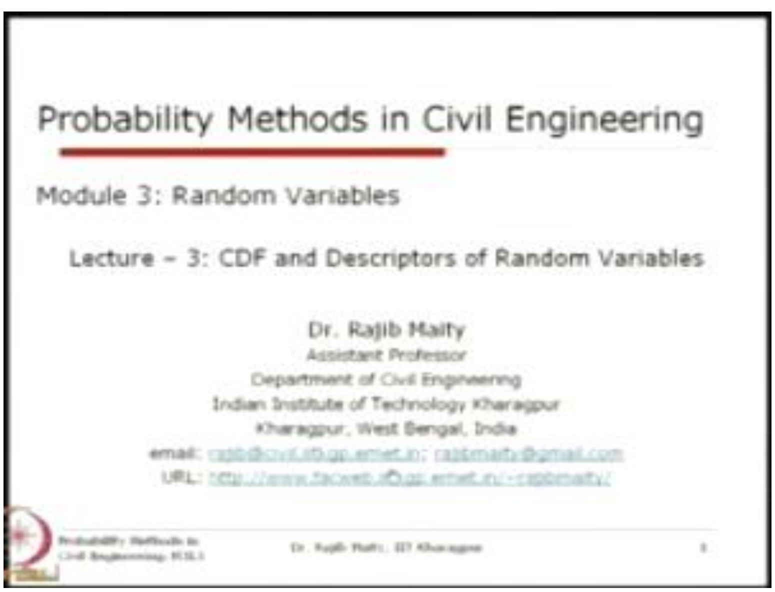

And in the last class we stop somewhere at while, we have started description on the CDF, that is cumulative distribution function. So what we will do we will start with that CDF the description of the CDF and we will see some problems on it, particularly pure CDF and we will also see some example of the mixed variable, where some part is discrete and some part is continuous, we will see one such example and then, we will go to some general descriptors of random variable.

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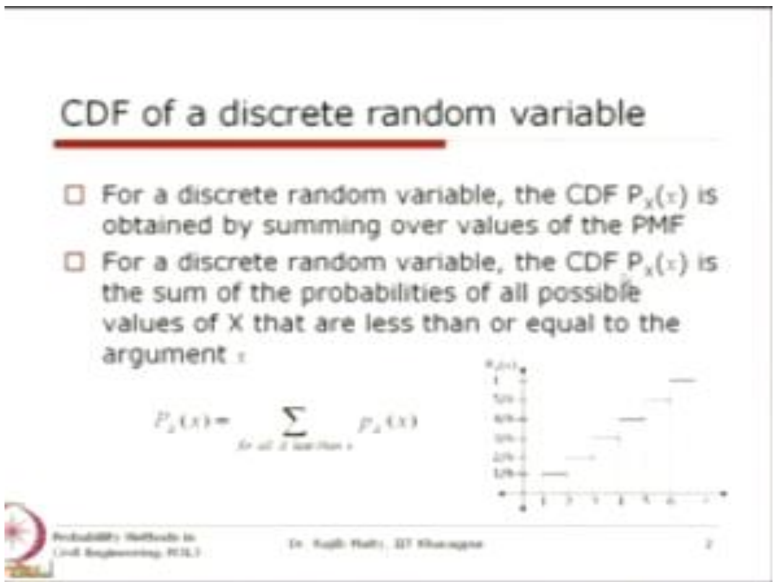

So where we stop in the last class is the description of the CDF for a discrete random variable, we know that there are two types of random variable, one is discrete and another one is continuous and for both this type of variables, we have seen what is their density function, that is probability density function for continuous random variable and probability mass function for discrete random variable.

Now we are starting that CDF that is cumulative distribution function for a discrete random variable. So for a discrete random variable the CDF that is P X (x) is obtained by summing over with the values of PMF that is probability mass function. For a discrete random variable the

CDF, PX (x) is the sum of the probabilities of all possible values of x, that are less than or equal to the argument x.

So if this notation that is P X (x) stands for the CDF, the value of the CDF at x which is nothing but the summation of their PMF for all possible x which is less than this small x. Now this is the, so if you take the one standard example of throwing a dice and getting that six different outcome 1, 2 up to 6, and if we say that all these are equally likely then the PMF says that at exactly at the point for exactly for the outcome 1 the probability is 1/6 and for all such outcomes the probability is 1/6. So this is now concentrated so in the last one we have seen that.

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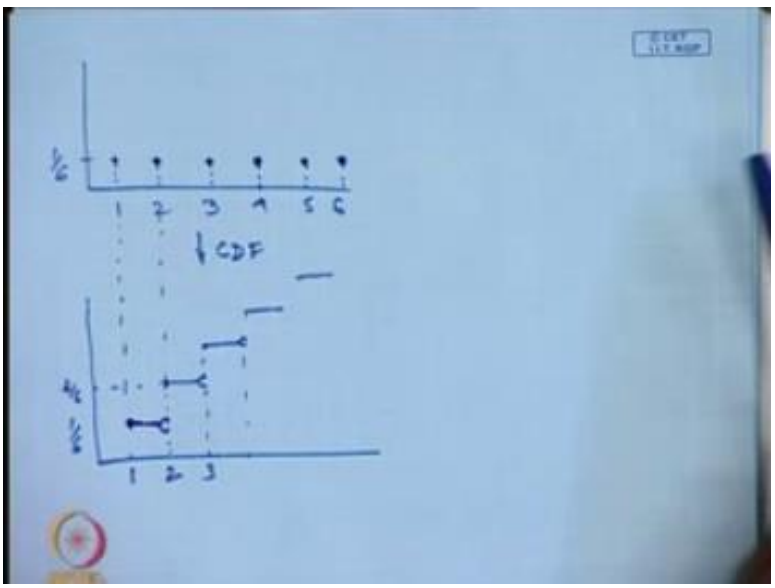

Its PMF looks like this where this 1, 2, 3, 4, 5 and 6 and all are the concentrated mass and these are all equal up to 1/6. Basically this should be the representation of this PMF and in some book we will see there is a stem diagram or kind of that but, this is as we are telling that these are concentrated mass of probability that is why the name PMF. So these dots are sufficient to declare that this is a PMF.

But just for the reference we can just put one dotted line like this just to indicate that this is referring to that particular outcome, if I want to know what is the CDF for this kind of descriptor then, you see that there is some two important thing, that we should we should keep in mind. The first one that is if I just take this reference line for this one, so this is this point is 1/6.

Now for if want to know what is the value for this 2 then I know, that for two this will be exactly 2/6 or 1/3. So this probability when it is exactly equal to 2 then, it comes the probability comes to be 2/6. Now what about for this line in between some number now for the for the CDF, I can say that any number the argument can take any number between 1 and 2 then less than that value say for example, 1.75.

So what is the cumulative probability up to 1.75 then obviously the probability is 1/6. So the probability that CDF remain constant starting from this 1 and going up to 2.Now as soon as it touches 2 it suits to 2/6. So generally in the representation we should not touch the line of 2, so this is a continuous line it will simply start from 1 and go as close as it can go up to 2 but it should not touch 2. Immediately when it touches 2, that is the cumulative probability at2, that is less than equals 2 that is why the less than equal 2, that is why they here the probability distribution function value of the CDF that is sorry the cumulative distribution function value of CDF at 2 it is 2 by 6.

Similarly, so it will be 1 line and it can go as close at as it can up to 3 but, it should not touch the line 3. So here, so like this so basically, we are getting some lines continuous lines a step lines like this and it is going up to 1. So this should be the representation so this is very important that it can go as close as to the next value but, as soon as it is touching the next value there is a sudden jump of this one.

So this kind of step 5 function that we can see for a discrete random variable, that presentation of representation of cumulative distribution function for a discrete random variable.

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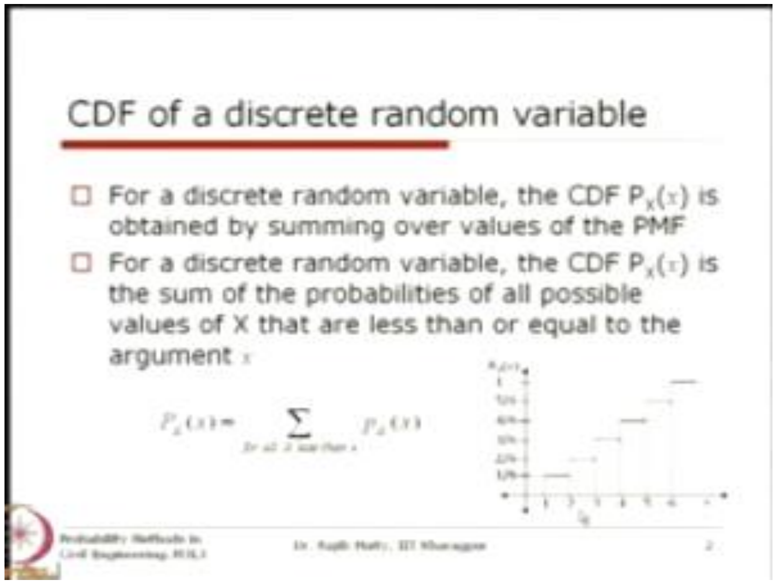

So in this figure you can see there are some these steps are shown and there is, so this line cannot touch this line and this dotted lines are nothing just the representative just to show that, this point corresponds to exactly equals to 2. So this CDF is represented only by some straight line parallel to the x axis in this six different steps and finally which is touching to that 1 okay. So this is the representation of the CDF for a discrete random variable.

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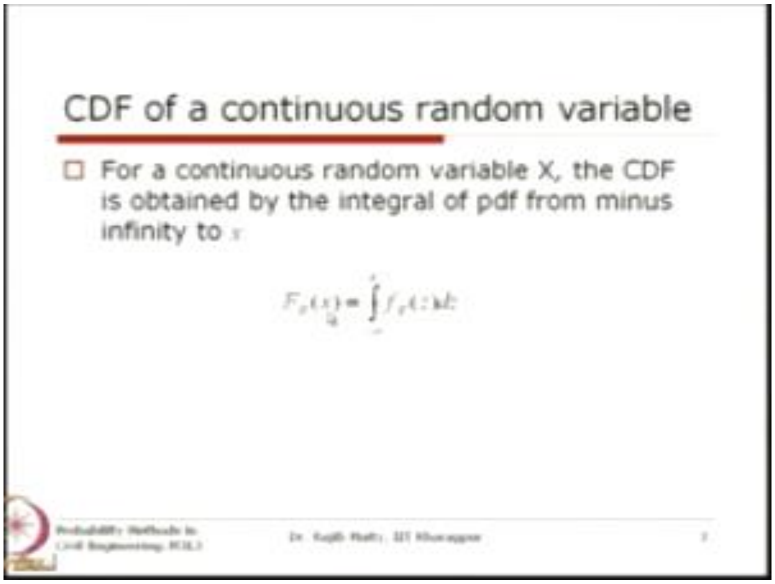

Similarly, if you see the CDF for a continuous random variable, where we know that, this particular function that probability density function that we discussed in last class where it is having a continuous distribution over the entire support of the x. So if you take that, now if you want to know the value of this CDF at a particular attribute equal to x, that means this F X (x) that means, the total value up to that particular point we have to see.

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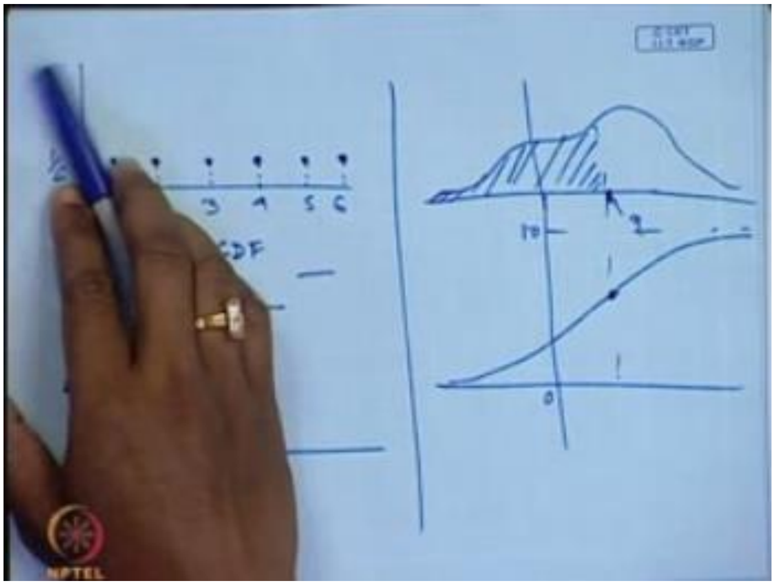

So here if we see that, for a continuous random variable this may be the distribution looks like this. So now at a particular value here if it is x, then this value if I want to draw that CDF here now, so at this value this point represents the total area from the left support up to that point x. So this value represents this one so this will be a monotonically increasing function, which will go and touch up to the maximum value that it can take is 1 and it should starts from 0.

So this value is nothing but the integration that means the total area of this function up to that point that means the integration of that function from the left support to that particular point. This is exactly mathematically represented here.

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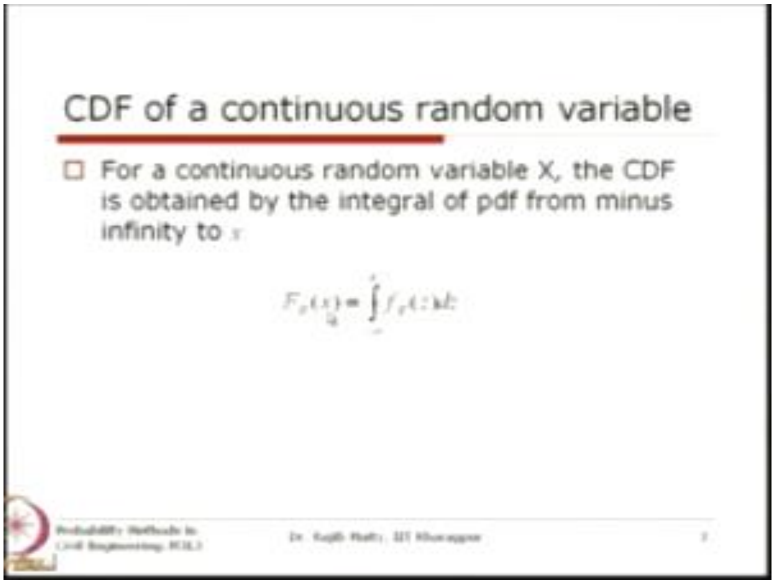

That you can see that f x equals to from this minus infinity to that particular point integration of that is PDF probability density function, and this will give you the CDF of a continuous random variable. One thing is important here, once again I am repeating the fact that this one will not give you the direct probability. So this is the representation on the probability density at a particular point, but not the probability.

But this function add that point it is representing the probability of x being less than equals to small x, that particular value. So if from this one again, mathematically we can if we just take the differentiation of this CDF cumulative distribution function, then we will end up to the probability density function.

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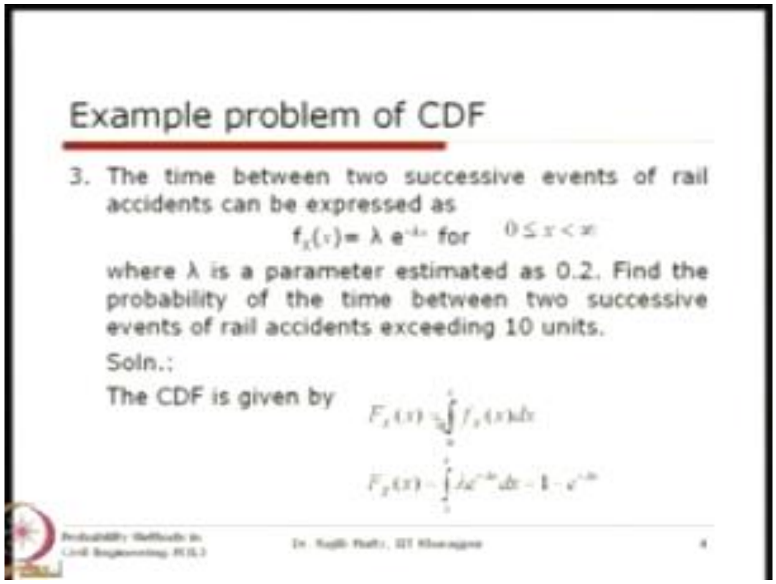

Now we will see two examples of CDF the first one is a straight forward one example of this exponential distribution, the time between two successive events of railway accidents can be expressed as this probability density function, if F X (x) = $\lambda e^{-\lambda x}$ which is the support is declared as from 0 to infinity. So where λ is called the parameter which is estimated to a 0.2, so these are the parameters it is estimated from we will just see and in the successive class how to estimate the parameters of the distribution.

So this with this parameter lambda equals to 0.2 we have to find out what is the probability of the time between two successive events of rail accident exceeding 10 units suppose that units suppose that units is not specified or mentioned here so such 10 units what is the probability that two successive events of rail accidents will exceed 10 units.

So first of all what we have to do from this probability density function we can directly get it from this implication, now if we want to know that what is this probability what is the cumulative density function of this one we will know that we have to integrate this probability density function from this $-\infty$ to x, so here it is from $-\infty$ to 0 it is 0.So, from 0 to x we will integrate it so to get that so if we do this simple integration we will get this$1-^{e}-\lambda x$ which is the cumulative distribution function for this probability density function.

Now if we put any particular value of x here that means we are getting the getting we are directly getting the probability of the random variable x being ≤ x. Now our question is find the probability of the time between two successive events of the rail accident exceeding 10 units now it is given that exceeding 10 units that means if I put here 10 that means I will get the probability of non-exceeding 10 units.

So if I want to calculate the exceeding than the total probability we know 1 so you have to deduct that particular value from the total probability to get what is the probability of exceeding 10 units.

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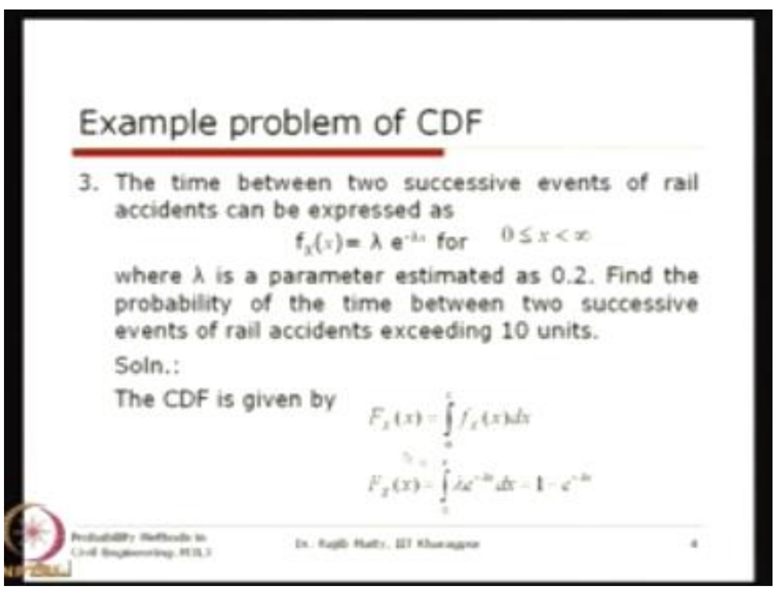

Exactly the same thing has done here that is probability of the exceeding 10 units is equals to probability x ≥ 10, so x ≥ 10 is nothing but, 1-x ≤to 10 and x probability x ≤ 10 is nothing but, F x 10. So this F x 10 we are directly getting from this our probability density function, this will be minus means after taking out this bracket this will obviously become plus, so this is eventually coming to this after putting this value of this λ this will eventually come e^{-10} x 0.2 that is a value of λ.

So this will be minus if you if you are putting this parenthesis if we take out this parenthesis obviously this will be the plus, so which is shown it here so the probability is 0.135 so 13.5% percent it is probability is that the rail accident sorry this will be rail accident exceeding 10 units for a particular for that case is 0.135. Now if you want to see how it is distributed this show where that probability density function is shown and this probability that cumulative density function is shown. Now if we want to see how it is varying over this x.

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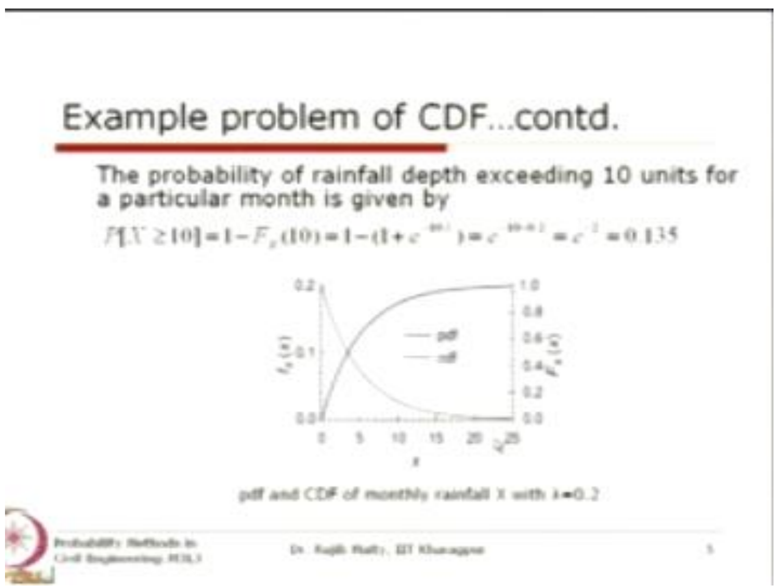

Then this is the graph so you see here so at x equals to 0 the value of these probability density function is 0.2 obviously which is the value of value of λ and gradually as it is goes to ∞, you see it is becoming it is coming down and being asymptotic it being asymptotic to the value 0, now if I want to get that CDF that means this CDF is this one so that CDF will be nothing but the integration of this area so gradually the that integration will increase and it is starts from 0 and will be asymptotic to the value 1 and it will be 1 at x equals to ∞.

So this curve that you see, this is your CDF and this one this is your PDF for the exponential. This is that typical example of the exponential distribution that is showed you in the last class, so these are these are how this PDF and CDF for this case looks like.

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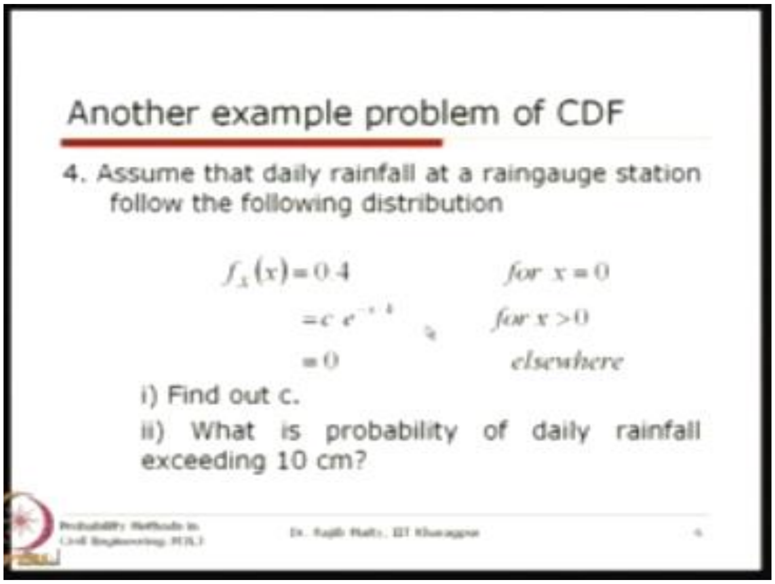

Another interesting problem is taken here this is on thus daily rainfall at a rain gauge station assume that, the daily rainfall at a rain gauge station follow the following distribution now the daily rainfall if you take from a from a particular station than what we see that most of the time there will be many 0 values and for some nonzero values it will come so what is done is what is done is first of all you just collect that reading.

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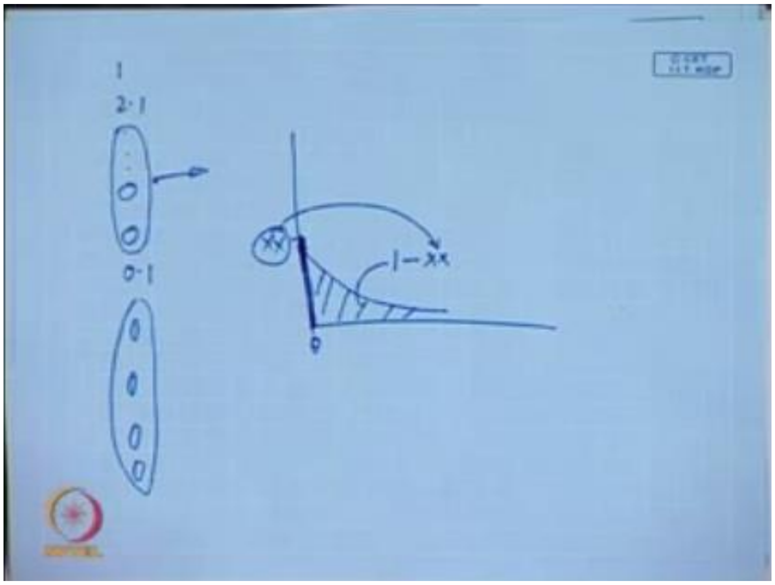

And in that reading we gets some non-zero values say 1, 2.1 like this we will get and then we get many 0 again some value 0.1and so, what is meant is there this daily rainfall value that kind of value is the series is having many 0 values, so if you want to calculate so those probability distribution function what we generally do is that we generally exclude these 0 value first and we see what is the first the probability of getting the value 0 and then we feed the another probability distribution for this non 0 value.

So this is a this is one example of one example of the mix distribution where the at 0 there is some probability is concentrated, so if you see this one so at 0 if it is 0 at 0 some probability is concentrated here and for this non 0 values it may have some distribution. Obviously, the more the that the magnitude of rainfall depth obvious the density will come down, so this is one example of, so here some probability is concentrated here and it is coming up so now, you can see that at 0 if some value is concentrated here.

So the total area under this graph obviously would be 1 minus what this value what is this value is concentrated as at 0. So this example, we have just we are considered here

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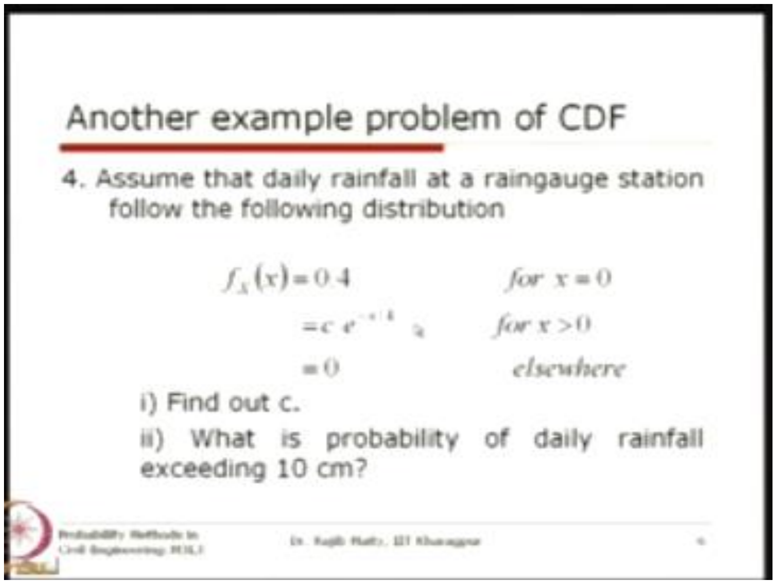

And we have just taken one example that this f x equals to here you see that 40 percent probability is concentrated at x equals to 0 so now that for x greater than 0 x greater than 0 the CDF c is c^{e} –x/4 elsewhere it is 0 that is why minus infinity to less than 0 this value is 0 so this the complete description of the CDF and this is the example of the mix distribution where there is a probability mass here concentrated at 0 and a continuous distribution for greater than 0 values.

So first we have to find out c and then you have to answer that what is the probability of daily rainfall exceeding 10 centimeter okay so this x is having the unit of centimeter here so first what we have to do so find out c so c is a constant here, so if we have to find out the proper value of c that is the total area under this curve should be equals to 1 that thing we have do we have to satisfy.

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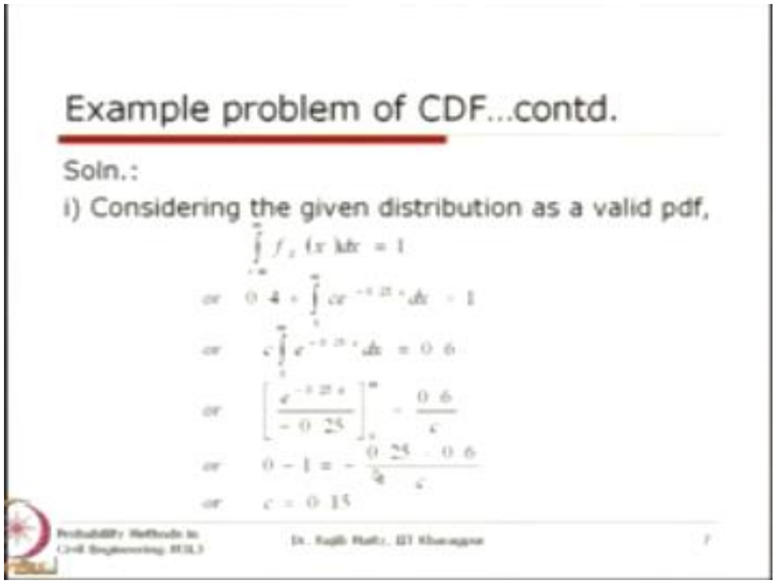

So, it is done it should from the minus infinity plus infinity, from the second axiom of probability we know that this total so the condition of this PDF that this total integration should be equals to 1, now I know that from minus infinity to less than 0 the value obviously 0, for at 0 the concentrated probability mass is 0.4 and from 0 to infinity this that function we know and this should be equals to 0.

So if you just if you just do this integration and solve this equation for this one unknown here, so we will after doing some step you will get that c +2 .15, so the value of c we got so, the complete description.

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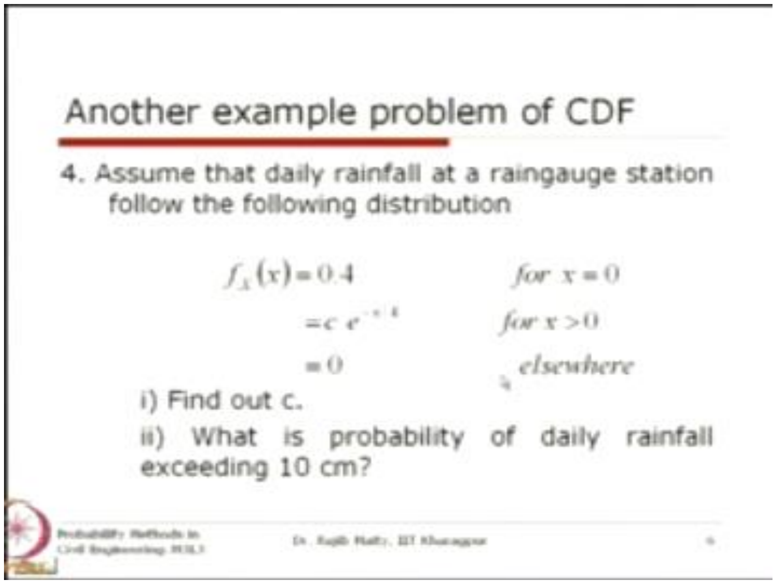

Of this probability distribution will be F_xx equals to 0.4 for x equals to 0 is equals to 0.1 5^e x/4 for x greater than 0 and 0 elsewhere.

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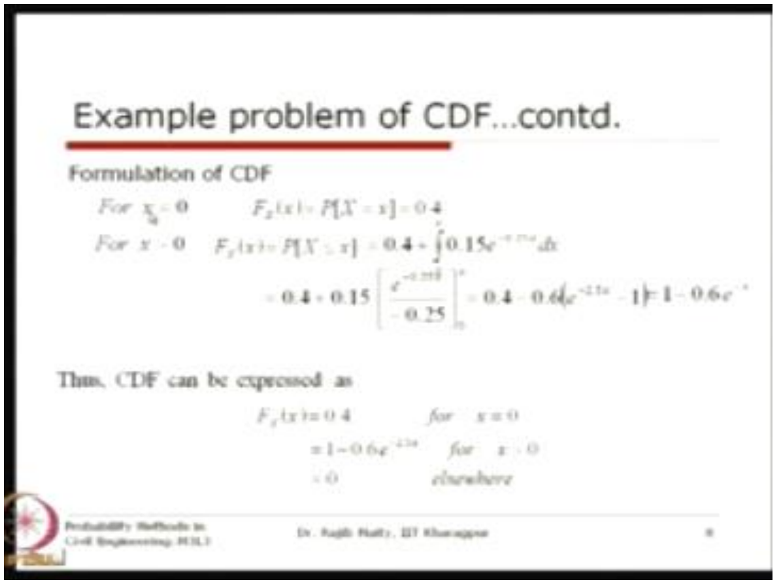

So here it is written that for x equals to 0 that now if I want to get the CDF that cumulative distribution function for x equals to 0, this F_x x that is x exactly equals to 0 equals to 0.4, now for this range that is x greater than 0, F_x x which is nothing but, the probability x less than equals to 0 it should be equals to that concentrated mass at x equals to 0.4 plus the integration from 0, to that particular point x of this function. So if you do this few step we will get the distribution function as that for this x greater than 0 in this zone is $1\text{-}.6^{-ex}$, now the once you get this CDF then the rest of whatever the answers we are looking for that is for example here we are looking the answer for this x greater than 10 centimeter.

So this is the final representation of this CDF that is f x equals to 0.4 for x equals to 0 1 - $0.6^{e2.5}$ x is greater than 0 and 0 elsewhere, so this will be 0.2 5x so instead of-x this will be 0.2 5x is the correction here needed. So this, with this final form of the CDF that we got now we will put that x is x equals to10 to get that.

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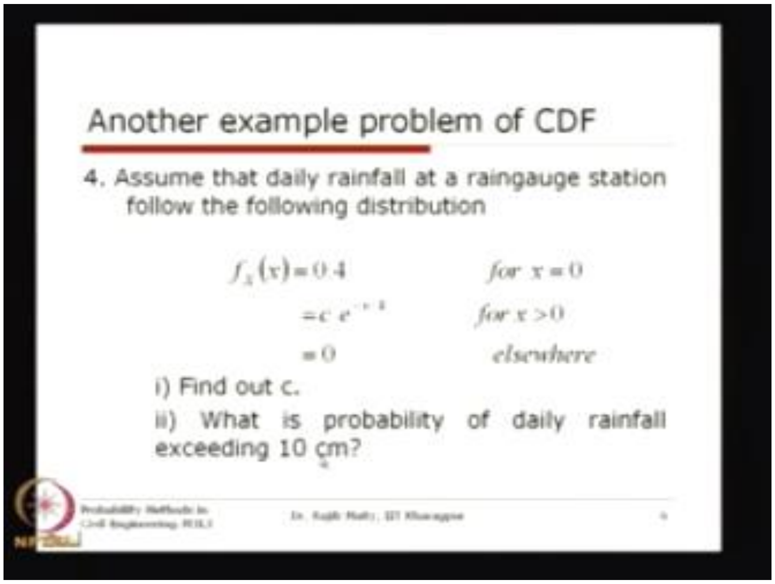

The probability here it is asking what is probability that rainfall again here it is shown that exceeding 10 centimeters so we will first calculate the probability from this CDF what is that probability of this daily rainfall less than equals to 10 centimeter than from total probability one if you just deduct we will get that answer.

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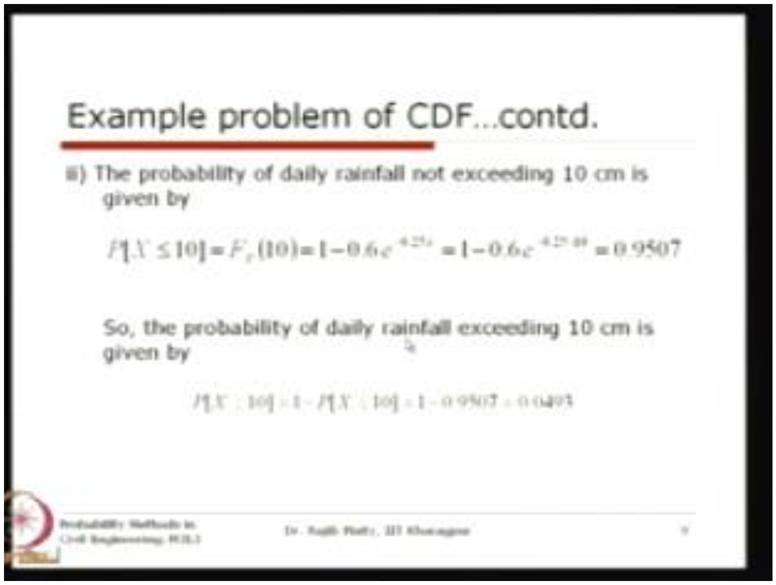

So from this one if you just put that x less than equals to 10 that is f x equals to 10 which is nothing but 1 minus this equation 0.9507 we will get from this one, so the probability of daily rainfall exceeding 10 centimeters obviously will be greater than 1, so 1 minus probability x less than 10 is equals to 0.0493 is this probability.

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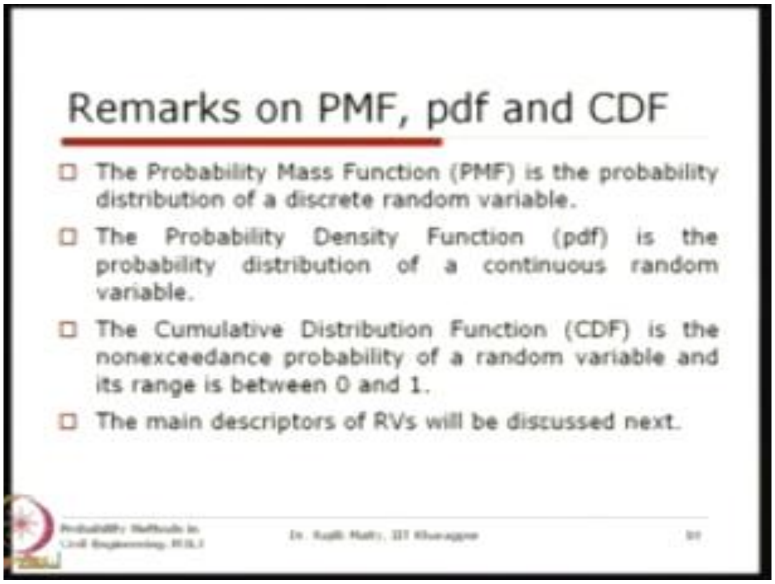

So up to this what we have seen is that we have just seen that PMF for which is for the discrete random variable that is probability mass function than we have seen that probability density function which is for the continuous random variable and last we discuss about this cumulative distribution function for both discrete and continuous random variables.

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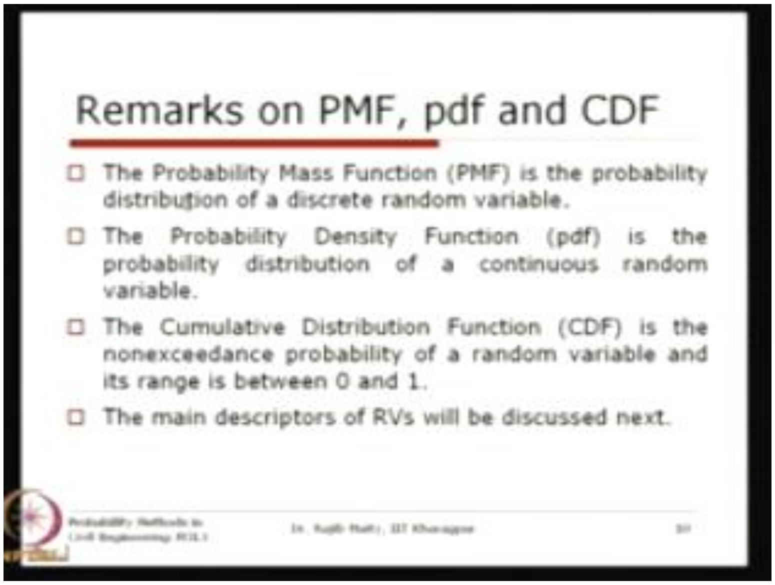

So some notes here on this the probability mass function PMF is the probability distribution of a discrete random variable, the probability density function PDF is the probability distribution of a continuous random variable, Cumulative distribution function is the non-accidence probability non-accidence probability of a random variable and its range is between 0 and 1.So now the main descriptors now sometimes if we do not get any particular close form of that close form of this probability density function or probability mass function of a variable.

Then from the observed data there are some descriptors of this random variable hat we will see, so that approximately with this descriptors a random variable the nature of random variable can be known to us, so this is our next focus to know that what are the different main descriptors of a random variable.

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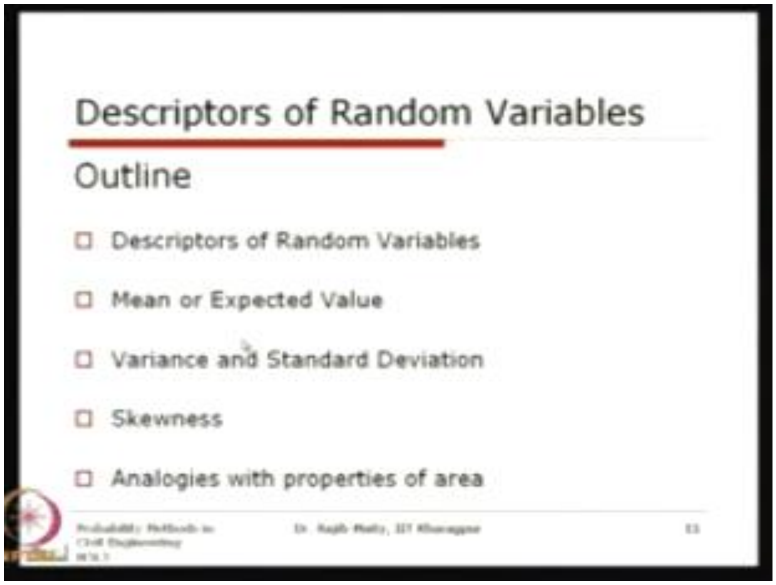

So what we will see that what is meant by this descriptors of random variables that we will see first then we will know what is mean or the expected value then variance or the variance and standard deviation then skewness then one where is there for that called kurtosis and then we will also see the analogies with the properties of the area that is area under the PDF probability density function in terms of incase of the continuous random variable.

That we will see just to rely that how this how this thing can be call as a moment that we will that we will see.

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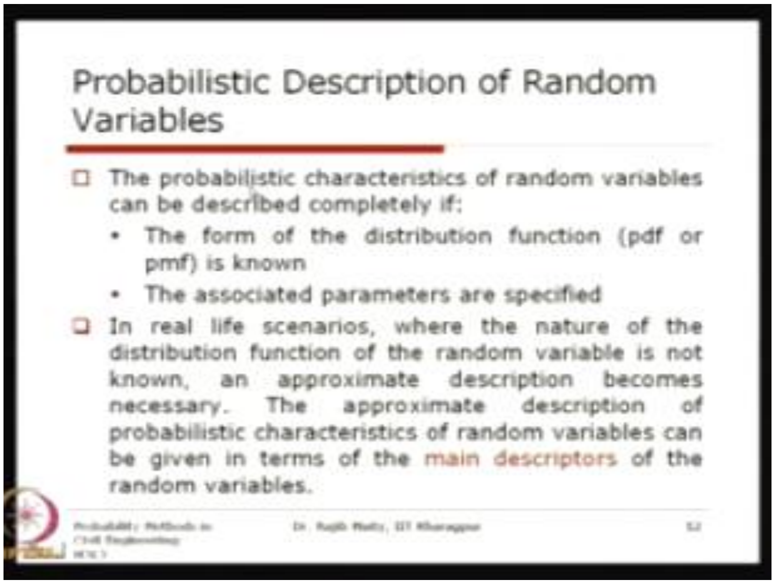

So probabilistic description of random variable if you want to see take some random observation of a random variables, some random sample data if we take then we will we can see these thing, that is the probabilistic characteristics of a random variable can be described completely if the form of the distribution function PDF or PMF is known obviously what we discuss so far and the associated parameters are specified.

For example, that a for the exponential distribution $\lambda e^{-\lambda x}$ so that λ is the parameter, so if you know that what is that density function PDF that is it is exponential form and if you know the value of lambda then it is completely known to us that the total description is given to us.

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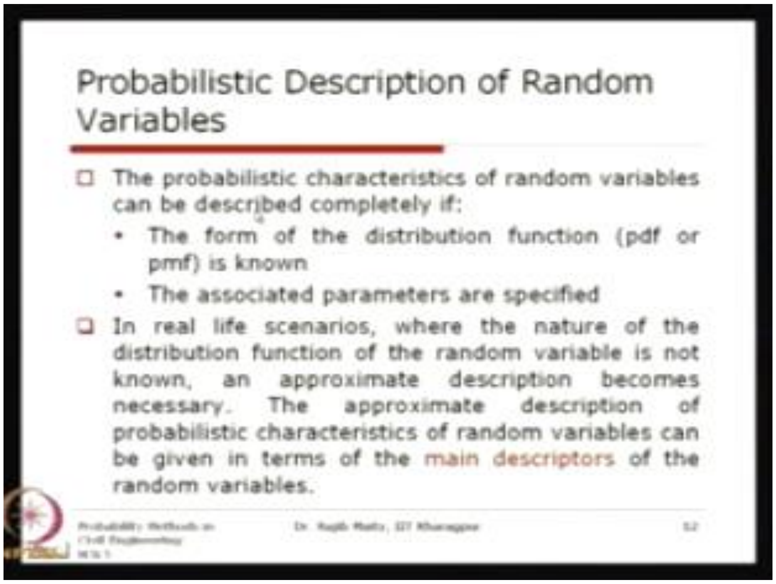

So this is what is its meant by this point, now in the real life scenarios where the nature of the distribution function of the random variable is not known an approximate description becomes necessary, the approximate description of the probabilistic characteristics of the random variable can be given in terms of the main descriptors of the random variables, so this is why so if we do not know the exactly the close form of this equation this is why this main descriptors becomes very important. Just to know the nature of the distribution of that particular random variable.

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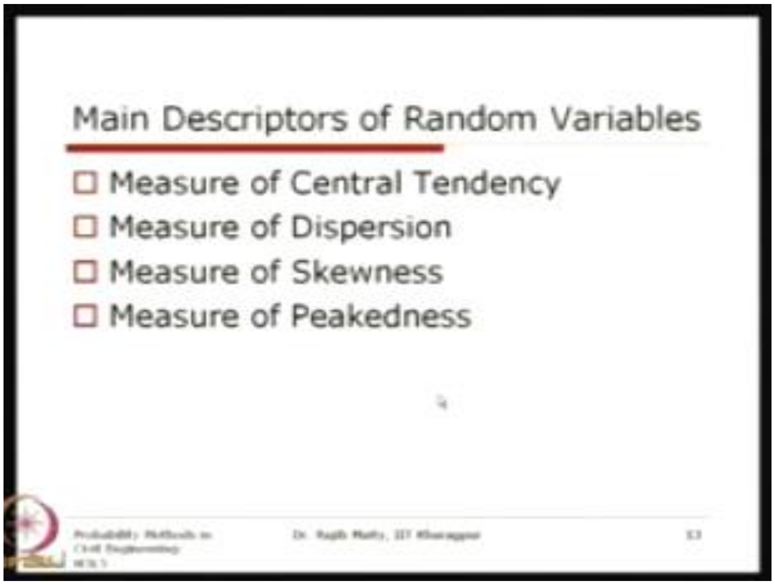

So there are the first four description that we that are very important and obviously there are higher side but this first four is very is important and mainly is residential for that nature of this distribution and the first one is the measure of central tendency, these measure of central tendency means where the centre is, now the centre is again a subject of importance in what sense we are looking for the centre.

So a given a random variable given the distribution where it is it is centre is that can be there are three different ways that you can say that what is the it is central tendency, second thing is the measure of dispersion, how so about that central point, how the distribution is disperses, how it is spread around that particular central value, then measure of skewness whether there is any skewed.

Whether it is skewed whether it is symmetric, so this is a measure of symmetry you can say that whether it is symmetrical or has some skewness to either to the left or right of this central point and measure of peakedness, so whether the of peak of that distribution is very high or low like that so you will go one after another we will start with this measure of central tendency.

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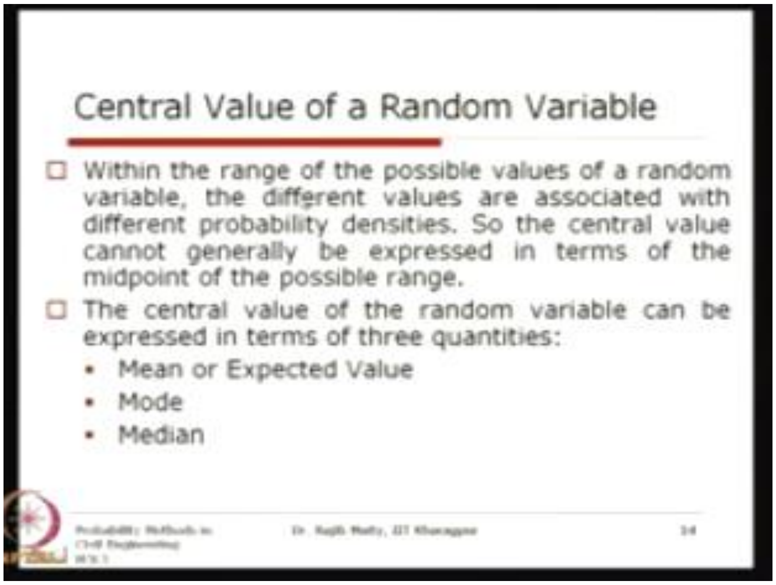

The central value of a random variable, so within the range of the possible values of a random variable the different values are associated with different probability different probability density, so the central value cannot generally be expressed in terms of the midpoint of the possible range. So just for a just for a small thing if we just say that.

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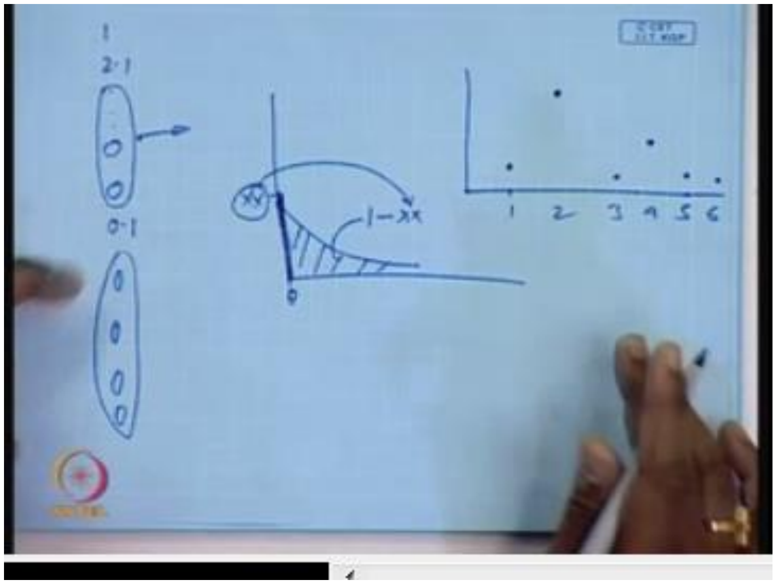

If we if we say that if we just take that particular example, say if I take the example of the discrete random variable first for the dice we have seen that 1, 2, 3, 4, 5 and 6 and if you see the all these things are equation probable then we can say the central is somewhere around 3.5 the outcome central tendency is 3.5. Now think in the situation where this outcomes or not equally likely.

So the mass is concentrated 1 is from for here for 2 it is here, 3 it is here, 4 it is here. So if it is like this then it may not be it may not be that midpoint of this outcome may not be the central tendency of this 1, obviously when I am putting this dot obviously I have kept in mind that the summation of all these should be close to one those exemptions what I mean is that obviously after satisfying all the axiom that is needed before I can declare that this is a valid PMF.

So if the what here it is it is it is understood the central tendency is that the central tendency need not be always the to the midpoint of the observation that we see, it depends on this how much density is associated to each outcome of that particular random variable, okay. So this is what is meant for this central tendency.

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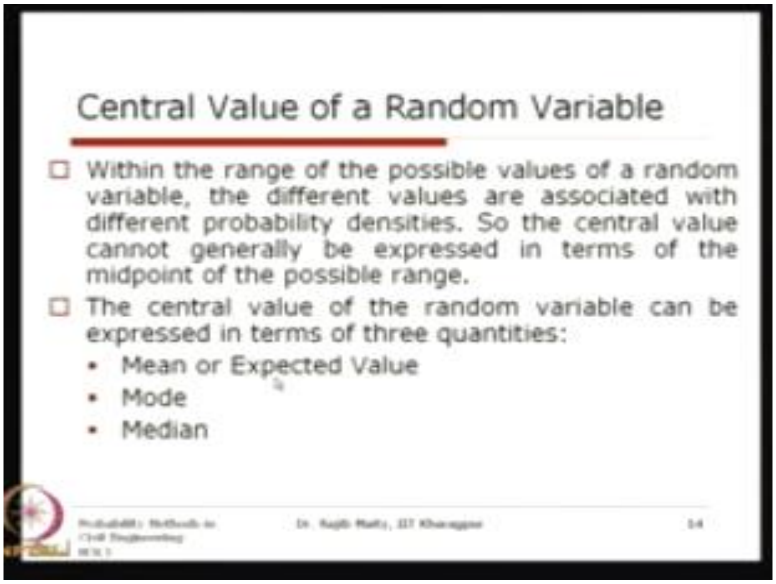

And there are three different three different ways how we can how we can say how you can express that the first the central tendency central value of the random variable can be expressed in terms of three quantities, the first is the mean or expected value second is mode and third one is median. So we will we will see the mean first which is expected value.

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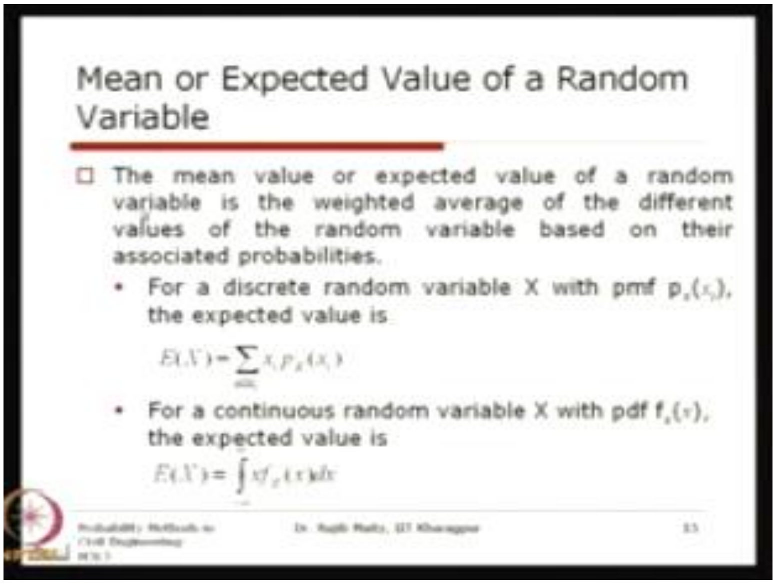

As well of the random variable, the mean value or the expected value of a random variable is the weighted average of the different values of the random variable based on their associated probabilities, for a discrete random variable x with PMF,PX, xi the expected value is Ex is equals to xi multiplied by that associated probability. So instead of just simply calling that this the average of this.

So the average of this outcome where what we are doing is that we are just taking the weighted sum, the weighted average, average is taken by the weight age of the associated probability so this is the expected value.

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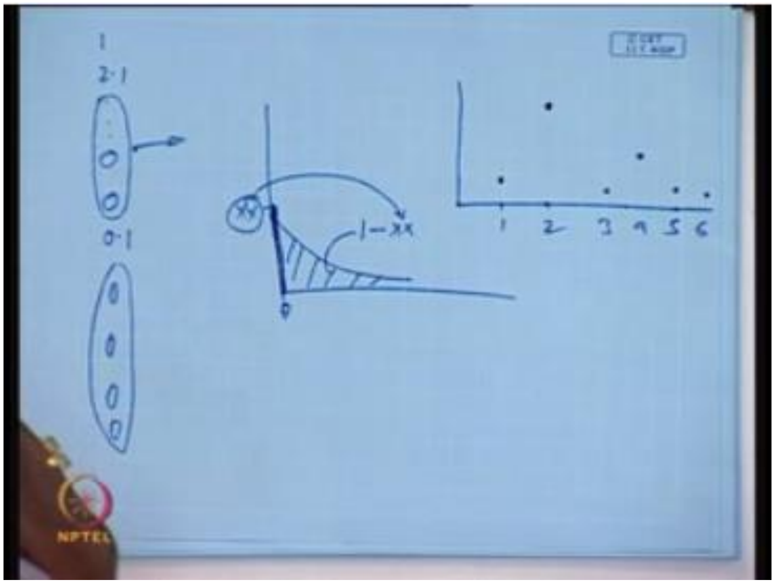

Now instead of multiplying the same values here that is the same values are 1/6 for each and every outcome, I am multiplying the each out come by their weighted probability, so this may be something but the when I am putting the weight for these two, this is obviously very high. So obviously this the central tendency will have the will pull that central part towards this particular observation. Because here the probability the concentration the mass is very high compared to the other outcome.

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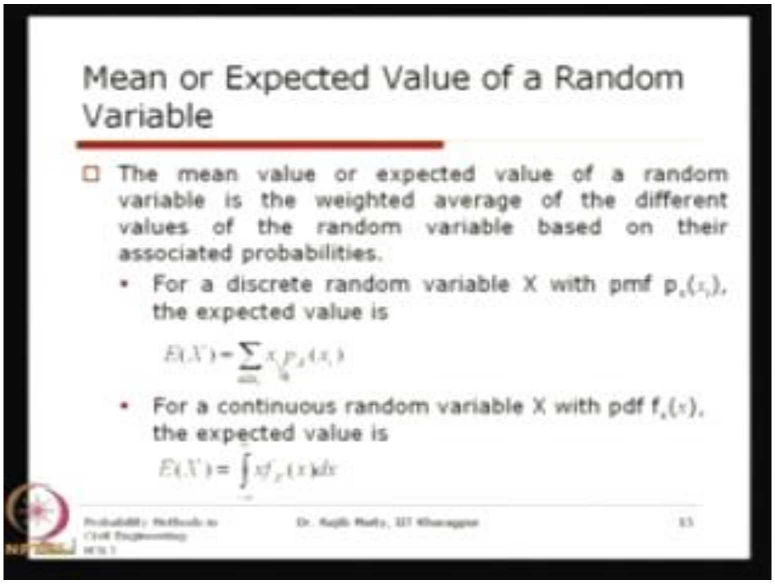

Now taking this same thing, same concept for this continuous random variable that for the continuous random variable x with PDF, F(x) x the expected value is that for the full range full range of this random for the full support of the random variable x that multiplied by x gives you that particular expected value of that random variable x, we will just discuss in a minute, how this is related to some kind of moment we will just come in that point a little later.

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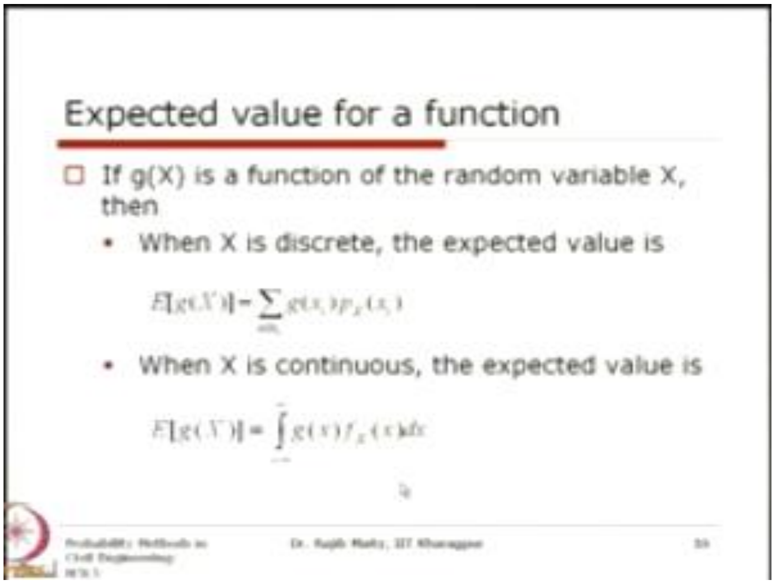

So as we have seen that for if we if when we are talking about the expected value of that particular random variable then we are multiplying then we are multiplying that variable with that.

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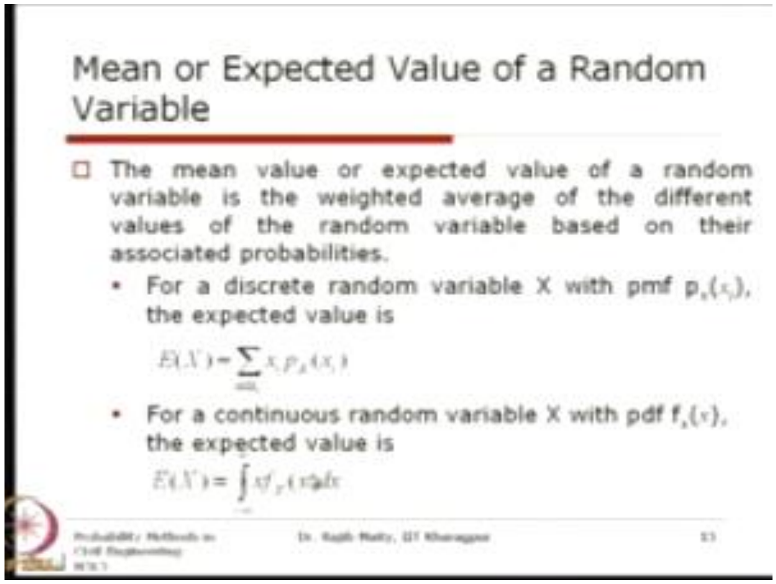

Particular value only with that x or here the outcome the x now the expected value instead of you can we can get the expected value of any function of the random variable the function of the random variable will be discussed in subsequent classes but here instead of only it is what we are trying to say here is that, so that expected value can be obtained for any other functions of that of that random variable as well. Simply by replacing this x by that function and this here x by that particular function.

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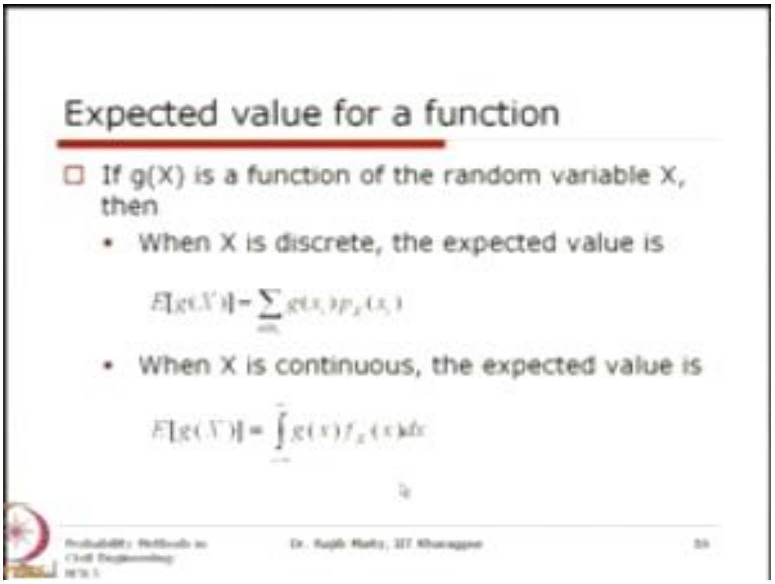

So here that is the g x here the g x is the function the random variable x then when x is discrete obviously we have to go for this summation multiplying that g x with that individual masses the probability mass for that particular outcome and when X is continuous then we are taking the integration instead of multiplying only by X by multiplying by this function that is g x into F x, x d x. So we get that expected value of that random variable of the function of the random variable g x.

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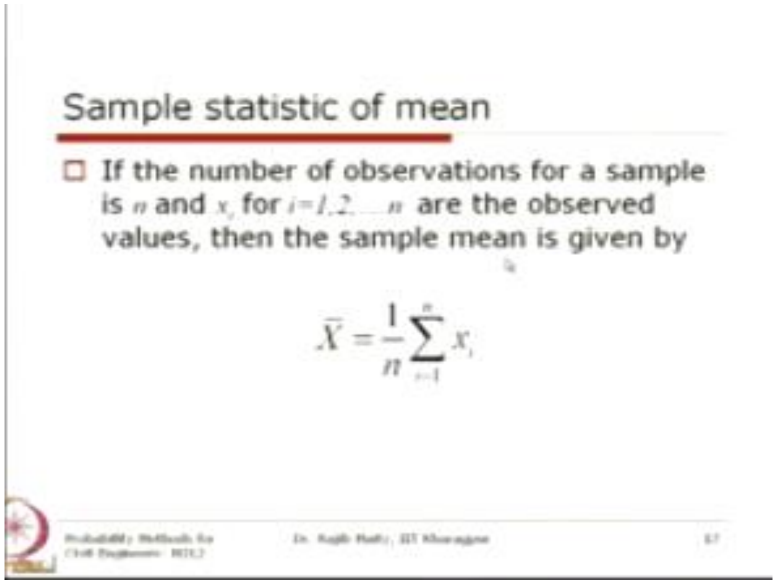

Now there are so if we know that probability density function then we can calculate that same for that mean now if we have some observations then from the observations if we want to get what is that sample estimate, now there are different criteria before I declare that, this is a particular estimated of that particular variable, that will be discussed later those are known as that consistency unbiased ness.

So this thing after satisfying those things if we have the different observation for a particular random variable then the mean of that particular random variable can be expressed as this where this n is the total number of observation is taken for the random variable, so sum it up and then divided by this total number of observation, that get what we get that is the mean of that the sample mean of that observation of that sample.

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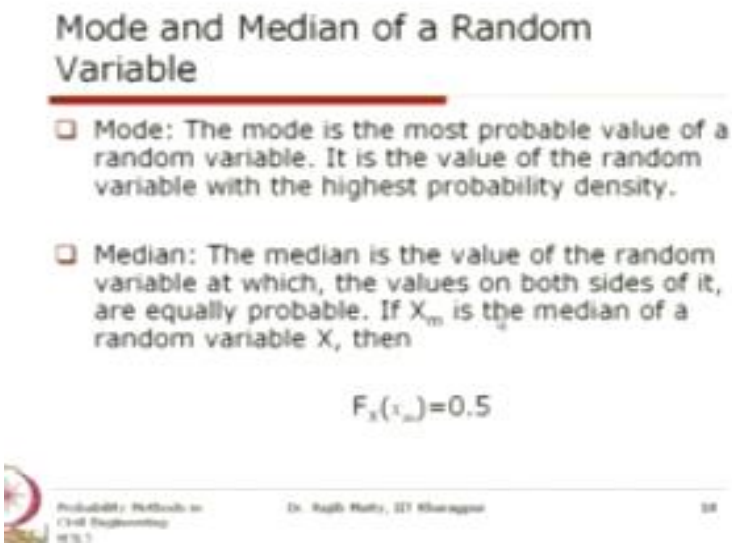

Other tools measure of central tendency is mode and median of the random variable, the mode is says that the mode is the most probable value of the random variable, so out of the different outcome in case of the discrete for this over overall range where the, which one is the most probable value for this one and this is the value of the random variable with the highest probability density.

Obviously when we are calling this probability density this is for the continuous, so if you say this one show, we see that.

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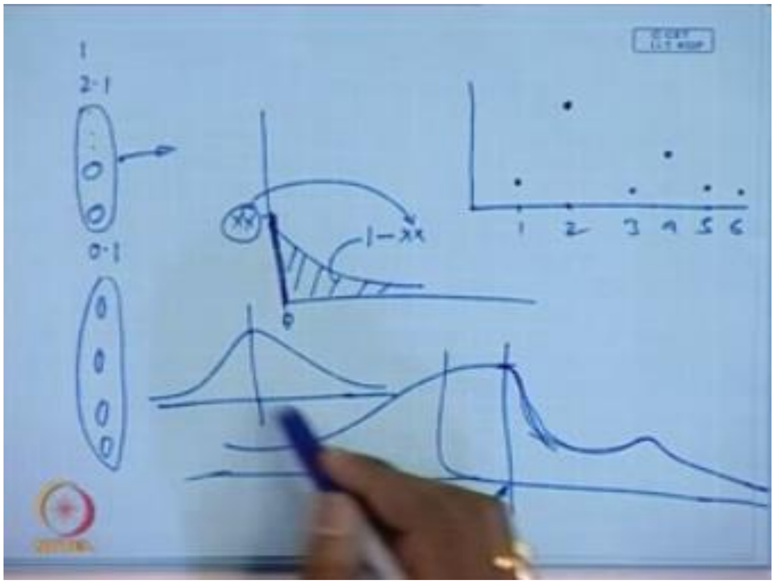

Here at 2 that the probability that mass is concentration maximum at 2, so obvious the mode here is2 and for if I just say that this is your this is your some probability distribution function then, obviously at this point where you see the density is maximum this your mode, again if you see that there are some values and there are some peak so there may be some other things so these are generally known as the bimodal or multi model.

So this is having more than1 1 mode so this is the secondary mode here, where there is a secondary peak for example the standard normal distribution if you see this is any modal and the mode is here at 0 and we will see that, what this moon that, this mean mode and median is same for this kind of symmetric distribution, that is a normal distribution this is same.

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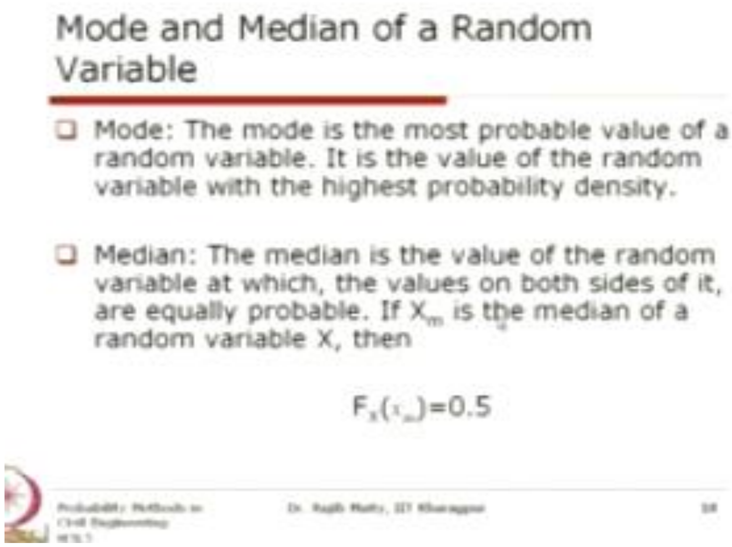

That we will see later.

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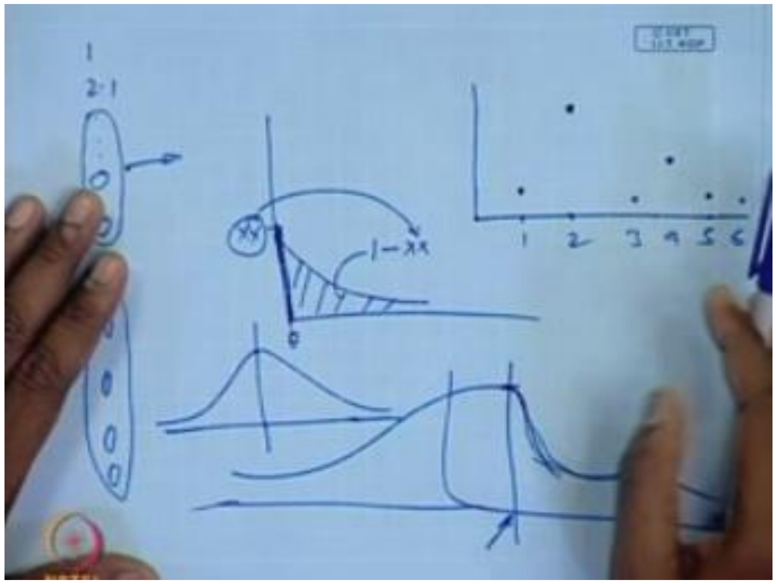

What we are mean here is that, for the discrete random variable the where the probability mass is maximum that is your mode and where the probability density is maximum, that is your mode for that random variable. And then it is median, the median is the value of the random variable at which the values on both side of it are equally probable if the X_m is the median of a random variable x then F_X x m is equals to 0.5.

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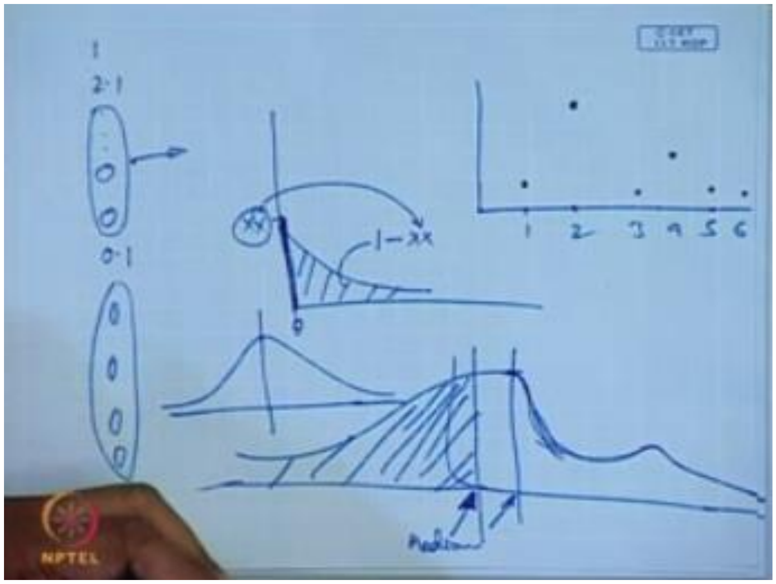

Now this is this is one thing that where should, when you will discuss this one first in case of this incase of continuous random variable, so I am just started going on integrating from its left support if it is minus infinity, from the minus infinity I am going on integrating and I will stop at some point where that, where the total area covered is equals to 0.5, so means this is the point where the probability less than that particular value is 0.5 greater than that particular value is 0.5.

So this value is your median similarly, you will go on adopt this probabilities and where it will where it will touch that0.5 for that corresponding CDF, the that particular value so if we will just add this plus this so where it will touch this 0.5, that particular value will be the median for this discrete random variable. So that median means that less than that particular value and higher and about that, the higher than the particular value both are equally probable as a total,

So left hand side total probability is 0.5, right hand side total probability is 0.5,that midpoint is your median.

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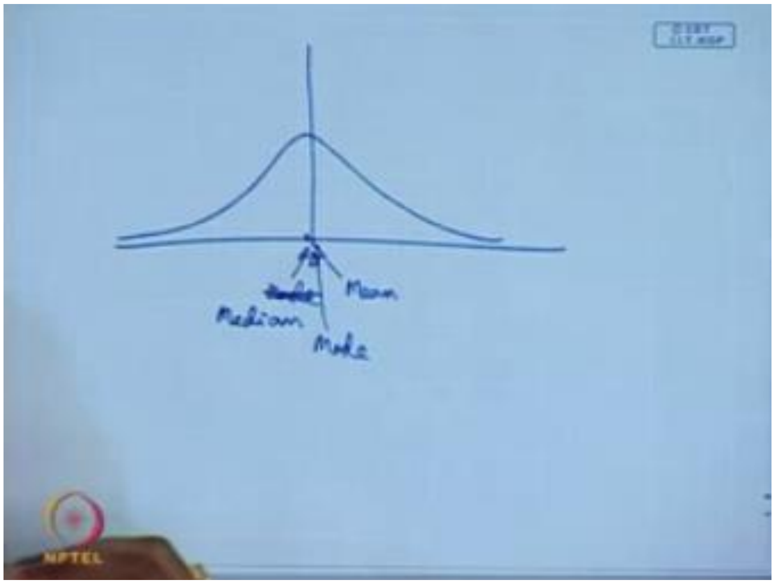

Now, what we are discussing in case of this normal distribution, so when we say that this is a normal distribution having a completely symmetric distribution and it is it is having some a mean here, so in this case you see that, exactly at that particular point where this touching the peak being the nature of the symmetry this is the this is covering the total area 0.5,so this is your this is your mode.

Now if you take that so if you take that integration take that mean, then you will see that exactly this point is becoming your mean and being this is the highest density point, this the same point is yours mode, so for the normal distribution, the mean mode and sorry median this is that where the50 percent probability is covered that is called the median so for this normal distribution this mean mode and the median are same point for this symmetric distribution that is a normal distribution.

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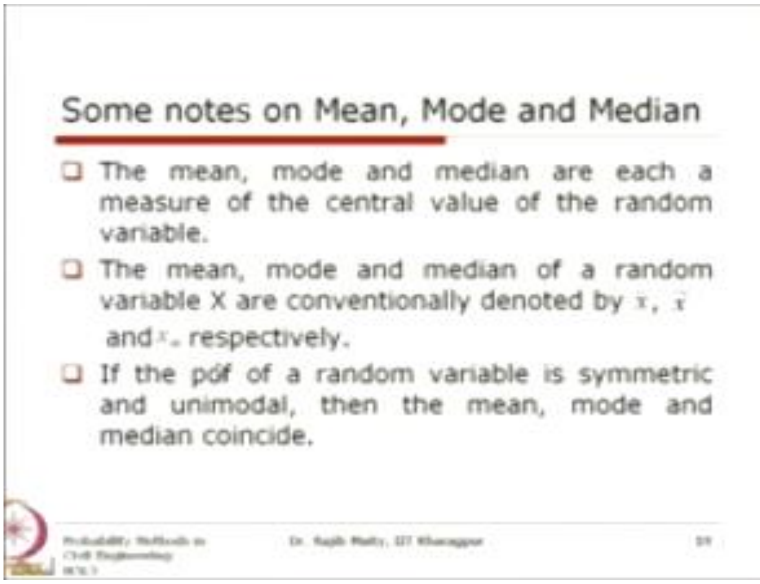

So this is meant that the mean mode and median are each a measure of central value of the random variable the mean mode and median of a random variable x are conventionally denoted by this x bar, which is mean this x tilde, which is mode and x m is the median of the particular random variable x. If the pdf of a probability density function of a random variable is symmetric and unimodal, obviously which is the normal distribution it is symmetric, it is unimodal that is having only 1 mode. Then the mean mode and median coincide, just what we discuss just now.

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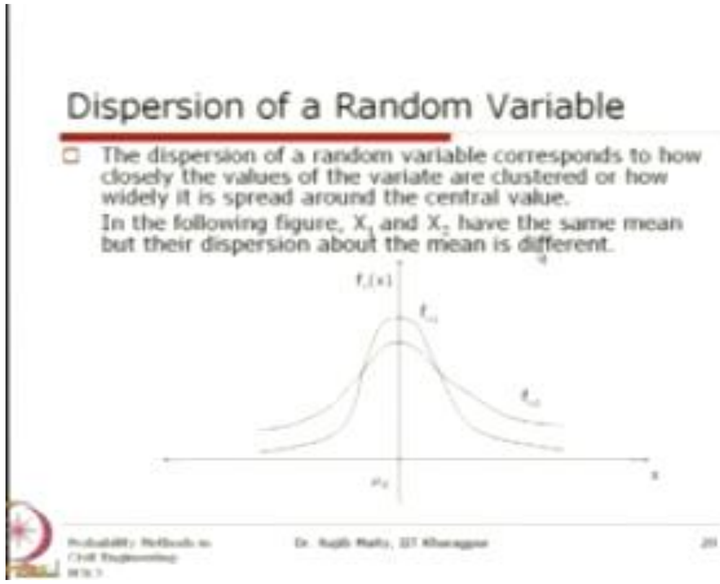

Now the second descriptor of this random variable is the dispersion, so dispersion what is meant is a spread over this mean, that is I know once I identify where it is tending towards the central I want to know, how it is distributed above that mean so the dispersion of a random variable corresponds to how closely the values of the variate are clustered or how widely it is spread around the central value.

In the following figure if you say that, this is the X_1 and X_2 have the same mean but, their dispersion about the mean is different, so this one obviously this one which is for this X_1, that is f X_1 this is the less dispersed compared to the which is that f x 2.Now, how to measure this one that is the, this measure of this dispersion.

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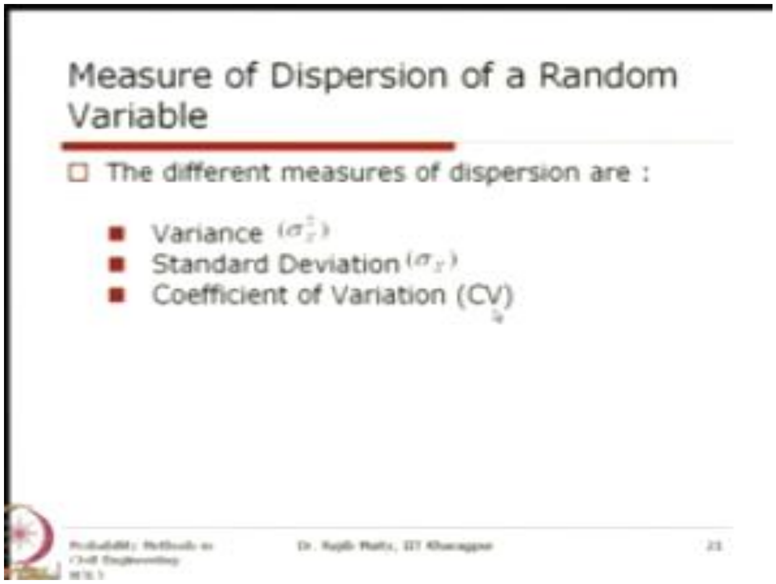

There are generally we use 3 different measure the, one is variance which is denoted as sigma x square, standard deviation sigma x, and coefficient of variation CV.

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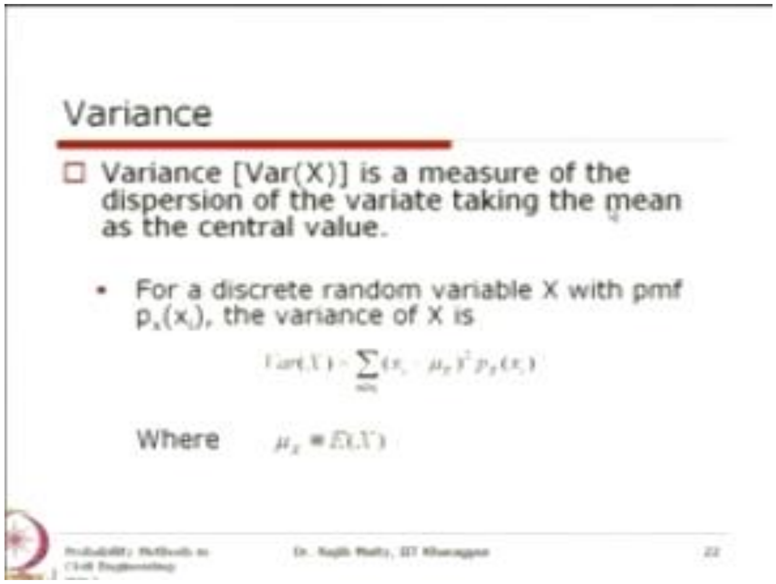

Now the variance, if we see that variance, which is generally also denoted that the var of x of this random variable X is a measure of the dispersion of the variate taking the mean as a central value, now when we are measuring, that spread around the mean out of this 3central out of the 3 measures of central tendency, if we pick up the mean and then if you calculate how it is spread around that mean, then that is known as the variance.

For a discrete random variable x with PMF, p_x X i the variance of the x is this variance is equals to summation of X i minus mu x, that square multiplied by p_x X i so this the measure of this variance, I will just explain how this things are meant with the context of this area with the context of this moment how it comes; now this mu x as I told mu x is nothing but, the expected value of that particular x.

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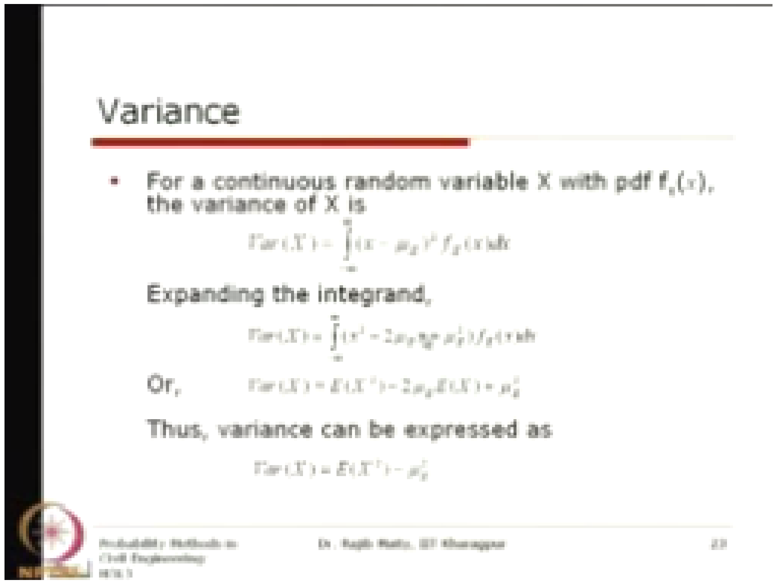

Now, now so let us first complete this one before I go for this pictorial representation, so this is for this is for the discrete random variable and this one is for the continuous random variable, now if the continuous random variable X having the pdf of this f x x than the variance of x is the we x minus µx which is the expected value that square multiplied by fx dx this is, this gives you that, that variance of X, now there are some if you just expand it a little, then we can come out to this one, just to expand this particular value and take this one as this that is, that it will come that x^2 multiplied by fx dx.

Now, you just recall that that expected value of the function, now here the function is x^2 , so function is x^2 multiplied by fx dx, which is nothing but, the expected value of x x^2, this is the function of the random variable, which is the x^2- 2 µ x; now this constant when we are taking this one, so this is already known this is constant so the 2 µ x can be taken out, 2 µ x multiplied by xfx dx, xf xd x is nothing but, that expected value ex plus this again the constant, so thus plus µ x^2.

Now after just doing this, so then again this ex is the µx, so minus 2µx square plus µx, so this is e x square minus µ x square so variance can also be represented like this.

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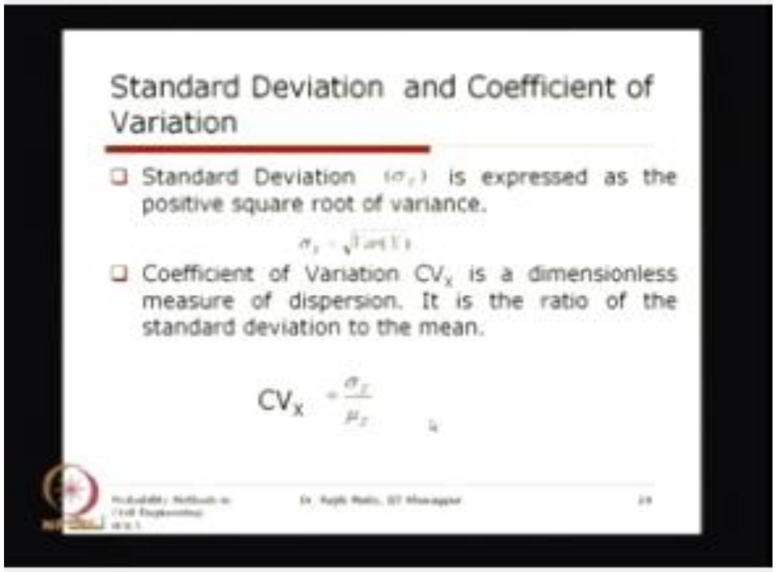

So, this standard deviation standard deviation is the another measure of this of this measure of dispersion where this standard deviation ∑ X is expressed as the positive square root of the variance f(x) which is the square root of this variance of X. And the coefficient of variation CV x is a dimensionless measure of dispersion it is a rescue of the standard deviation to the mean, so this CV x is equals to sigma by μ x. Now from this for all other higher thing that is the first the how it is becoming the higher thing that, I will just explain in the explain it here.

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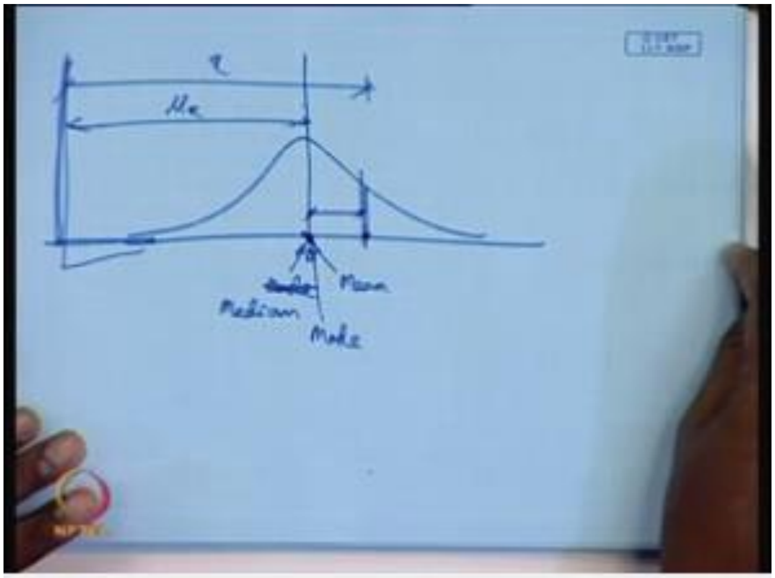

That is what you are doing when we are when we are measuring this that spread that is a that is that is the spread around this mean what we are trying to take is that, first what we are doing we're taking this is your mean and we are taking that X minus μ x. That is a particular value x if I take that minus μ x suppose that this is somewhere where is the where is the origin is here, so I am taking the value x here, this is your μx this is your μ x that is I know that is your mean, so I am taking the x minus μ x that is nothing but, from the distance from the mean to that particular value.

Now this one I am just multiplying it with that particular density at that point that is the dx and multiplying that one with the distance to this one, this is a kind of the second moment from starting from this mean of the particular distribution.

So, we will see in a in a moment, how it can be represent, how it is represented as the second moment about this mean and we can go for this, so the mean is nothing but, the first moment with respect to the origin and variance is the second moment with respect to the mean; now like that I can go to the third moment with respect to mean fourth moment with respect to the mean fifth, sixth in this way we can go and each and every moment will give some property of the distribution.

For example, the second moment that is this one with respect to this mean is giving you the measure of the dispersion, how it is dispersed around the around the mean. So we will come that 1 how it is represented in case of the μ particularly.

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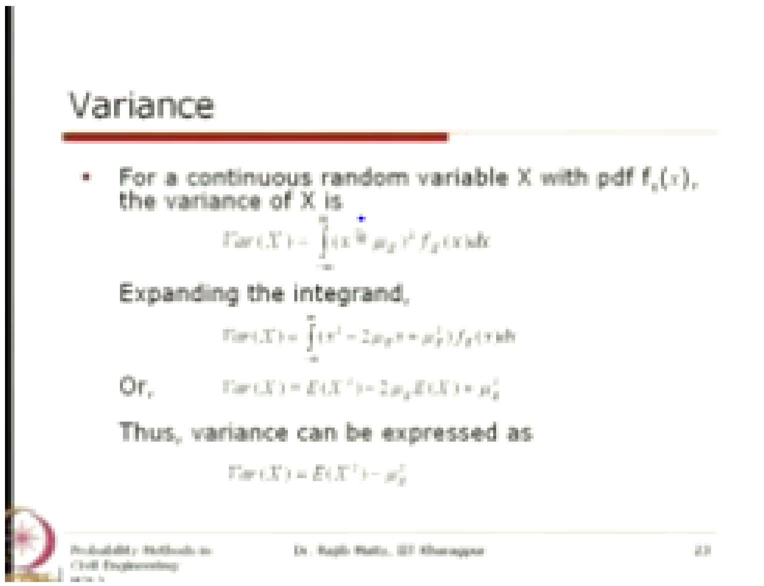

So before that we will see, so here that that this distance square multiplied by this density that is f(x) multiplied by this dx the small inferential small area.

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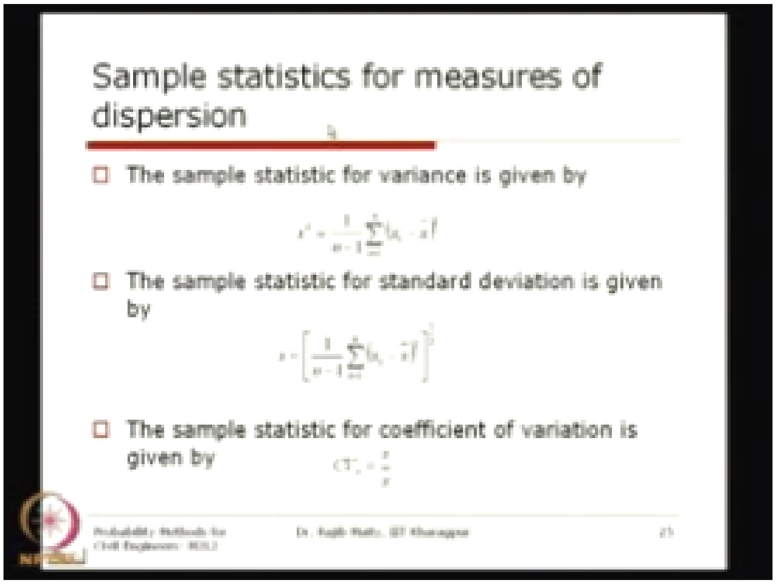

Now, if I want to know the sample statistics of this measure of dispersion the sample statistics of the variance is given by so this sample statistics means again for the random variables some observations are taken xi for n if the number of samples are n, then xi minus x bar which is mean an of that particular ever that mean of that sample mean that minus this one square summing up them and divided by n minus 1 this n minus 1 minus 1 is due to the to make this estimate as un biased.

The sample statistics for the standard deviations we told that is the positive square root of this variance, so this full quantity power half that is square root of the this one which gives the standard deviation, for this standard deviation we go for this first root but, for the higher moments when you go for this measure of that skewers and all we generally do not take that the third root of that one to this one, this is this is here for the for the first one that is for the measure of variance we take the root just to see that if you see this expression, then you will see that unit of the standard deviation is equal to the unit of the random variable itself.

So, this is at his is sometime has greater help so that is why we take that 1 square root and we declared that 1 as a standard deviation, which we do not do for the higher order moments and he sample estimate of this coefficient of variation is the sample estimate of the sand this x bar is that sample mean of this to get that coefficient of variation.

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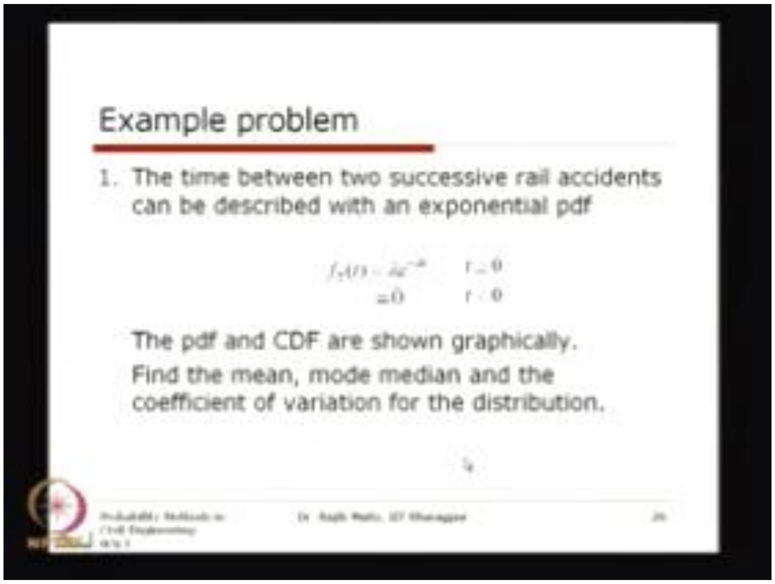

So, one example problem we will take for this thing that is whatever we got the mean and this standard deviation and variance, so the time between two the same example that we discussing this in the context of this CDF the time between two successive rail accidents can be described with an exponential pdf that is fTt equals to lambda e power minus lambda t greater than or equal to 0 and 0 for this other areas that is less than 0.

The PDF and CDF are we have seen it earlier that is how it looks like pdf and CDF in such cases, so we have to find out the mean, mode, median, coefficient of variations, so whatever the statistics we have seen just so far we will just see, how it is for this exponential distribution, how it can be calculated for this kind of distribution.

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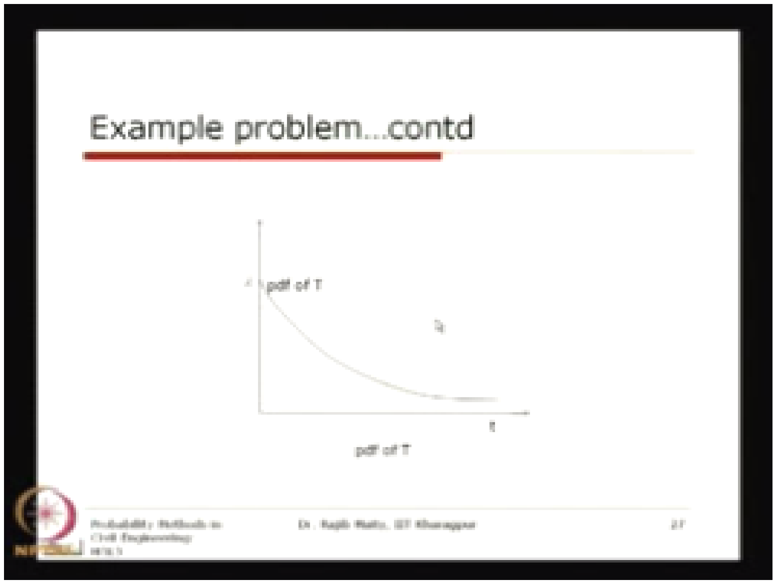

So, this is the pdf of this T which we have also seen that, this value is lambda and it is gradually coming down and getting asymptotically to this 0 at ∞, so this is the pdf for the random variable T.

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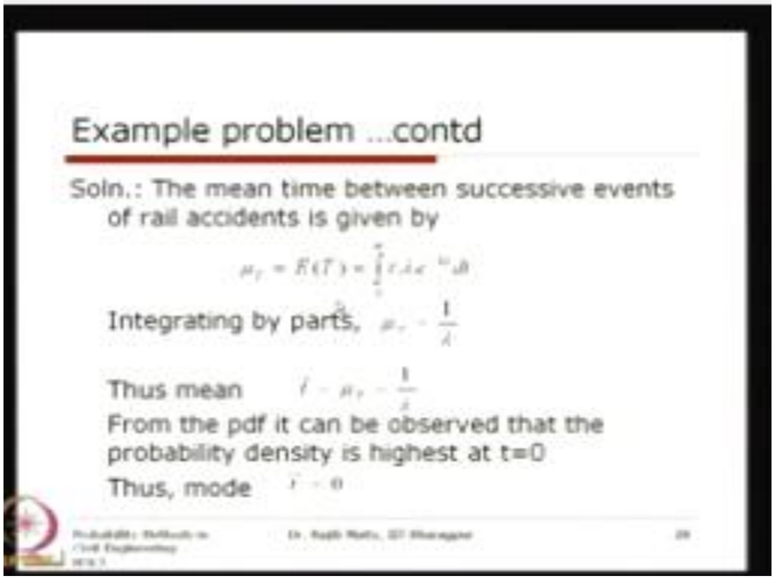

Now, the mean of this mean time between the successive events of this rail accident, if this is the distribution then, we know that µ t which is the expected value of this tis the over the entire range of this pdf that multiplied by lambda e power minus lambda d t, if we just do this small this integration by parts, then we will see that this µ Tis equals to1 by lambda. Now, here one thing you can see that so this is that this is that lambda that, we are getting here and even we have seen that the sample estimates.

So, if you take some sample of this particular event then if we calculate their mean the sample mean and if you take the inverse of that sample mean then we will get the measure of this lambda, so this is how means we method of this is called the method of moment to get that estimate of this parameters, just few slide before we are discussed, we are taking that how to estimate the parameters it is one of the method, that even how to get that one but, that is not the context what we are discussing here, now we are just getting that for this distribution what is the mean.

So the mean are the expected value of the random variable is 1 by lambda. So, this mean t bar μ t is 1 by lambda, from the pdf it can be observed that the probability density is highest at t equals to 0.

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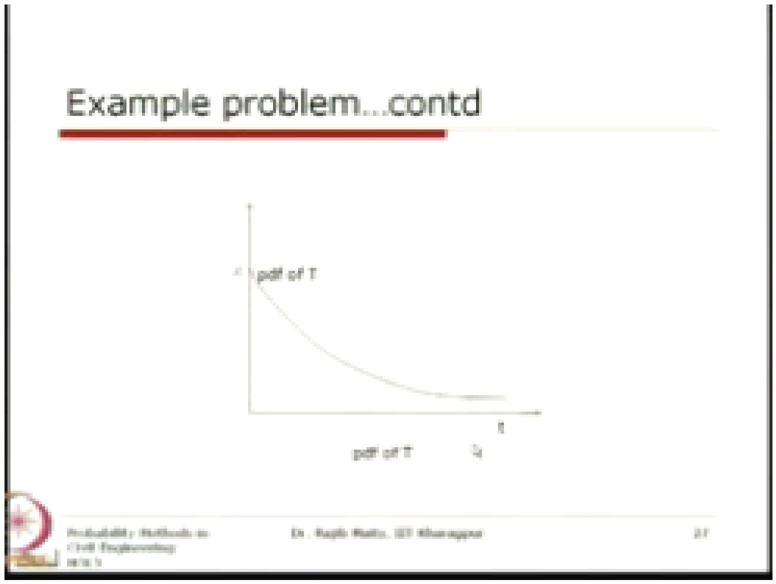

So the density values if we see if we just look this density value so we see that at t equals to 0 itself thus value the magnitude of this d is maximum which is lambda.

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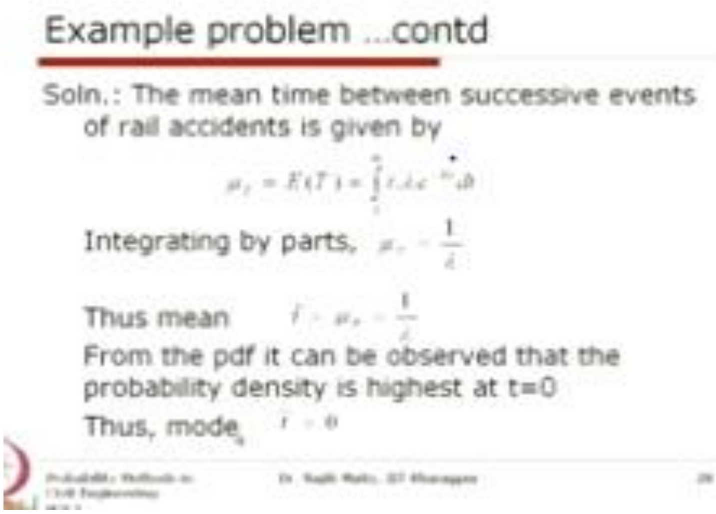

So the from the definition of the mode we can say that a mode that is t tilde is equals to 0, so modest at 0, mean is at 1 by lambda. Now we will see where the median is, so median means.

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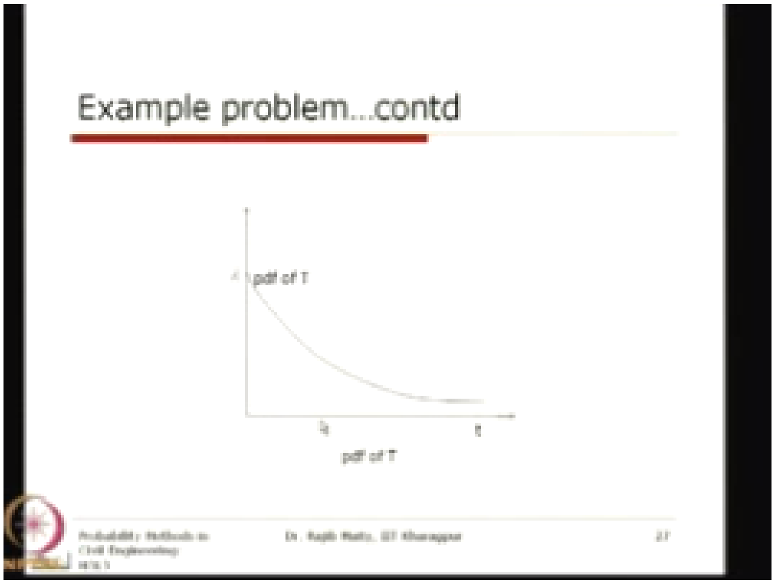

We have to calculate that, that we have to integrate this one and get some value where it is covering the 50 percent of the total area, total area is 1, we know, so 0.5 we have to integrate it from the 0 to some value x, where it will be equal to 0.5 to get the median.

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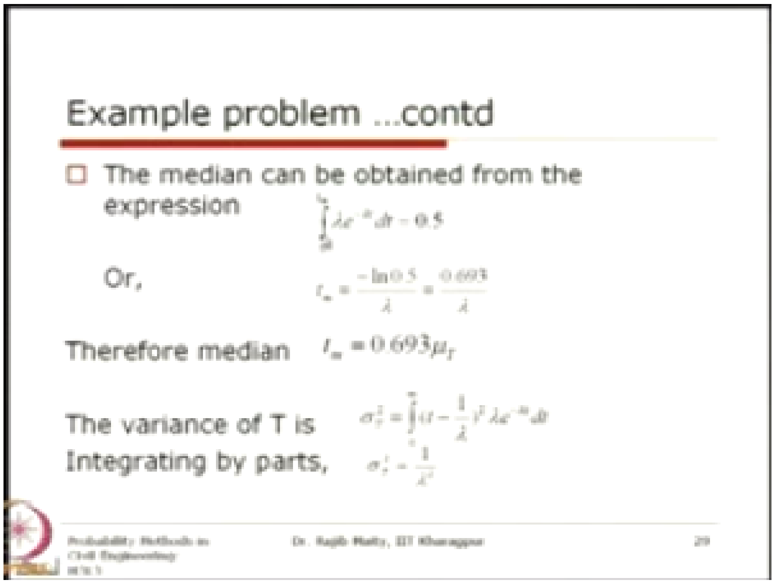

So exactly the same thing is done, so for the 0 to t m that is the value of this median of this value should be equals to 0.5. So if we do this integration we will get that the mean becomes the tm equals to this and tm equals to 0.693 μt, that is 1/λ the variance of this t, now if you want to get that variance of that one so again we will use that same expression that is random variable t-1/λ, 1/λ is nothing but, we know that this mean that is expected value of this t. So if we take the $t-1/\lambda^2$ then multiplied by this probability density dt then we do this integration by parts and we see that this sigma t square which is variance is equals to $1/\lambda^2$.

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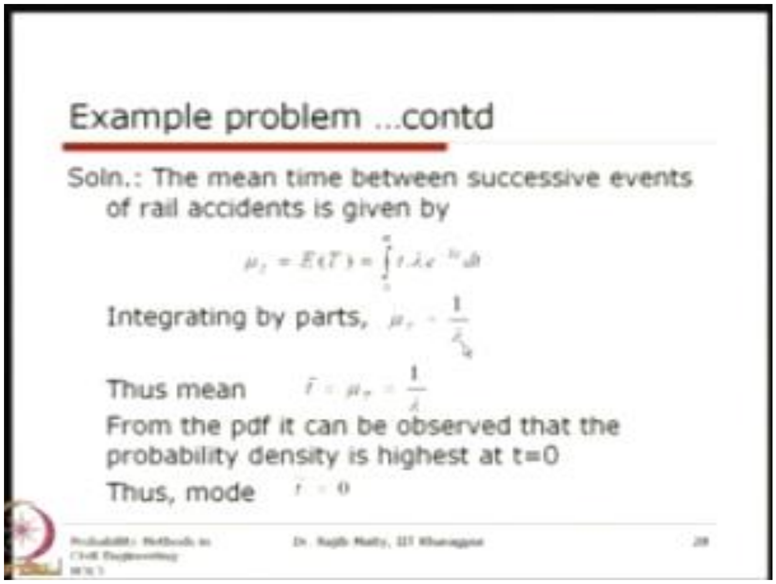

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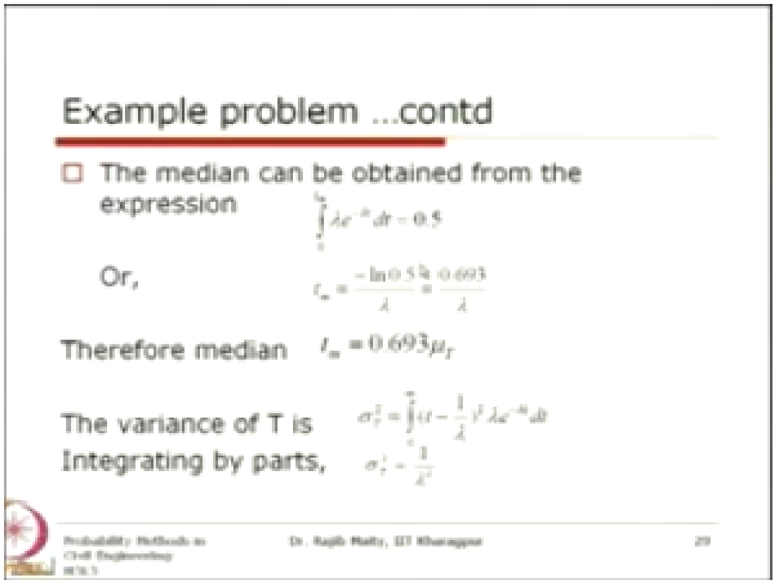

So we have seen that mean is $1/\lambda$ for this exponential distribution and the variance is $1/\lambda^2$.

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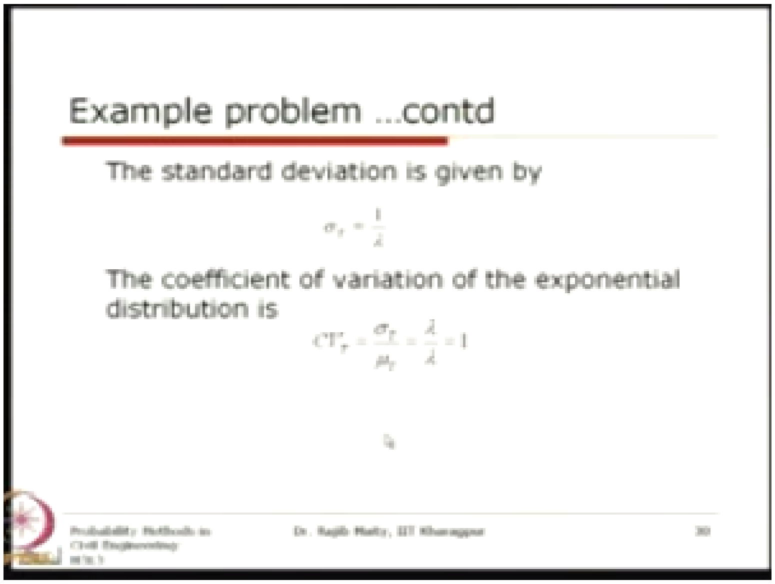

Now the standard deviation again we know the standard deviation should be positive square root of the variance, so the standard deviation is again equals to 1/λ, so standard deviation the magnitude of the same the mean and the standard deviation which is again that 1/λ. So the coefficient of variation if you want to calculate coefficient of variation of the exponential distribution is equals to sigma T by μT, we know that so this is 1/λ/1/λ = 1. So coefficient of variation is equals to 1.

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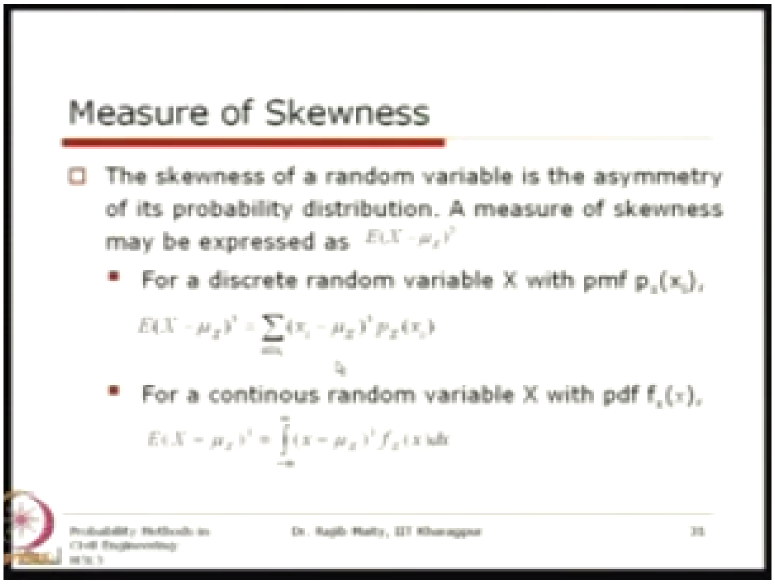

So there, are two other measure of dispersion and one is that measure of skewness.

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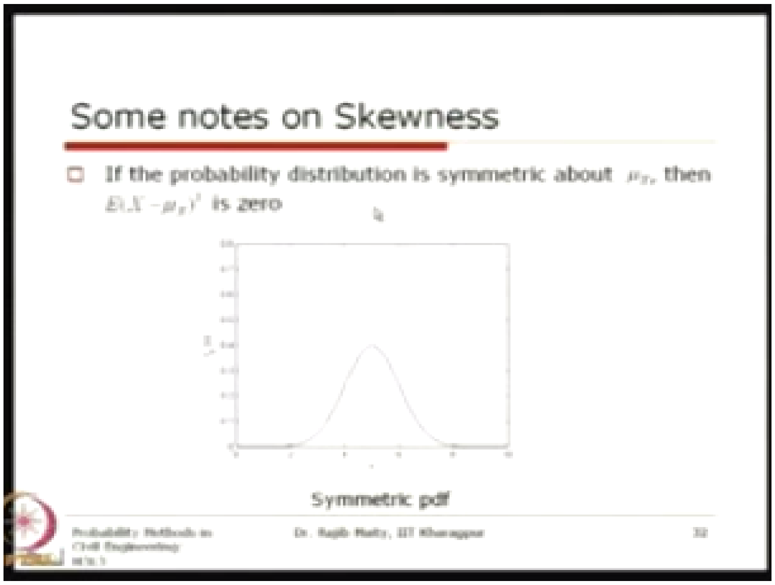

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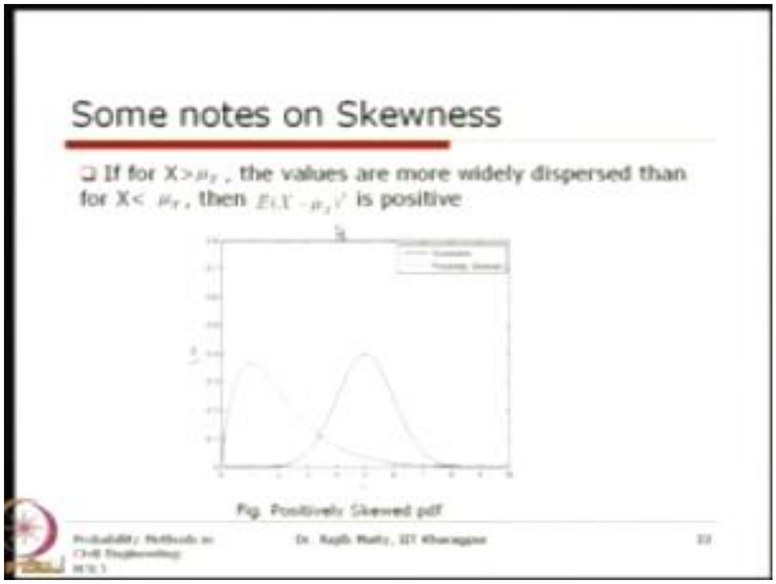

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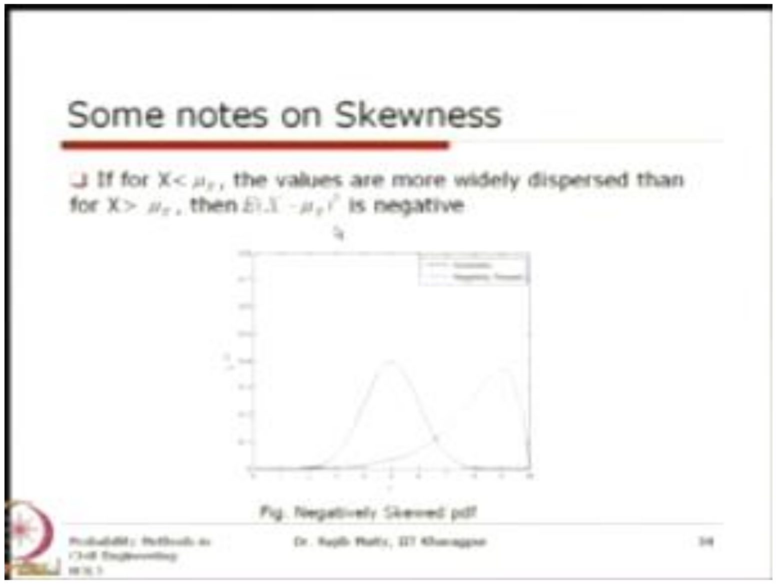

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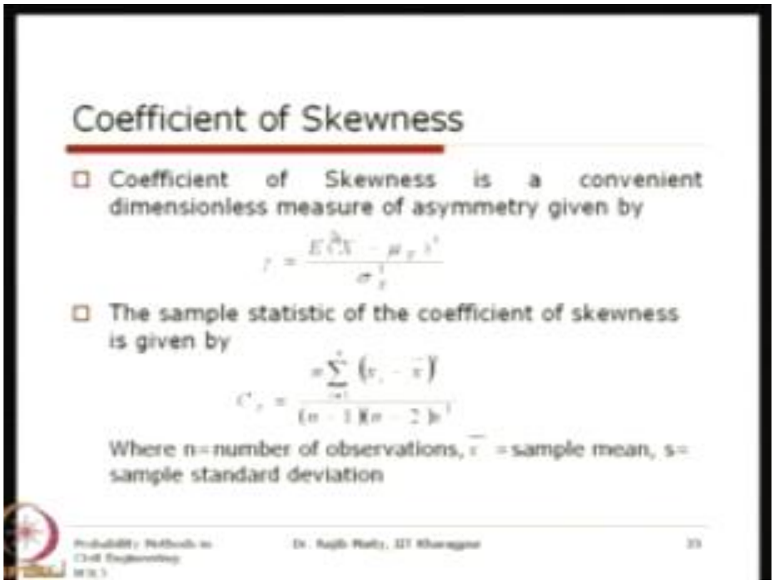

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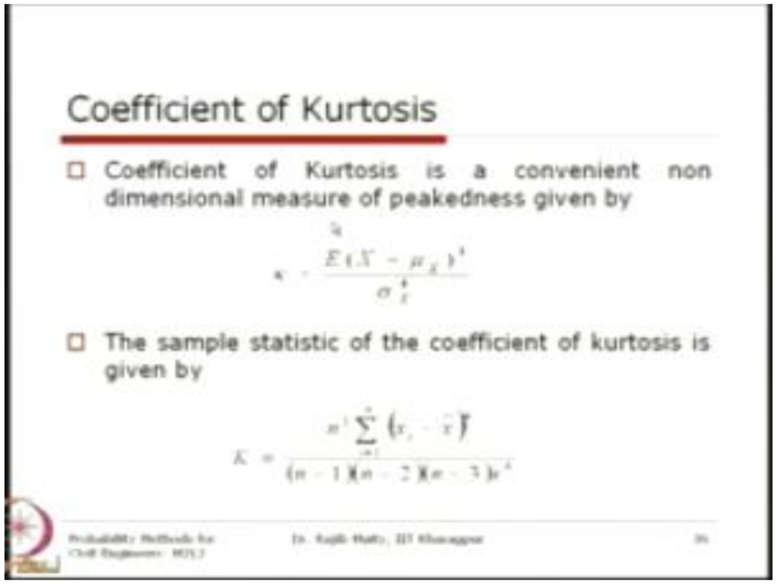

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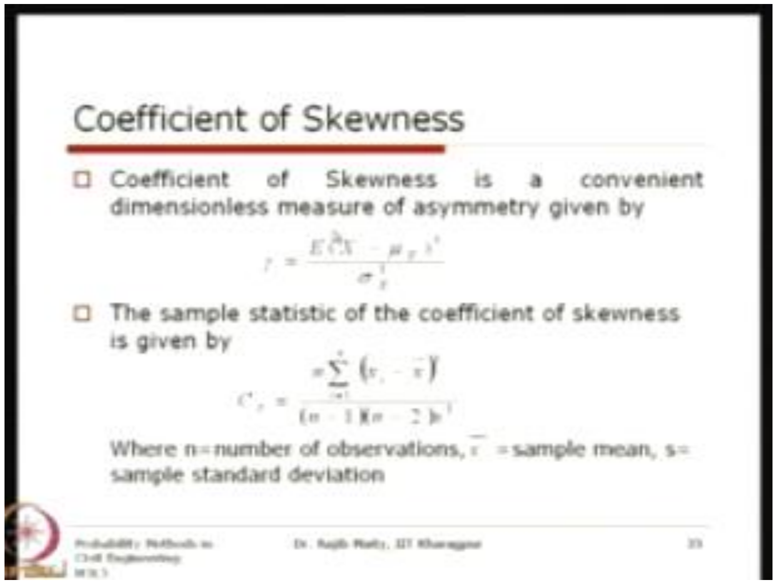

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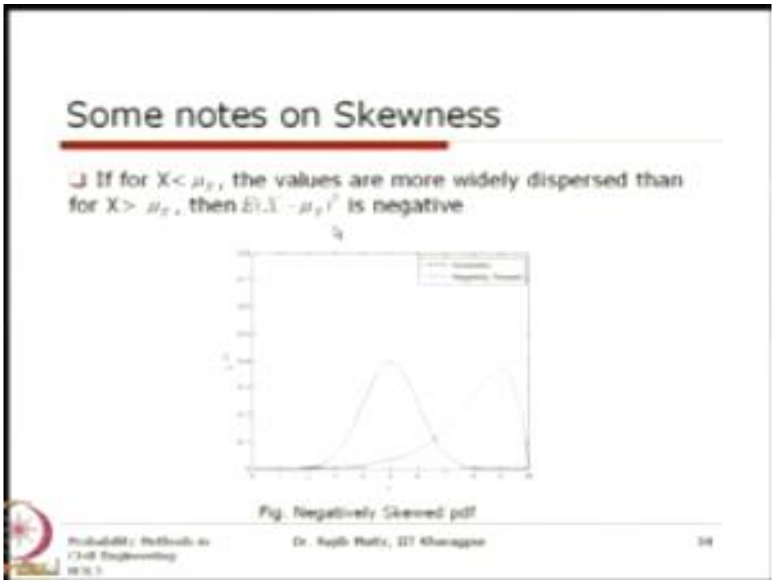

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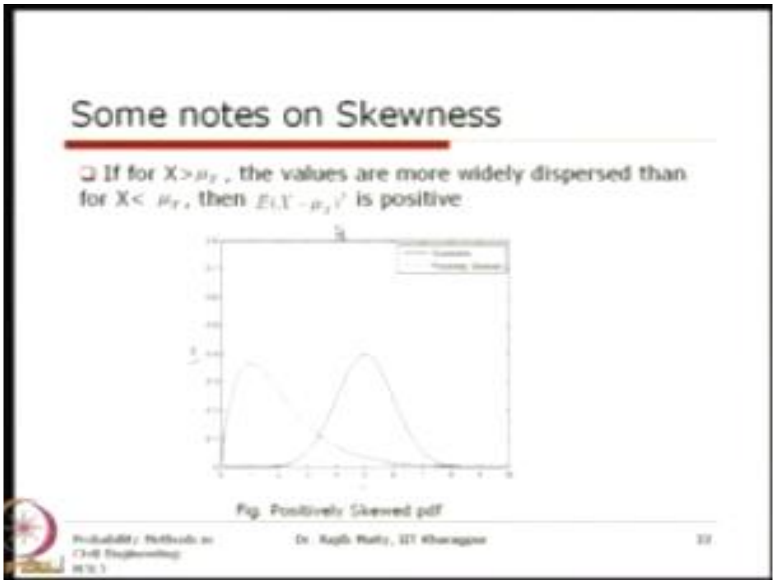

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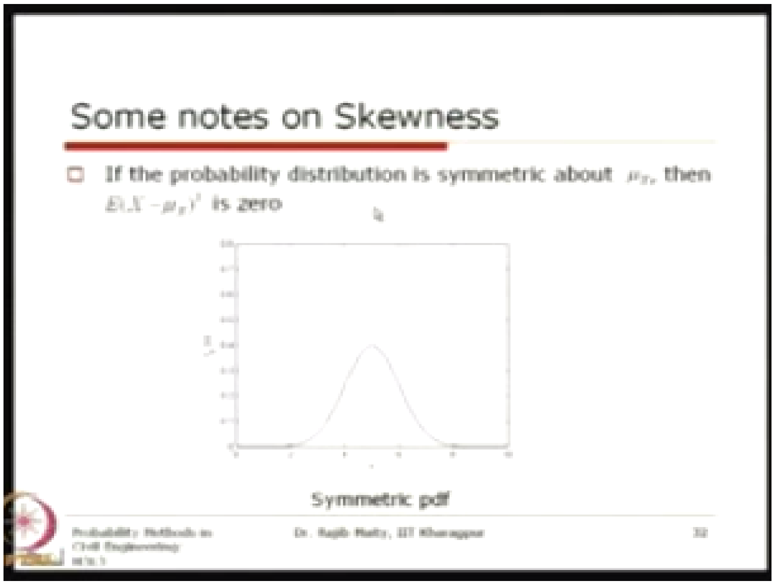

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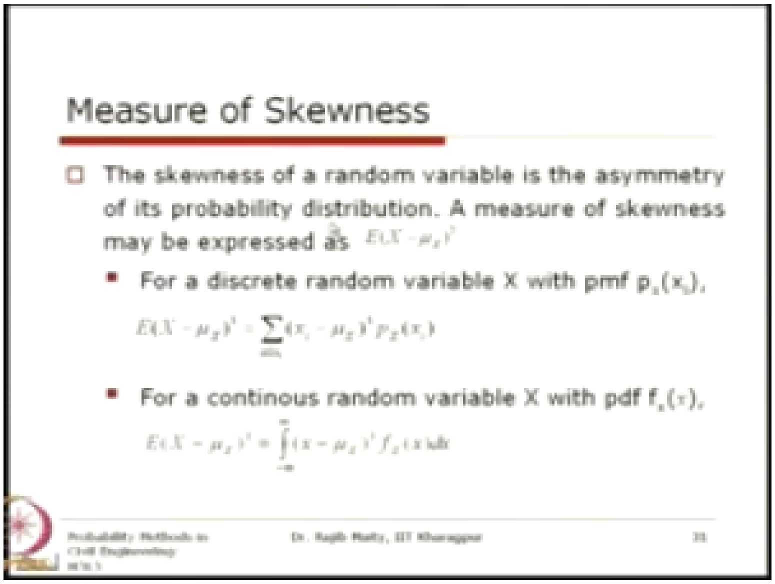

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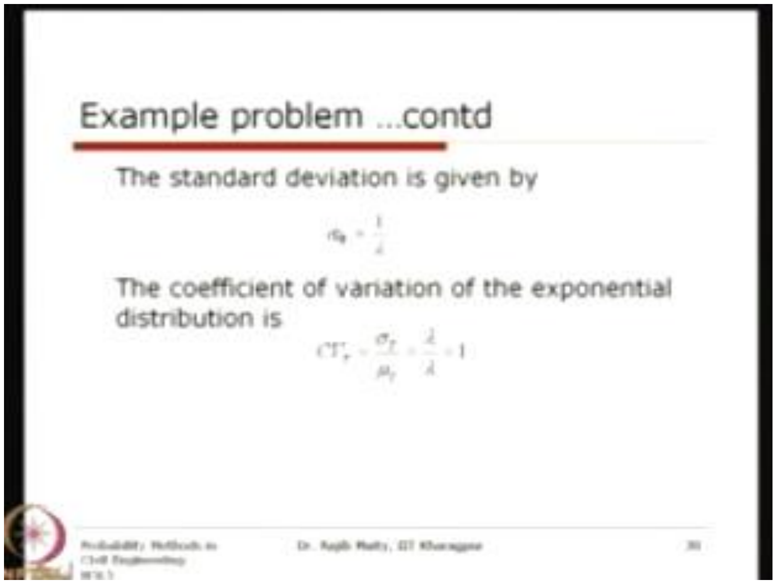

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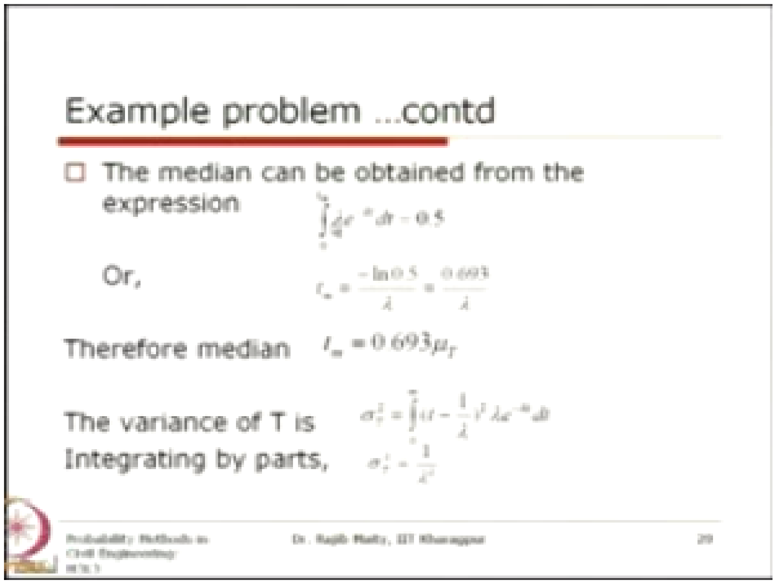

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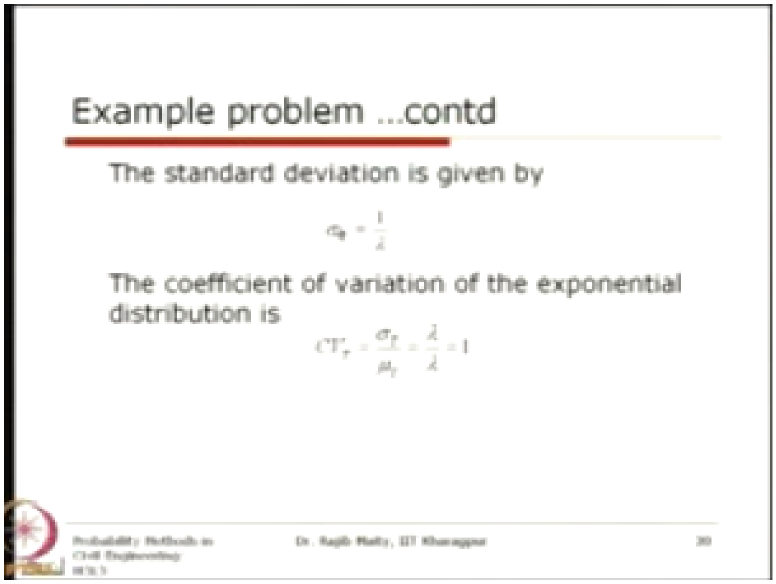

And other one is measure of measure of peakedness and we will see this one later. These are the these are the higher moments what we have seen today is that is that how the first moment about that, about the origin and in the next class as well what we will se.

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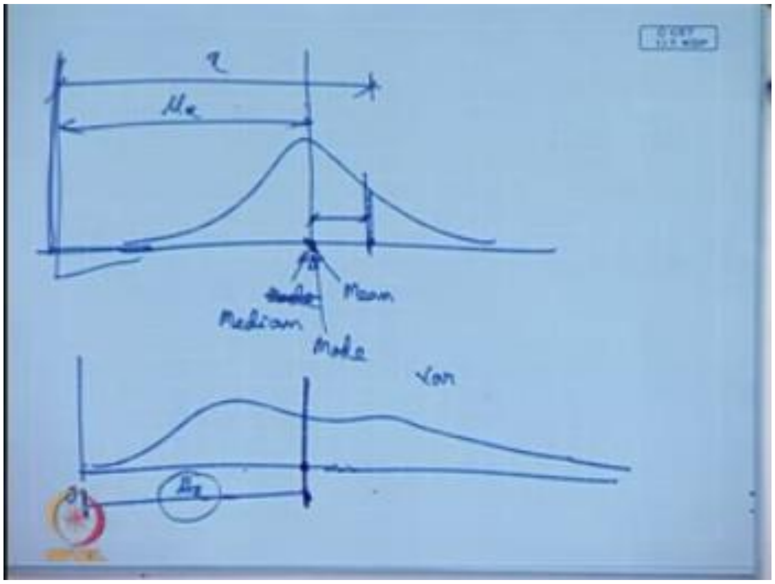

That we will see that, how it is becoming that mean from this, so one particular distribution if you take and how this moments are actually coming from this one with respect to the origin. So this is basically your μx, how it is means the moment with respect to the origin, and the once we get the moment with respect to the origin that means we are getting the value of mean and all other higher moments we calculate with respect to that mean.

The first that we calculate is that with respect to the mean that is a second moment with respect to the mean what we got that is called the variance. One open question that I can put before I conclude today's class is that, as I told that second moment with respect to the mean is variance, what is the first moment with respect to the mean.

So the first moment with respect to the mean that means I have to take that particular value minus that mean minus that mean multiplied that density and that we will get. So interestingly or this is mathematically very easy to state that the first moment with respect to the mean is always 0. Because you are basically whatever the positive side of that mean and whatever the negative side of that mean both are cancelled out to results in that the first moment with respect to the mean is equals to 0. So we start with this second moment, the second moment is the measure of variance third moment skewness, fourth moment peakedness like that fifth moment, sixth moment are there.

Basically up to the fourth moment is sufficient to describe a particular random variable. Today's class we discuss up to the variance. Next class we will start with the description of the skewness and the kurtosis. And we will also see in more details how this can be related to as moment. So we will that, will be the starting off again for the first moment and that moment generating function for a random variable. So thank you for today's class.

Probability Methods in Civil Engineering

End of Lecture 08

Next: "Further Descriptors of Random Variables" In Lec 09

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