INDIAN INSTITUTE
OF
TECHNOLOGY
KHARAGPUR

NPTEL
National Programme
on
Technology Enhanced Learning

Probability Methods in Civil Engineering

Prof. Rajib Maity

Department of Civil Engineering
IIT Kharagpur

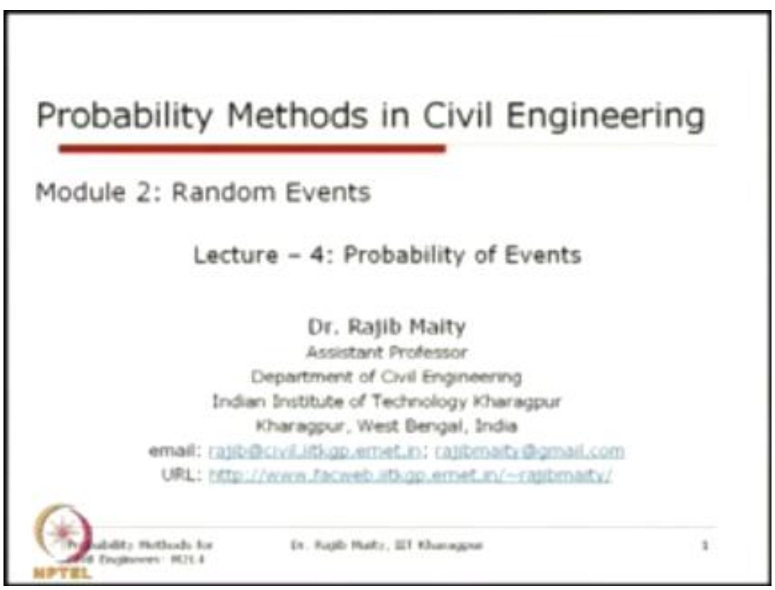
Lecture – 05

Topic

Probability of Events

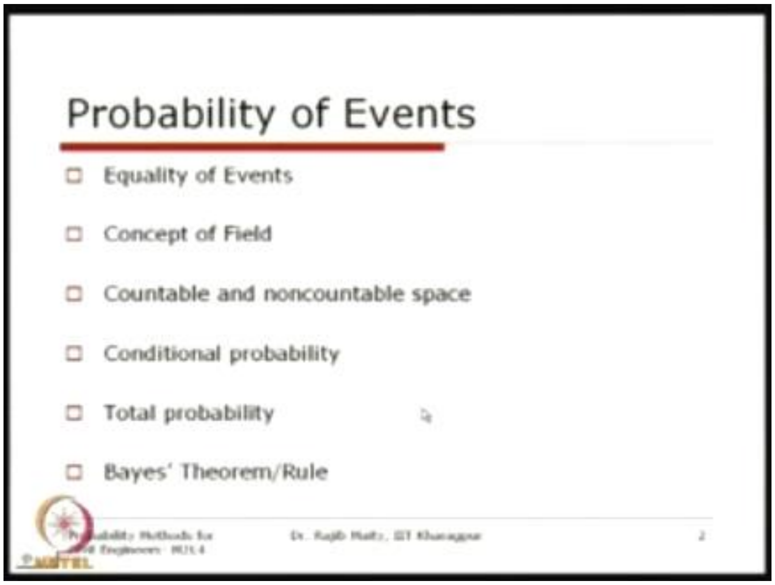
Hello and welcome to the lecture in the course probability Methods in Civil Engineering.

(Refer Slide Time: 00:23)



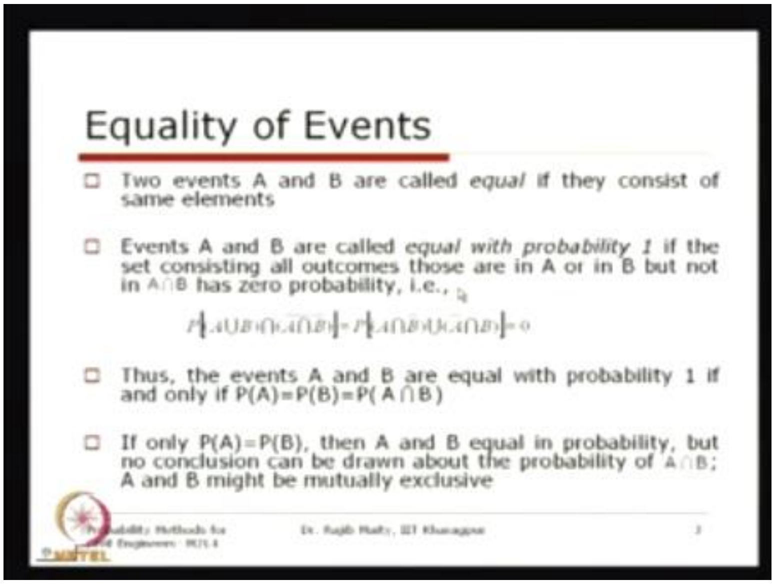
Today, we will cover the Probability of Events which is very useful for the different applications in the problems related to Civil Engineering.

(Refer Slide Time: 00:36)



In this lecture, we will first touch a few basic concepts that is equality of events and concept of field, which are useful particularly when we deal with the most of the problems in the Civil Engineering. Then we will touch the countable and non-countable space followed by we will go to the conditional probability and with the help of this we will try to explain the total probability and related theorem of total probability and after that we will cover this Bayes' theorem and rule. And finally, we will see some of the application problems for applying to this particular concept, and we will go one after another starting from this equality of events.

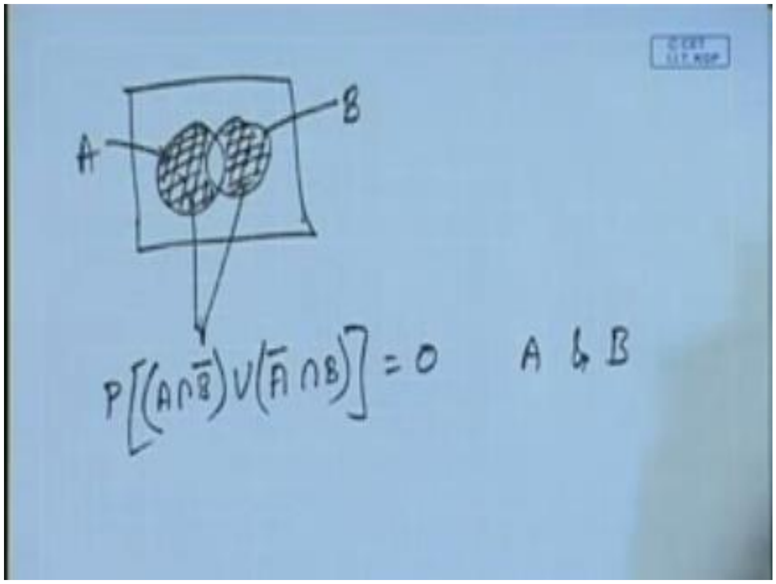
(Refer Slide Time: 01:27)



This equality of events says that the two events A and B are called equal if they consist of same elements the event A and B are called equal with probability 1 this equal with probability 1 is important because then we can say that all these elements of both the events are same and that is if the set consisting all outcome those are in A or in B but not in A ∩ B has 0 probability that is, a probability of these which is equal to this one, as I discuss in the previous class this probability should be equal to 0.

This second part point if I want to explain in graphically, then it looks like this suppose that this is.

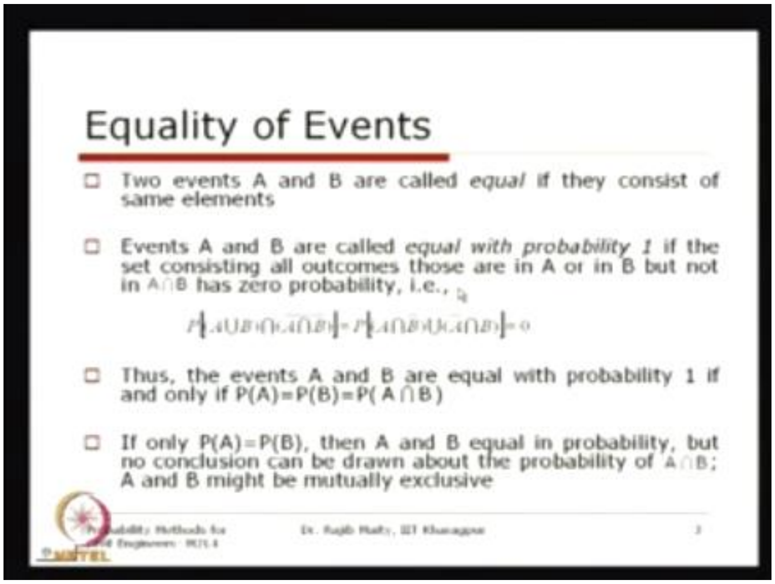
(Refer Slide Time: 02:21)



One sample space and in which there are two events, one is A this one is your A and another one is your B. Now what it says that if these A and B are equal with probability 1, if the sets if the set consist of all the outcomes those are in A, but those are in A as well as in B, but not in the A ∩ B. So, what we mean is that these two areas one is this in A or in B, but not in their intersection. So, the probability of these two events probability of this area should be equal to 0 so, this is exactly what is what it is meant. So, these two area is nothing but you are A ∩ B prime, which is union with A prime, A complementary with intersection B.

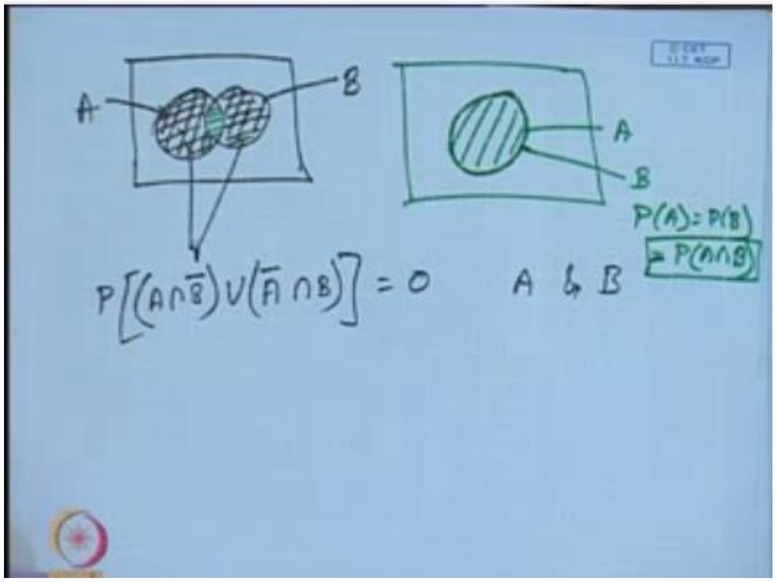
So, this is the area and if we say that there is no such element in this area then that means what we are trying to say is that this probability of this event if it is 0 then we can say that this A and this B these two event are equal.

(Refer Slide Time: 03:56)



So, coming back to this point again that this event A and B are equal with probability 1, if the set consisting all outcomes those are in A or in B but not in A ∩ B, has zero probability that is probability of that this is a thing I explained which is obviously equal to this one also so these two events are referring to the same event whose probability if this probability is 0 then we can say that this event A and B are equal. Thus the event A and B are equal with probability 1, if and only if, the P(A) is equals to P(B) is equals to P(A ∩ B). This is important once again if we just refer to that particular Venn diagram here.

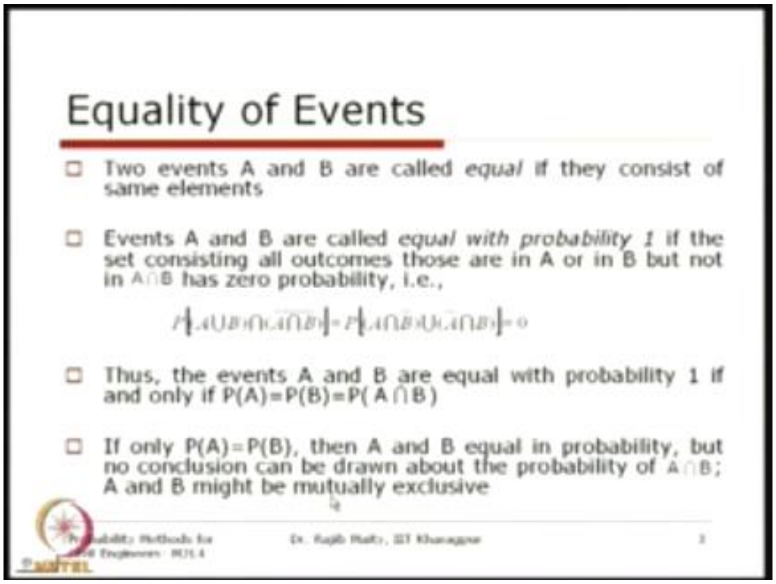
(Refer Slide Time: 04:50)



That is if this P(A) and this P(B) is equal to the probability of their intersection which is nothing but basically, we are just pulling these two events to be on the same to be on the same event that is this as well as this means this is your A as well as A, this itself is your B so then already you can say that P(A) is equals to P(B) equals to P(A ∩ B). This All these three elements of this equation is important because if we do not consider these if we say that P(A)equals to P(B) that does not mean that these two events are same.

So, this must be there for example, that if we take the if we take the example of throwing of one dice and if we say that the probability of getting 1 or probability of getting 2, both are same or if I say the probability of getting an even outcome and probability of getting an odd outcome so, one is event A another one is event B the probability of these two events are same but we cannot say that these two events are same. So, this one this intersection this is a last part this P(A ∩ B) is also is important to declare that these event A and B are equal thus, once again.

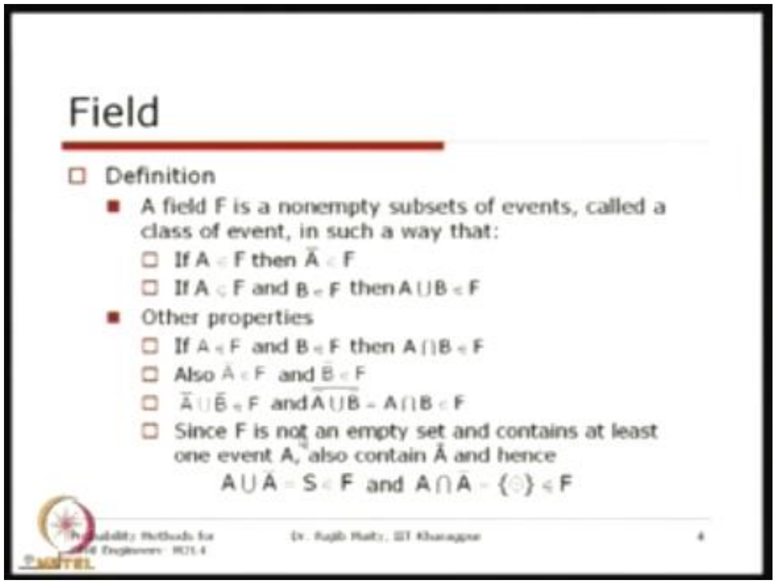
(Refer Slide Time: 06:36)



If we thus, it is stated that the event A and B are equal with probability 1, if and only if their individual probability is equal to their intersection. If only, that is what just now, we discussed if only we say that P(A) is equals to B, then the, then A and B are equal in probability but no conclusion can be drawn about the P(A ∩ B) the example that we are telling, that getting a dice throwing a dice and getting the even number and odd number. So, these two events are equal in probability, but there, the probability of their intersection is 0.

So, that is why the statement states clearly that if only P (A) = P(B) then A and B equa lin probability but no conclusion can be drawn about their P(A ∩ B) A and B might be mutually exclusive.

(Refer Slide Time: 07:50)



Then a concept is important which is known as field the definition of fields says that a field F is a non-empty set of events, non-empty subset of events, called a class of event. Now, a class of event is again another definition where this class means that we are considering instead of considering each and every event of a sample space we are considering only particular subset of the whole sample space and that particular subset is generally denoted by the class of event. So, this field what we are now trying to understand this field F is a non-empty subset of the event this is called a class of event in such a way this F is defined in such a way that if any event A if that any event A belongs to F, then its complementary is also belongs to F.

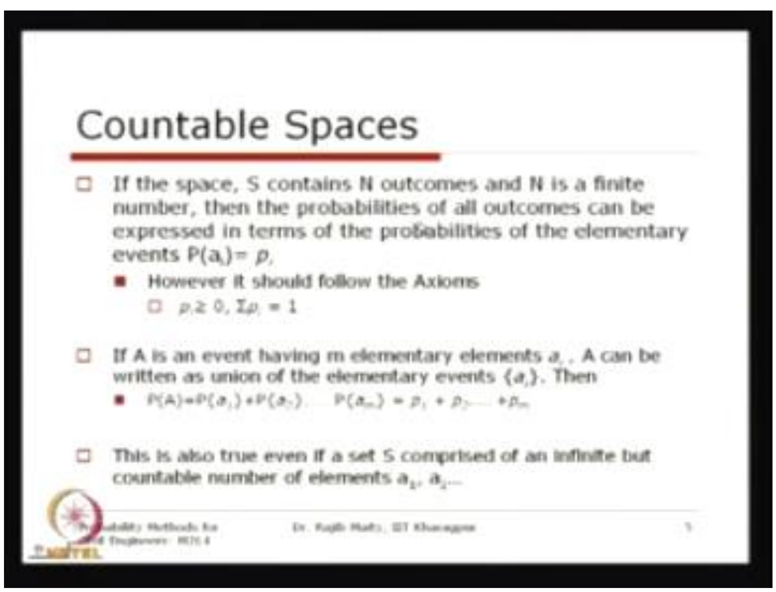
If A one event A belongs to F and another event B belongs to F, then their union is also belongs to F so these are the two their minimum criteria to define one field which is a non-empty subset so based on these two there are other properties as well which are which can also be drawn. The other properties of this field that states that if one event belongs to that field and another event belongs to that field then their intersection also will be in that field. Also if the complementary of one event belongs to F and the complementary of another event belongs to F then we can say the complementary the union of their complementary that is complementary A union complementary B also will belongs to F.

And the complementary of the union of individual complementary that is A complementary union B complementary full thing their complementary which is nothing but equal to A ∩ B, is also belongs to F.

Last one since A since F is an nonempty sorry for this mistake, F is a nonempty set and contains at least one event A also so it also contains that A complementary that is it contains at least one event which is denoted as A here so it also contains that A complementary thus the A union A complementary which is nothing but the full sample space.

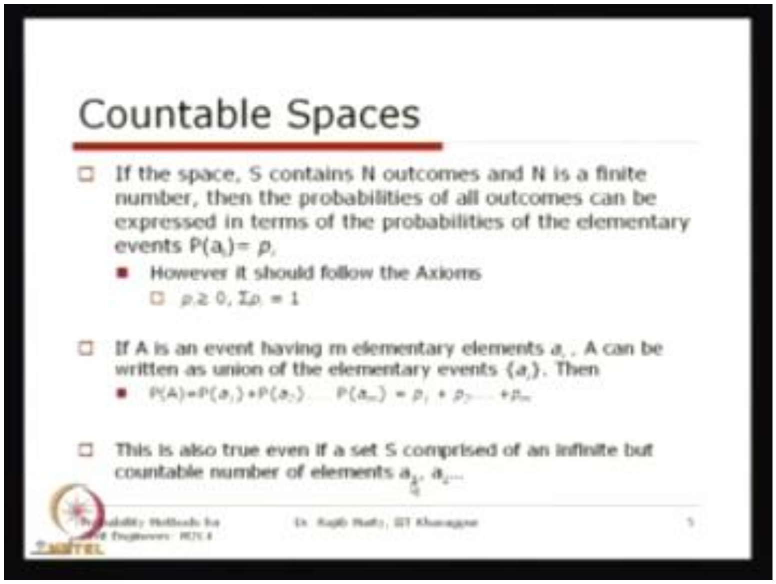
So that is also belongs to that field and A intersection A complementary which is nothing but a non-linear event so that is also that is also belongs to that F. So this so this S is nothing but almost a certain event and this is almost this is the impossible event that is a null set these two are also these two extremes are also belongs to the field.

(Refer Slide Time: 10:57)



Next is countable space sometimes in the space S contains N outcomes and N is a finite number then the probabilities of all outcomes can be expressed in terms of the probabilities of the elementary event, probability of Ai = Pi. So if there are N finite N countable or countable events are there in the space then their probability can be defined by the individual events.

(Refer Slide Time: 11:39)

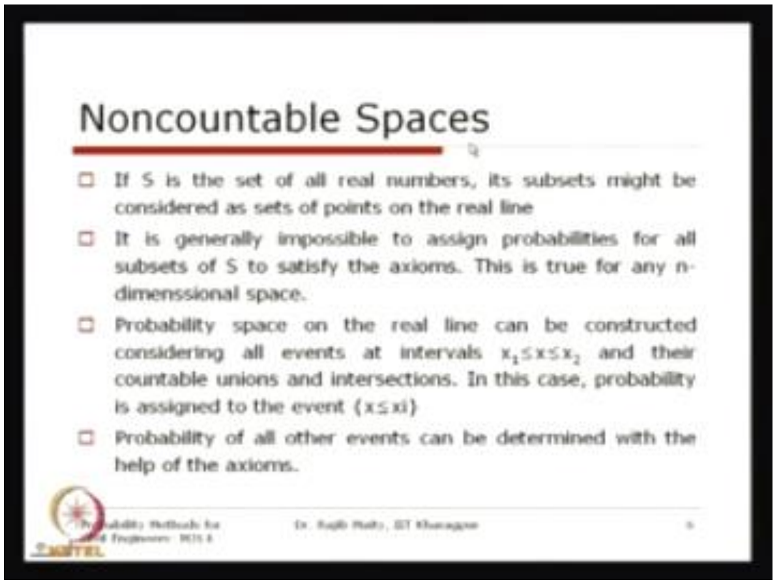


For example here it is shown that P(Ai) = Pi however it should follow the axioms that this each the probability of each the probability of each and every events should be greater than equal to zero and their summation should be1 which is directly following from the from the axioms of the probability that we will discuss in previous classes. If A is an event having m elementary elements Ai.

A can be written as the union of the elementary events Ai, then the probability of A is nothing but the summation of their individual probabilities which is nothing but p1+ p2+…..pm. So there are m elementary events are there if we just add up we will get the probability of that of that event A. This is also true even if the set S comprised of an infinite but countable number of elements A1, A2 in this and so on.

So even though I am talking about this countable space if it is true when the S comprised of an infinite but countable number of elements in such a way, that A1, A2 and in this way, so on. So a contrast to these countable space which is which is more important.

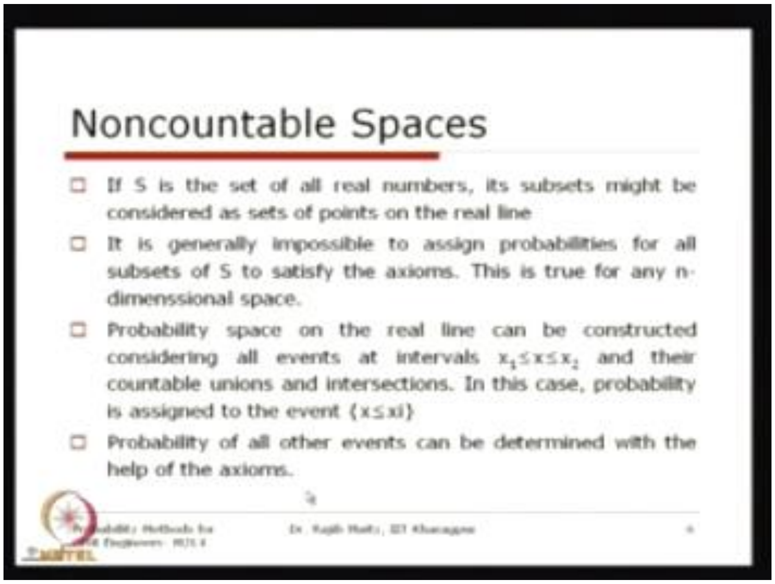
(Refer Slide Time: 13:09)



In particularly for these applications in this Civil Engineering where most of the cases we will see which is the non-countable space. In many cases we have seen that the total sample space cannot be cannot be defined just in terms of few elementary events rather it should be expressed in terms of a non-countable set. For example, if we take the example of the real line then whatever the number that lies on this real line it consist of this full sample space.

Now for such cases how to define the probability that is now our now we are going to understand. So it is not only for the real line which is a one dimensional picture it can be that the concept can be extended to any n dimensional space. So, it can be the two dimensional where it refers to the areas or three dimensional which is referring to the volume and in this way it can be explained that the concept can be extended to any n dimensional space. Now here we will discuss about the one dimensional that is the real line.

(Refer Slide Time: 14:20)

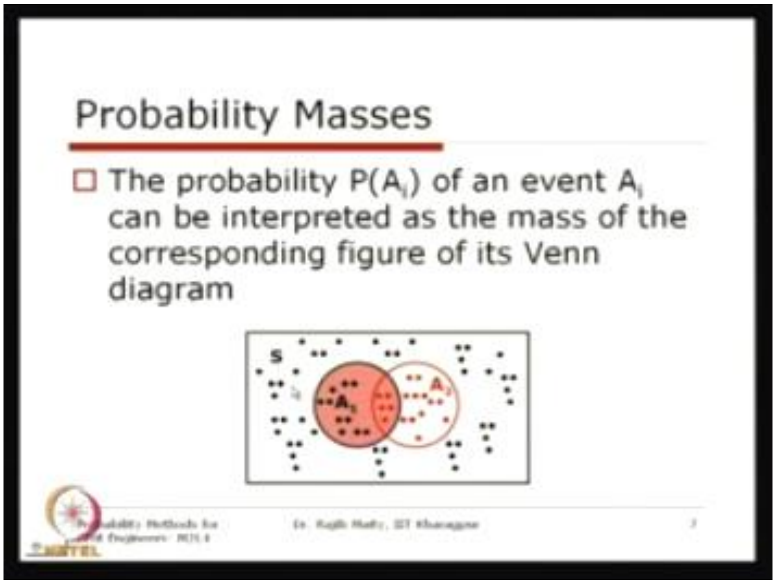


So if the if S is set of all real numbers its subset might be considered as a set of points on the real line. This is generally impossible to assign the probabilities of probabilities for all subsets of the S to satisfy the axioms it is true for any n dimensional space just now what I have discussed. So the probability space on the real line can be constructed considering all the events at any intervals.

Where x lies between x1 and x 2 and x 1 and x 2 can be of any real number on the real line and their countable unions and intersection. So in this case particularly so what I mean is a real line that one dimensional case the probability is assigned to the event which is x less than equal to x i, so xi is any number on this real line. So if we just define the probability that probability of x less than equals to x i.

Then this is sufficient to explain the entire set of this probability for the entire sample space the probability of any other event all other events can be determined with the help of the probability axioms.

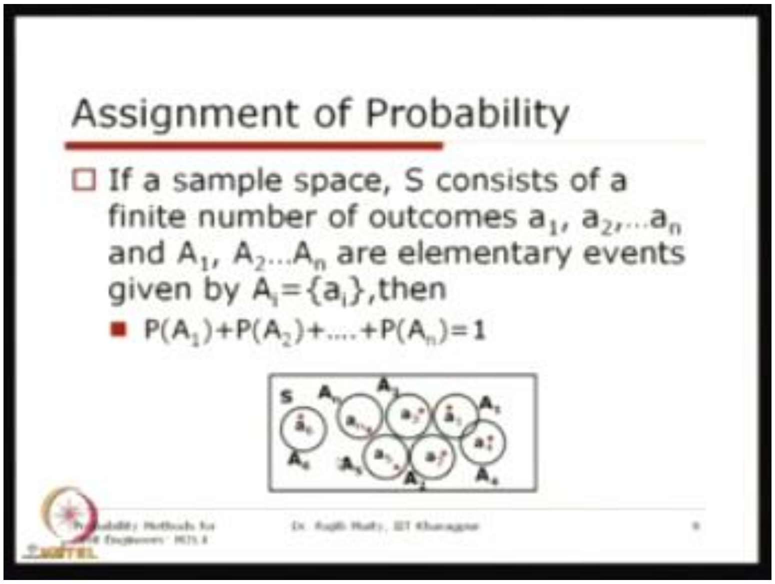
(Refer Slide Time: 15:45)



Now probability masses, now this the probability P(Ai) of any event Ai of an event Ai can be interpreted as a mass of the as the mass of the corresponding figure of its Venn diagram here whatever you can see here that this is a Venn diagram that is shown. Now if these dots are the outcome of the experiment and all these dots are consist of the sample space, now the probability can be treated as a concentrated mass to these points to this outcome.

Now here if I just extend this one to this to this continuous field suppose that instead of being an elementary event if the set consist of this consists of an continuous event then what will happen we will just discuss in a in a minute.

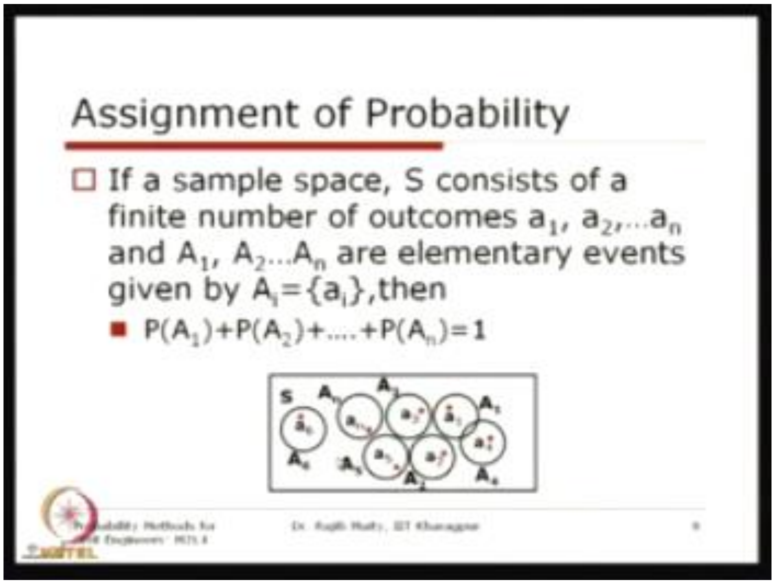
(Refer Slide Time: 16:41)



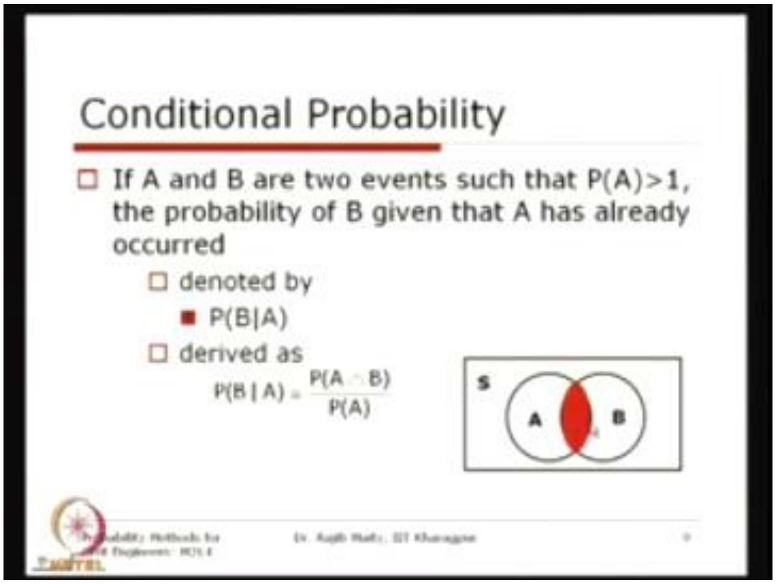
The second thing is that for these cases where it is an elementary A events, the if a sample space S consist of this finite number of outcomes A1, A2 up to An and this A1, A 2… A n are the elementary events then by in that this Ai corresponds to the event A the Ai then this probability of all these events should equal to the 1 which directly follows from the axiom of this probability.

Now what just now I was telling was that instead of being this discrete point if it is a continuous point this is important in the sense that what we can in that case what we can imagine that this probability is a mass in where in this field that is that can be expressed in terms of the density. Now if I take one elemental area of that particular sample space then the total mass that is the total area multiplied by the density. The total mass will be give you the probability for that particular for that particular event.

(Refer Slide Time: 17:50)



(Refer Slide Time: 17:53)



Now in the previous class also we have discussed about the concept of this conditional probability. So here just to recall the fact that if there are two events one is A and an other one is B and these events are taken in such a way that the probability of A is sorry for this mistake this should be greater than 0. So if we say that the probability of A greater than 0 not 1 this is greater than 0.

Then the probability of B given that A has already occurred. So this is expressed in terms of the probability B on condition A so this is known as the conditional probability. So which is differed from this probability of this B in the sense that when there is no other information is available this is simply the probability of one event B so probability of B. Now if we say that A has already occurred.

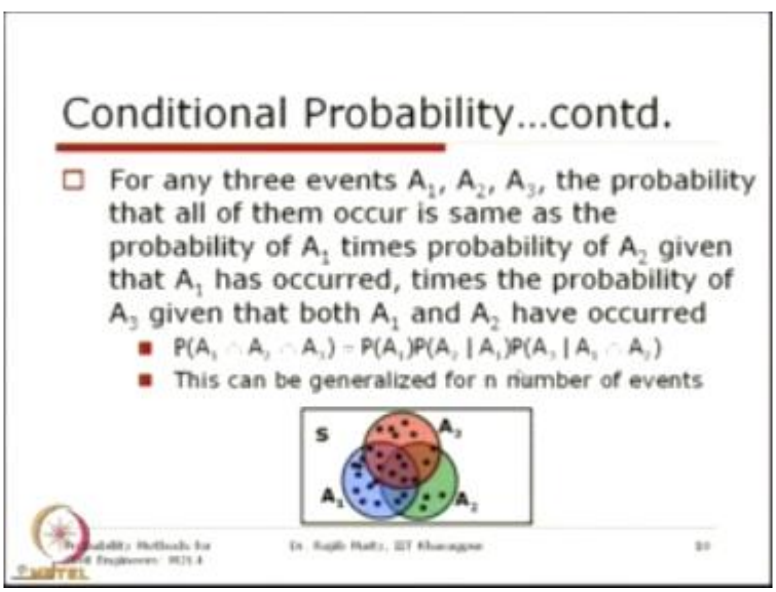
Now that so one information is available. So based on the available information whether the probability of the other event may or may not change and this is known as the conditional probability which is denoted as like this B on condition A, which is derived as the probability A on condition A which is equal to the P(A ∩B)/P(A). So here so if we see if we refer to this Venn diagram.

Then what if we simply say what is the probability of B? Then we will just concentrate to the event B which is shown by this circle. Now if we say that A has already occurred then we know that our sample space or our total feasible space as is now within this zone which is denoted as the event A. Now the success of this one so what is the probability of B? So the success area is highlighted in these area in this way which is the intersection of these two event that is $A \cap B$ that is why this is the success this is the area where the success is realized to declare the probability of B.

Now the total feasible space as we have seen that as A has already occurred so the probability of A comes here so it says that conditional probability of B given that A has already occurred is equal to probability of $A \cap B$ divided by probability of A.

We will use this relationship to form our base role which is very important so far as the application is concerned we will refer to this one before that we will try to understand how this probability of particular event can be derived in terms of the other events however still before that discussion we will.

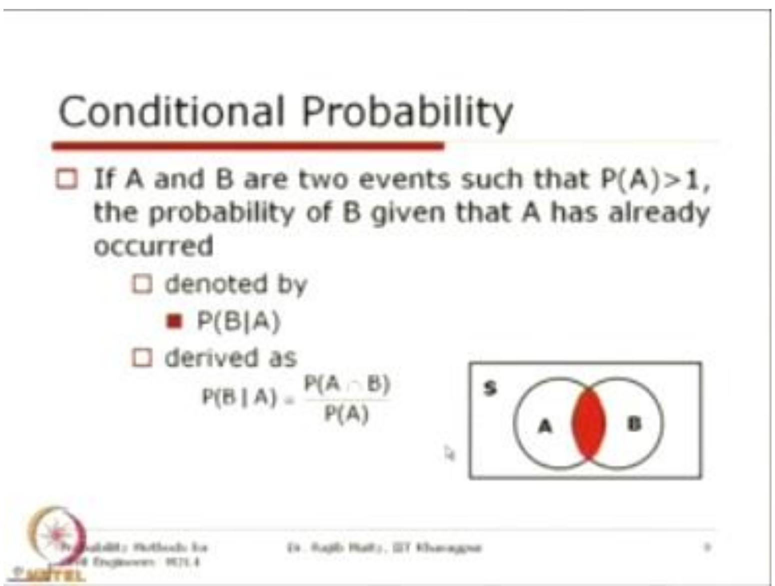
(Refer Slide Time: 20:59)



Discuss about the same that conditional probability when we are talking about the more than two event so for any three events that is A1, A2 and A3, the probability that all of them occurred is the same as the probability of A1 times probability of A2, given A 1 has occurred times probability of A3 given that both A 1 and A 2 has occurred so this if you want to if you know that there are three events A1, A2, A3 if we say that - what is the probability of the simultaneous occurrence of all these three events.

This can be expressed in terms of probability of A1 x by probability of A2 on condition A1probability of A3 on condition A 1 and A 2 both has occurred this is just followed from this two event.

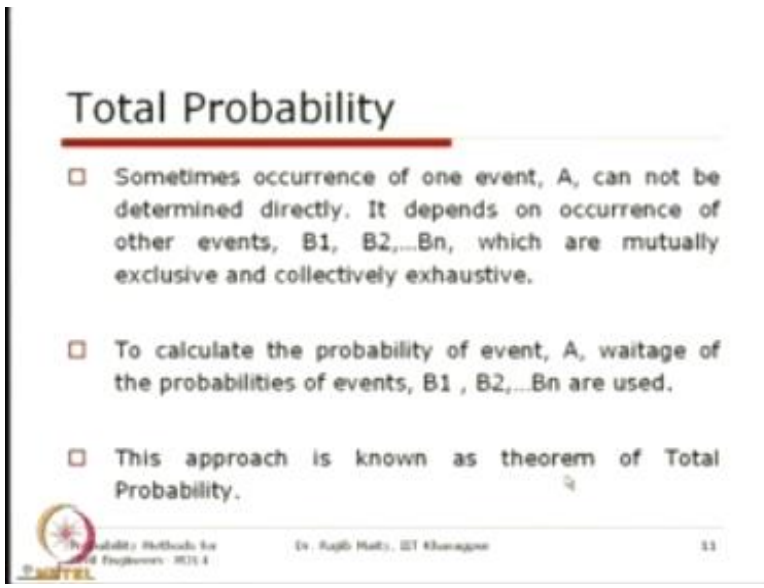
(Refer Slide Time: 21:53)



As from this conditional probability case that is probability of A on condition B is nothing but equal to probability of A multiplied by probability of B on condition A now extending the same thing. We are getting here for these three events that is probability of A1, A 2, A 3 first we take the first event that is probability of A1 multiplied by probability of A2 on condition A1 probability of A3 on condition A1 and A2 both has occurred so extending this same thing this can be generalized for this n numbers of events also so probability of A 1 ∩ A 2 ∩ A 3 intersection A 4 up to if we go ahead like this then we will say that probability of A1 multiplied by probability of A2 on condition A 1 probability of A 3 on condition A1 and A 2 multiplied by probability of A4 on condition A 1 A 2 A 3 all three has occurred.

And this will go on in the same way as it is go as it is going on for this N numbers of different events.

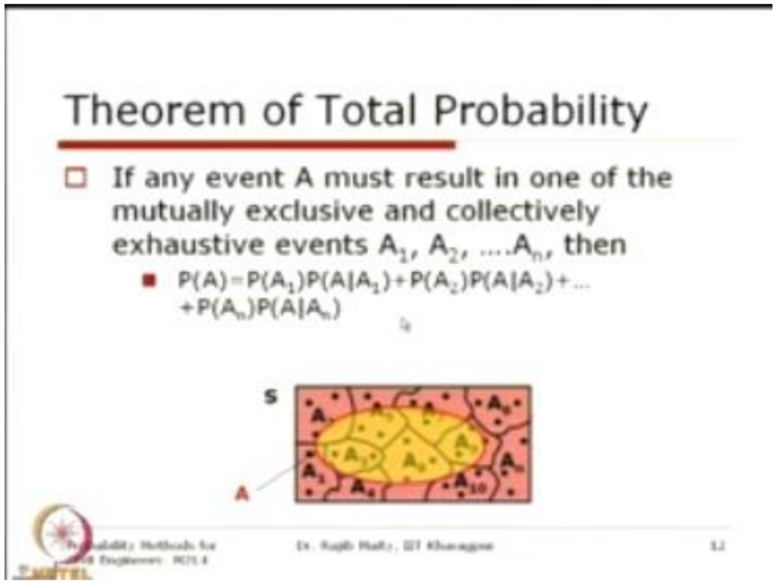
(Refer Slide Time: 23:06)



Next is the concept of the Total probability sometimes the occurrence of one event A cannot be determined directly It depends on the occurrence of the other events such as say B1, B 2 up to B n which are mutually exclusive or collectively and collectively exhaustive so now I just spoke these condition here which are mutually exclusive and collectively exhaustive which is generally leading to the Theorem of Total probability so even I do not know the probability of a particular event but from the experience if you know the probability of the other events.

Then this probability can be expressed in terms of this one to calculate that probability of this event A the waitage of the probabilities of the event of the event B , B 2 and Bn are generally are used and this approach is known as the Theorem of Total probability so this Theorem of Total Probability.

(Refer Slide Time: 24:12)

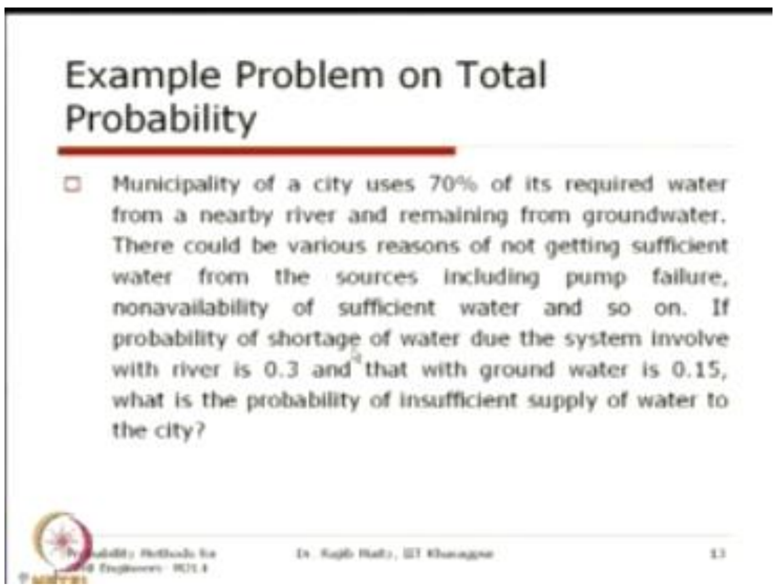


Says that if any event A must result in one of the mutually exclusive and collectively exhaustive events A1 to An I take a minute to explain one second the mutually exclusive and collectively exhaustive - this means the occurrence of one event for example, this A 1 to An what is shown here the occurrence of any one event implies the non-occurrence of all other event this is meant by the mutually exclusive and collectively exhaustive means the probability of A1 p+ probability of A2 + up to in this way probability of A1 = 1 so if we see here if this full rectangle is your sample space.

Then this is these are the events which are non-overlapping to each other this A 1, A 2, A 3, A 4, up to An so these events are known as mutually exclusive and collectively exhaustive so if these are the events then the probability of another event A which is overlapping with all these events is equal it can be expressed as probability of A is equals to the probability of A1 multiplied by probability A on condition A1 + probability of A2 on condition probability of A on condition A 2 in this way it will go up to probability of A n multiplied by probability of A on condition A.

So this theorem is known as the Theorem of Total probability so these the events this probability of A1 probability of A2, probability of An generally as known from the experience and probability of A1 on condition A on condition A1 probability A on condition A2 also known from the previous experience when both these information is known to us then if you want to know what is the total probability of the event A then these probabilities these conditional probabilities are waitage to the individual probability of the individual events A1A2 up to A n this is the basis of this total probability theorem.

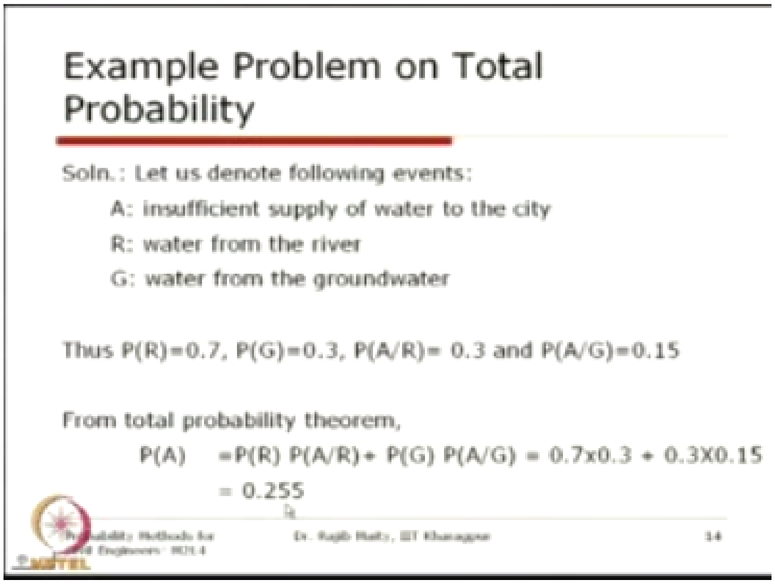
(Refer Slide Time: 26:44)



Now if you see one example problem using this total probability problem this is an water supply problem Municipality of a city uses 70 percent of its required water from a nearby river and remaining from the ground water that is 30% is used from this groundwater now there could b various reasons for not getting sufficient water from the sources including the pump failure non-availability of the sufficient water and so on so the failures can be of I can I am just dividing the failure of not supplying to not supplying sufficient water into 2 parts.

One is which is related to the supply from the river another one is related to the supply from the groundwater so if the probability of the shortage of water due to the system involved with the river is 0.3 and that with the ground water is 0.15 what is the probability of insufficient supply of this water to the city now here in this problem we can see that this probability of insufficient supply of the water is my total probability that I am looking for which is depending on this here it is 2 and that can depend on the many factors so now if I just define the.

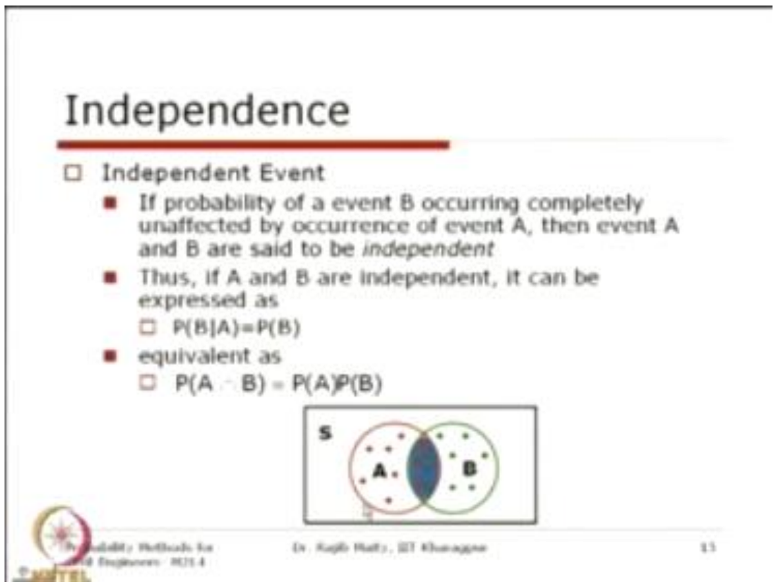
(Refer Slide Time: 28:08)



Events like this that A event is the insufficient supply of the water to the city and R is the water from the river and G is the water from the groundwater. Now from the problem we have seen the probability of the water that we get from the river is 0.7 from the groundwater it is 0.3 now probability of the insufficient supply in the case of the river it is 0.3 and probability of insufficient water in case of the ground water it is 0.15 then so from the total probability theorem that is probability of the insufficient supply of the water to the city this equals to probability of R.

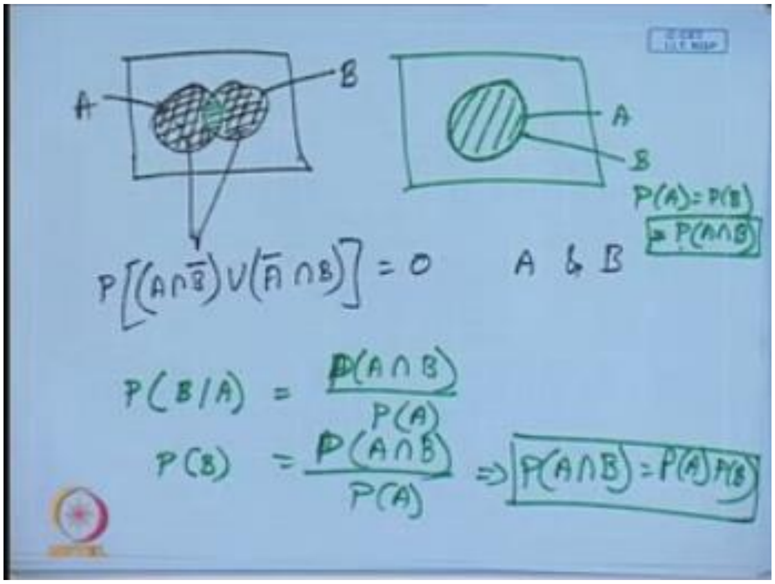
Weighted to the probability of insufficient supply in case of river and similarly for this groundwater so which is equals to your this calculation which comes that the total probability of insufficient supply of the water to the city is 25.5 percent.

(Refer Slide Time: 29:11)



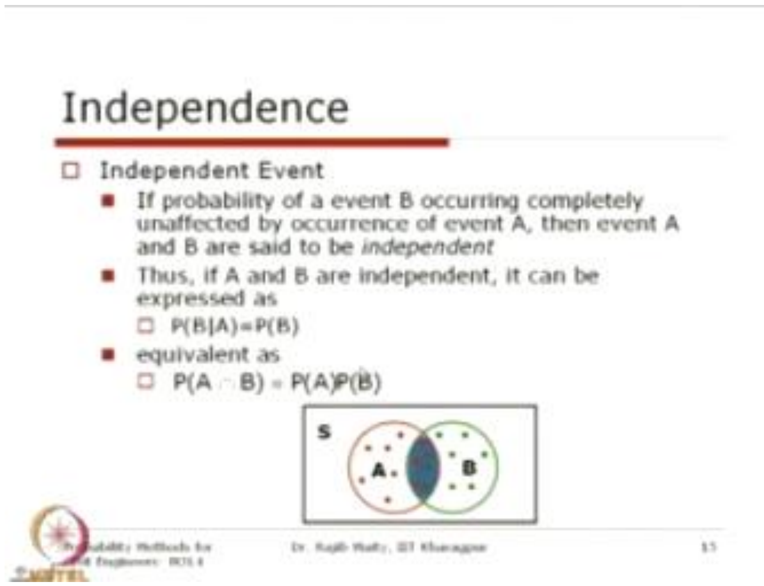
Now another important concept is known as independence which is which is relevant here to discuss is that independent event if the probability of a event B occurring completely unaffected by the occurrence or non-occurrence of the event A then event A and B are said to be independent thus if A and B are independent then it can be expressed as probability of B on condition A is equals to probability of B. So whether the A has occurred or not it does not have anything to change the probability of B so which is equivalent as the probability of A intersection B is equals to probability of A multiplied by probability of B. So, which is directly following from the conditional probability which is just we have seen the --

(Refer Slide Time: 30:12)



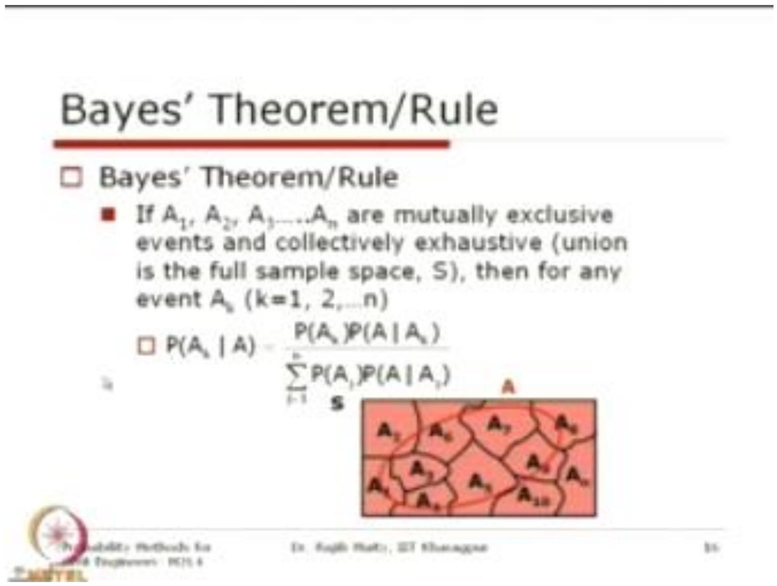
Probability of B on condition A is equals to probability of A intersection B, probability of A. Now this is probability of B on condition A if these two are independent then this is nothing but equal to probability of B. So, A intersection B divided by probability of A which… So this is generally the basis to declare in mathematically the two variables, to random variables are independent which will be used in from the next class onwards.

(Refer Slide Time: 31:09)



So this is forming the mathematical basis to declare two events to be independent it is said that, if and only if the probability their joint probability is equals to the multiplication of their individual probability, then we say that these two events are independent.

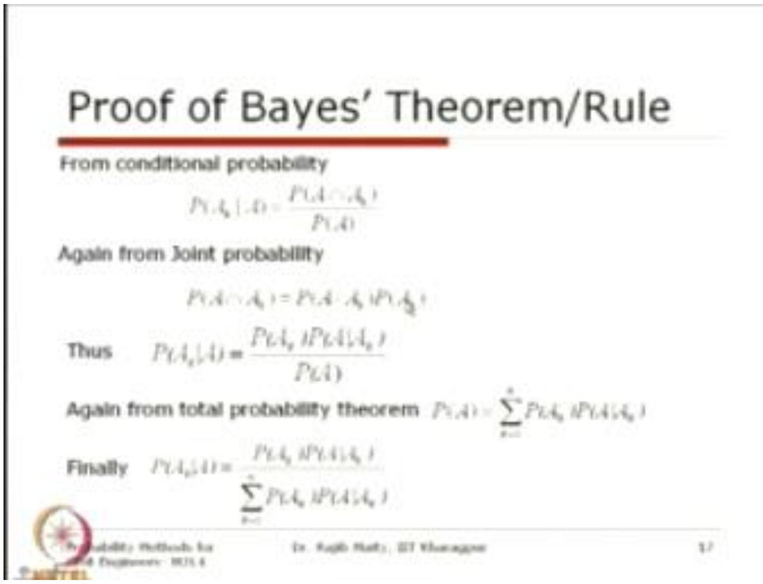
(Refer Slide Time: 31:32)



Bayes' theorem or rule if A_1, A_2, A_3, A_n are mutually exclusive events and collectively exhaustive just now we have discussed about this mutually exclusive and collectively exhaustive; then for any event A_k, k can range from 1to n, what we can say that this probability of A_k on condition A is equals to probability of A_k multiplied by probability of A on condition A_k divided by summation of all these probabilities which probability of A j multiplied by probability of A on condition A_j.

This is known as this Bayes' theorem which we will just see that this comes from this total probability theorem that is if we know the probability of individual event and after that we know the occurrence of one particular event A, which is shown as a red ellipse here, then, what are the probability of these different sub-events which are mutually exclusive and collectively exhaustive is generally obtain from this Bayes' theorem or Bayes' rule.

(Refer Slide Time: 32:49)



The proof of Bayes' theorem can be explained like this, that is, if I take any particular event A_k, k can be from 1 to n; So, A_k,a particular event on condition that A has occurred can be expressed in terms of, just now we have seen the conditional probability, so this A intersection A k divided by probability of A. Now, again from this Joint probability this probability A intersection A_k can be expressed as probability of A on condition A_k multiplied by probability of A_k.

Now, from here if we just replace this one to this part, then we see that probability of A_k on condition A is equals to probability of A_k multiplied by probability of A on condition A_k divided by probability of A. Now, this probability of A can be expressed in terms of this total probability theorem. Then from this total probability theorem this probability of A can be expressed as this for all this k, it should be the summation of the probability of A_k multiplied by probability of A on condition probability of A k.

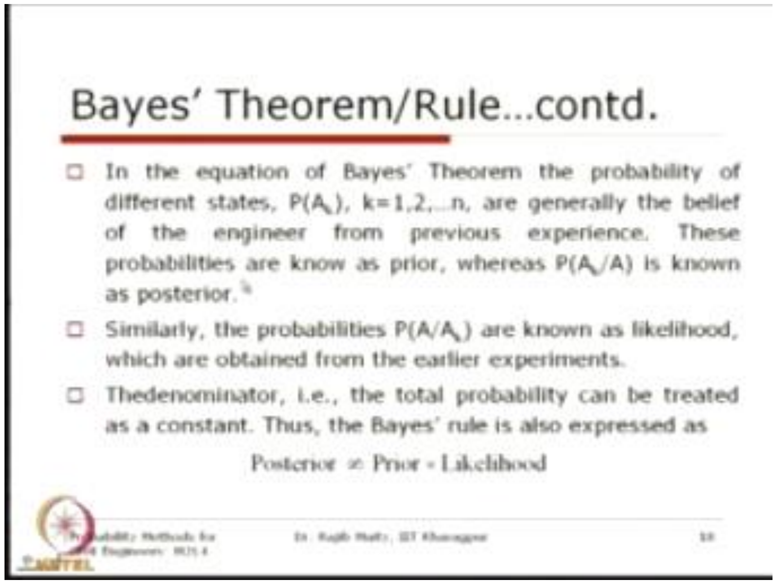
Now if we replace this one here, so, probability of a particular event on condition A is equals to probability of A_k multiplied by probability of A on condition A_k divided by summation of for all k probability of A_k multiplied by probability of A on condition A_k. Now these probabilities these individual probabilities have is generally having different meaning first of all this probability of A_k so what we are doing is that if we say that, this left hand side is unknown and right hand side is known, then what we are trying to do is that, we know the probability of A_k without any condition what we are trying to understand, what we are trying to get is of course the probability of the same event here is A_k, here is also A_k but here the condition is that occurrence of the particular event is known.

So, this one, this probability of this individual events which are mutually exclusive and collectively exhaustive, these events are my prior knowledge. Now from this prior knowledge due to the occurrence of the particular event I want to update the knowledge of the probability of A_k. So, this is known as the posterior, so, probability of A k is your prior and probability of A k on condition A is your posterior, posterior.

Now this probability of A on condition A_k, that is if we have the data available to us, then these probabilities can be calculated from the previous experience and the previous experiments. So, these are, this is known as the likelihood of that particular, of that particular event, so this is known as the likelihood. Now, this denominator part is coming from this Total probability theorem, which is equals to the probability of the, probability of A.

So, as compared to these probabilities, this can be treated as a constant. So as this can be treated as a constant, if we take this out then this equality sign will convert to proportional sign, so this if we just want to discuss it.

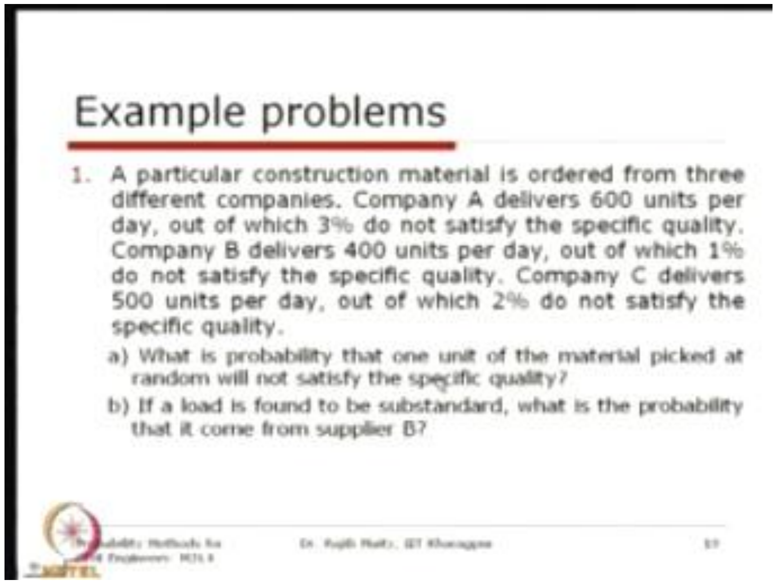
(Refer Slide Time: 36:31)



Then in the equation of this Bayes' theorem we have seen that this probability of different events, that is probability of A_k are generally the belief of the engineer from the previous experience. These probabilities are known as prior. Whereas the probability of that particular event on the condition that one event, one particular event has already occurred, this is known as the posterior.

Similarly, the probabilities of this A on condition A k are known as a likelihood, which are obtained from the earlier experiments. In the denominator, there would be one space here the denominator that is the total probability can be treated as a constant. Thus the Bayes' rule is also expressed as so left hand side that that is your posterior, which is proportional to the prior multiplied by the likelihood.

(Refer Slide Time: 37:29)



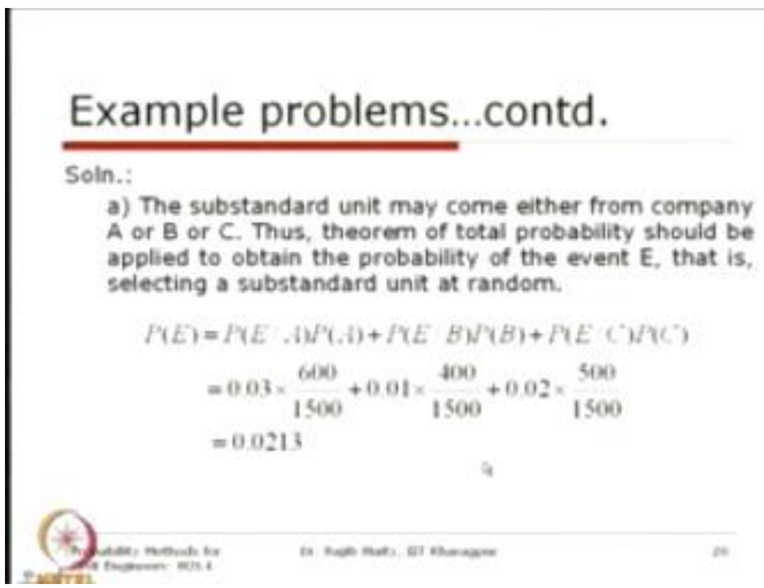
So with this, we will take one particular example where a particular construction material is ordered from 3 different companies. So the company A delivers 600 units per day out of which the 3 percent do not satisfy the specific quality. Company B delivers 400units and out of which 1 percent 1 percent does not satisfy the specific quality. Now, the company C delivers the 500 units per day,

So out of which 2 percent do not satisfy the specific quality. So the total units per day is being supplied is, 1500 units are being supplied by 3 different companies. Now we want to know at the construction site that what is the probability that 1 unit of the material picked at random, this picked at random is important, so I do not know without knowing the knowledge that which company has supplied to this one, if you do not know that information, that is why it is picked at random will not satisfy the specific quality.

So to at this problem we have to use the theorem of total probability the total probability of getting one particular unit which is defective or not satisfying the specific quality. Now on the other hand if a if a particular unit is found to be the to be substandard, then what is the probability that it has come from supplier B, now here we are giving one condition that one unit has found to be substandard that is already fact.

Now depending on that fact, depending on that event, what is the probability that it is being supplied by B, so we will see this two problem? The first one will be solved by this Total probability theorem and the second one will be solved by this Bayes's rule.

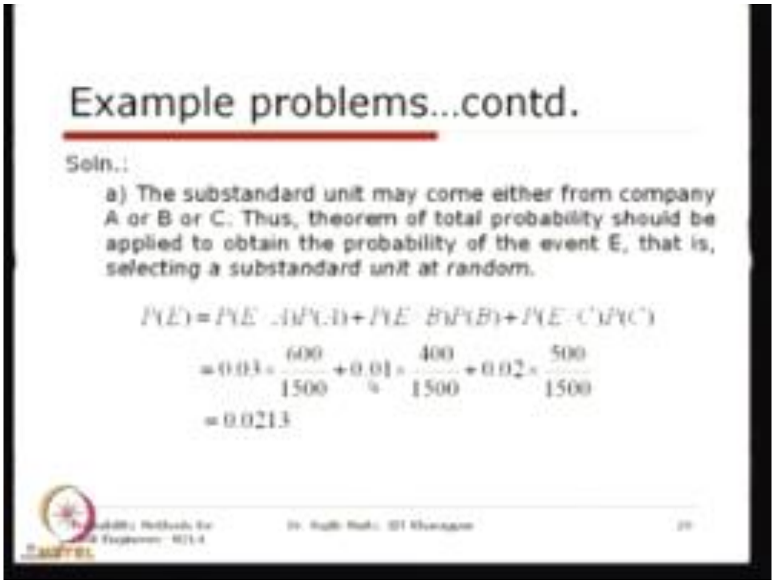
(Refer Slide Time: 39:42)



So to answer the first one the substandard unit may come either from the company A or B or C. Thus the theorem of Total probability should be applied to obtain the probability of the event E, that is the selecting a substandard unit at random. So, this E is denoted as that the probability of the event, that is this selecting a substandard unit at random. So, this E is denoted as the, as that the probability of the event, that is this selecting a substandard, substandard unit.

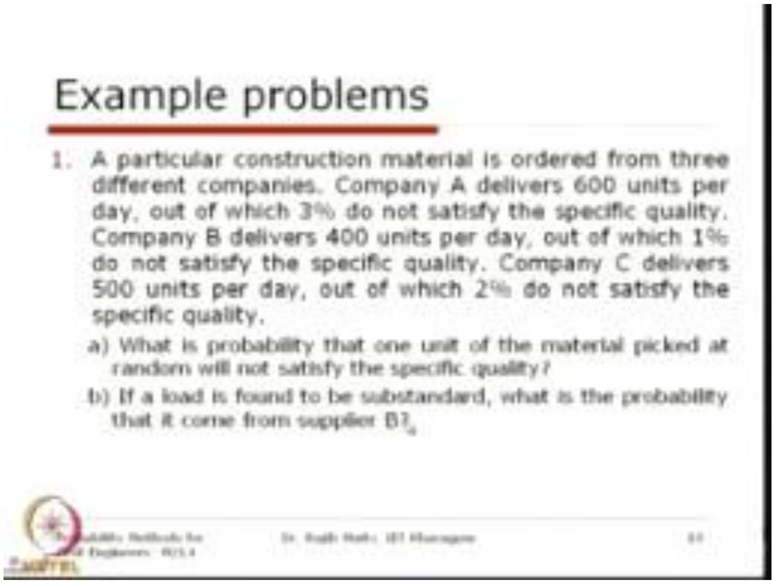
So, probability of E which should be expressed in terms of that probability of A, that is, it is supplied by what is the probability of supplying A, probability of supplying B and probability of supplying B ,supplying C and what are their chances of supplying the defective units. So, this first one is 0.03 which is, and their probability of supplying by, by event A is600 by total units being supplied in a day is 1500.

(Refer Slide Time: 40:51)



And, the second one is 0.01 and the probability it is being supplied by company B is 400 by 1500, and the probability that it is being supplied by C, company C is 500by 1500. So, if we do this calculation, it comes that the total probability of getting a substandard unit is 0.0213. So, this is the total probability that we get.

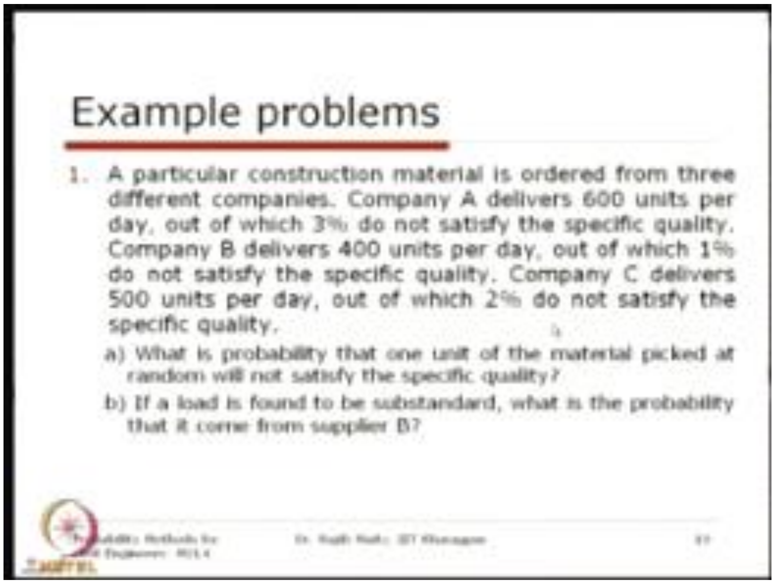
(Refer Slide Time: 41:21)



Now, if we just see the second question, if the unit is defective, then what is the probability that it has come from company B? So, once it is known that the unit is substandard, the probability of the unit being supplied by a particular company is not the same as that when the information of the substandard unit was not known.

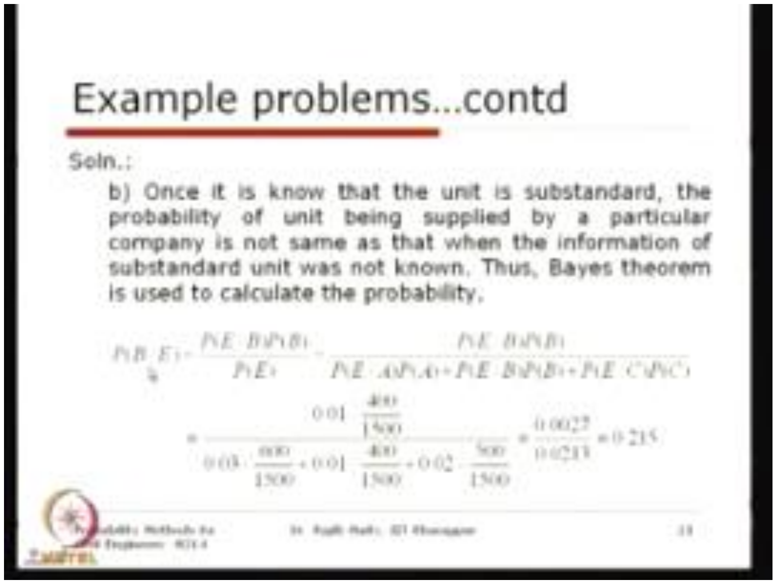
So, what I, what is mean there is that, if, if this information was not available, then probability of supplying the substandard unit by with the company B is known to us, which is nothing but, here 0.01 supplied in this, it from the earlier, earlier experiences.

(Refer Slide Time: 42:02)



Company B delivers this one, out of which 1 percent do not satisfy the specific quality.

(Refer Slide Time: 42:11)

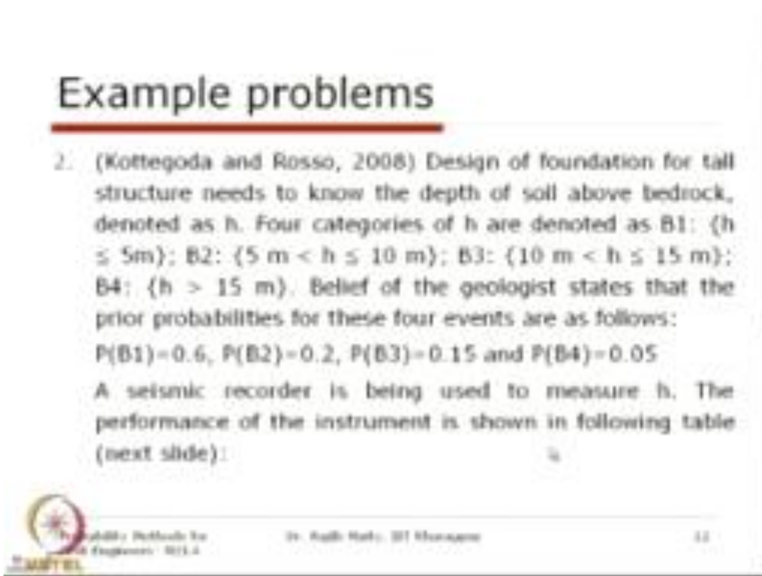


So, now we have to update that information, that is the probability of B on condition that one substandard unit has come. So, this is equals to probability of E on condition that probability of E on condition, it is supplied by B multiplied by the probability of B, this one; which is now this A is again expressed in terms of this total probability which is expressed in this form - Probability of E and condition A multiplied by a probability A, similarly from probability B, similarly from, from company C .

Now, if we just put this forms, then it comes that probability B is 400 /1500. Similarly, the total probability as we have seen in this last slide that this quantity comes to 0.0213and this is 0.0027, a ratio gives that 0.215.

So, so, the first one, we have solved from this Total probability; and second one, we have solved from this Bayes' rule.

(Refer Slide Time: 43:12)

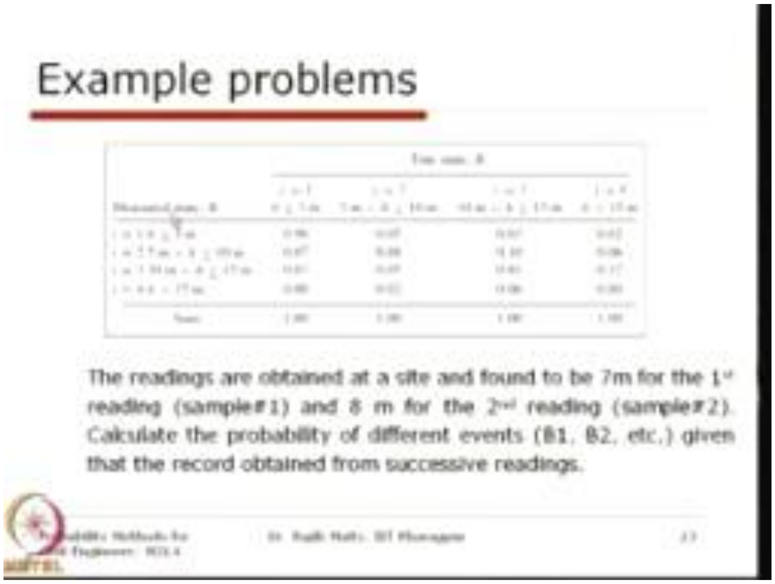


We will take another interesting problem which is taken from Kottegoda and Rosso, 2008, from that book. Or, this is a basically a geotechnical problem, or the design of, the design of foundation for the tall structure, needs to know the depth of the soil above the bedrock which is denoted as H

.Now, four categories, four categories of H denoted as B1; so, there are four different categories of this depth. The first one which is less than 5 meter and second one B2 is5 meter to 10 meter; third one is 10 meter to 15 meter and B4 four is greater than 15meter. So, belief of the geologist states that, the prior probabilities of these 4 events are as follows: the probability of B1, that is the, that is the bedrock should be within the 5 meter depth, is equals to 60 percent; probability of B2 is equals to 0.2; probability of B3 is equals to 0.15 and probability ofB4 is equals to 0.05.

Now, a seismic recorder is being used to measure this H. Now, obviously any instrument that we will use; this type of, this type of measurement is generally not always perfect, this is also having some certain percentage of error. So, this is coming from this earlier experiment, where both the true depth, as well as the record both, are available to us. So, the performance, so, which is denoted the performance of this instrument, the performance of the instrument is shown in that table in the next slide which is here .

(Refer Slide Time: 45:09)



So, this is the measured state which is BI and these are the true state which is B So, if the measured states generally say that, it is within the 5 meter, so, there is 90 percent chance that it is actually in the first state; there is still 5 percent chance it is in the second state, 3 percent chance it is in the third state and 2 percent chance - this one.

So similarly, in this way for all these cases, if it is measured to this, this particular fact and if the true data is also available, then we can complete this particular table here. See this is quite…There are two important thing that should be observed here; one is that, if the instrument is perfect, then we can say that this one, if it is measured in the, in the, in the state one, then this should be 100 percent probability and all other probability should be 0.

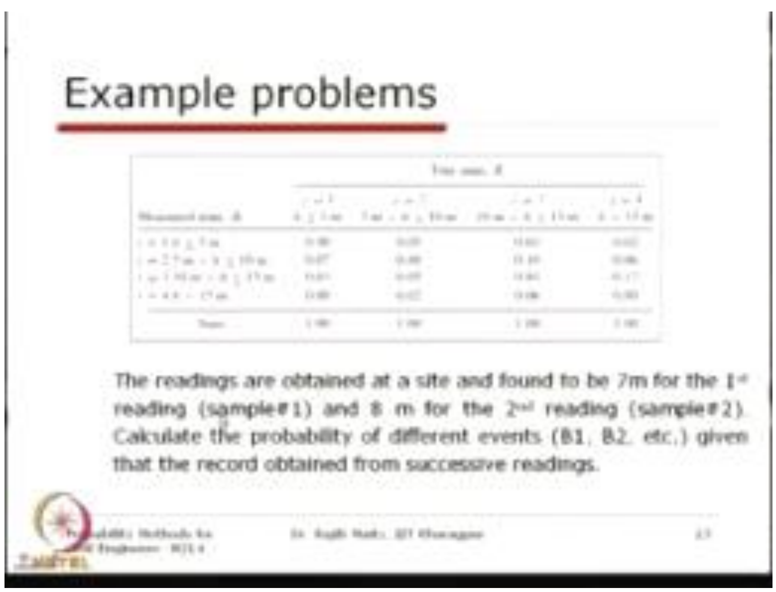
If it is true, this is also the true state will be true, so, this should be a perfect one, that is, 100 percent probability and other should be 0. But, as this instrument is, is not perfect, that is why we are getting this distribution of these probabilities from the earlier experiments; and obviously, this diagonal, diagonal is heavy diagonal, that means, most of the time it generally measures the true fact.

Another thing, that is, if it is measured in state one or a particular state, and there are the probabilities, the, where this state will be; so, it should be exhausted. So, the state one whatever the probability is, if we just add up the probabilities in a row-wise, row-wise fashion or in a column-wise fashion, this should be equal to 1.

So, this is denoting that this is collectively exhaustive. Now, the readings are obtained at a site, so, now, the same instrument, once we know the performance of this instrument, the same instrument is being used to know the depth of the bedrock at a particular site. Now, the readings are obtained at a site and found to be 7 meter for the first, first reading which is denoted as sample 1 and 8 meter for the second reading.

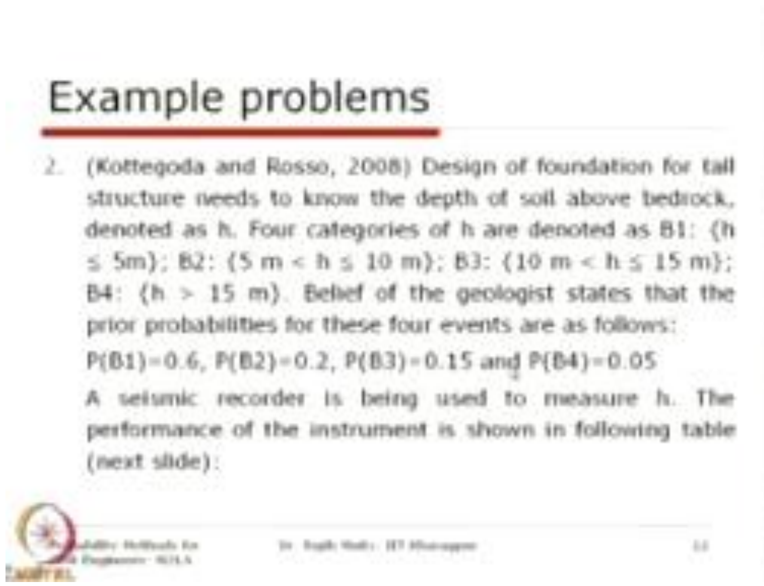
Now, we will have to calculate the probability of the different event, that is B1, B2, B3 given that the record obtained from the successive readings. So, if the no reading is taken, then the probabilities are listed here. So, probability of being it in the B1 state, that is, below 5 meter is 6.6, 60 percent, 20 percent, 15 percent,5 percent.

(Refer Slide Time: 48:01)



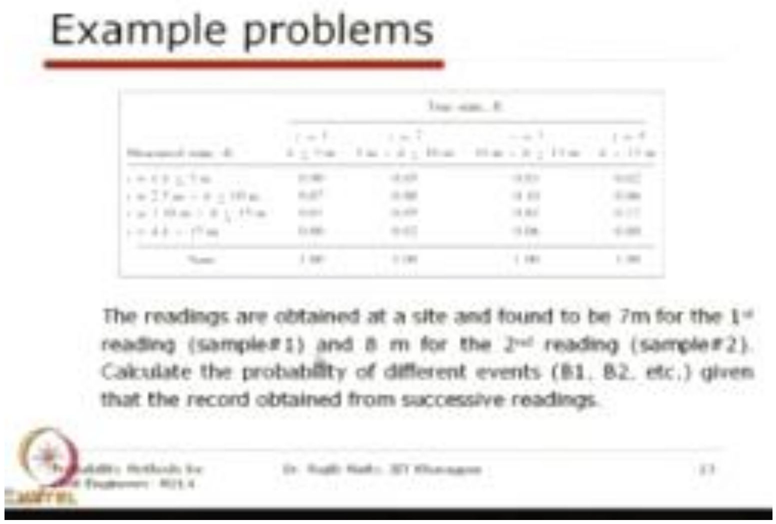
Now, I got one sample which is7 meter depth from this instrument which is obviously not perfect, erroneous instrument. So, after getting that sample1 from that instrument, what are the, this probability.

(Refer Slide Time: 48:16)



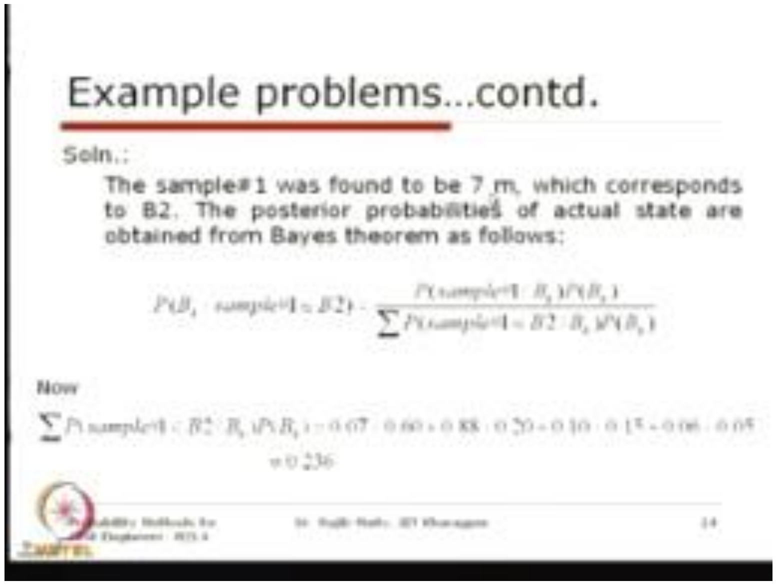
How these probabilities are being updated

(Refer Slide Time: 48:20)



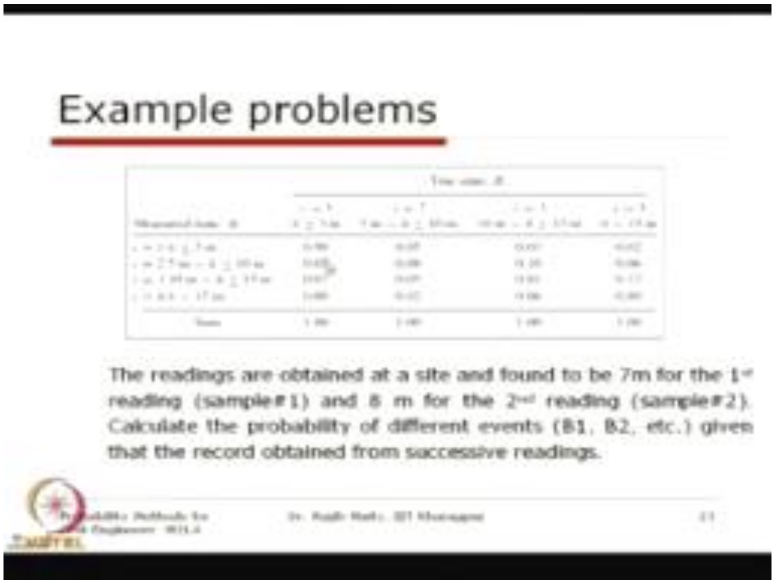
So, to know this fact, once we know this, after this sample one, then another sample has been taken which is sample2. So, these probabilities will be updated after sample1.Again it will be, again updated after taking this sample2. How these probabilities are changing? We have these successive probabilities; that, we will see now in this problem's.

(Refer Slide Time: 48:42)



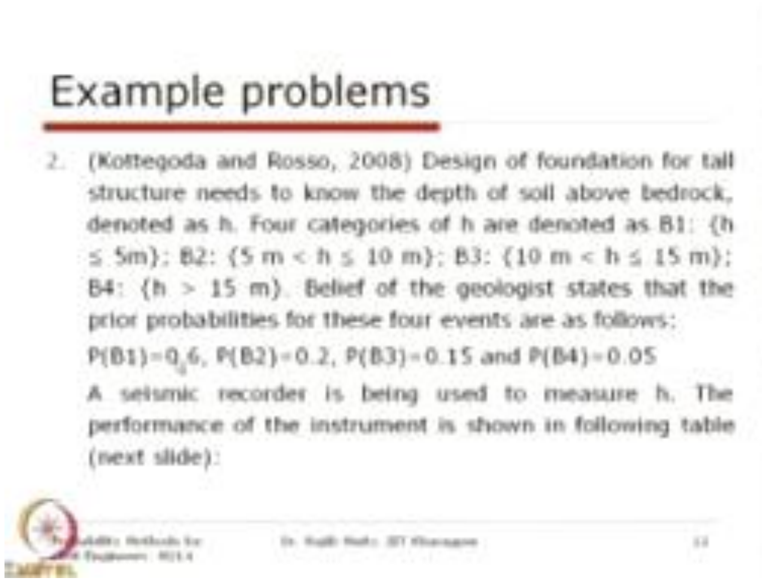
The sample#1 was found to be 7 meter which corresponds to the B2. Now, the posterior probabilities of the actual states are obtained from Bayes' theorem, that is, what is the probability of a particular state B k? Now, here B k means that B1, B2, B3, and B4 on condition that sample#1 is in B2. So, this is from this, derived from the Bayes' rule, we can, we can state that, this is the probability of sample#1 belonging to B2 on condition BK multiplied by B k; and the, and the total probability is the probability of sample#1belongings to B2 on condition Bk probability of Bk. Now, this total probability is calculated from this, this one, that is from, if I just take that, that sample is two, what is on condition that it is in actually in one and multiplied the probability one, which is 0.07multiplied by 0.6.

(Refer Slide Time: 49:45)



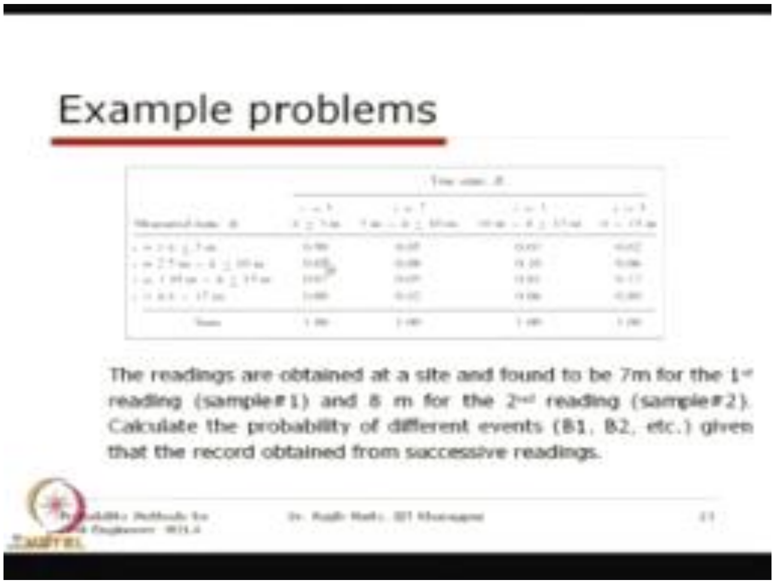
Now, if you see this slide that, this is your 0.07; it is in measure state two. So, this is in 0.07.

(Refer Slide Time: 49:55)

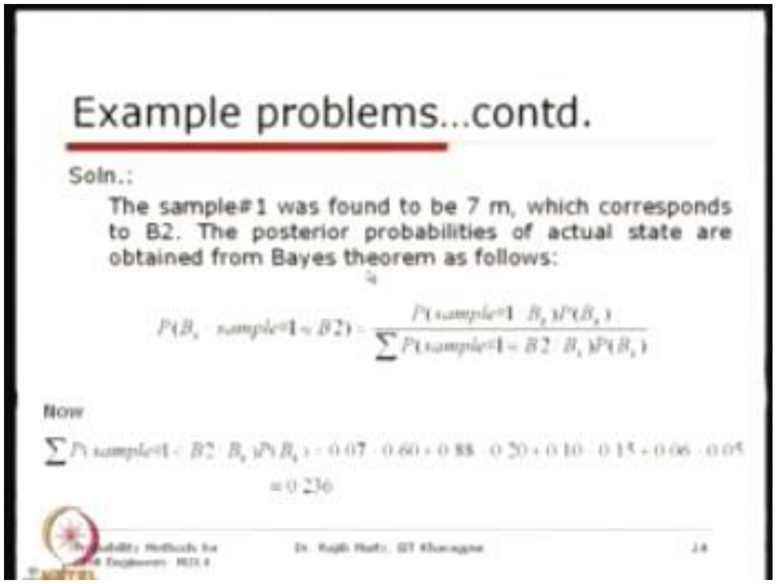


And the probability of the prior knowledge of B1 is 0.6.

(Refer Slide Time: 49:58)

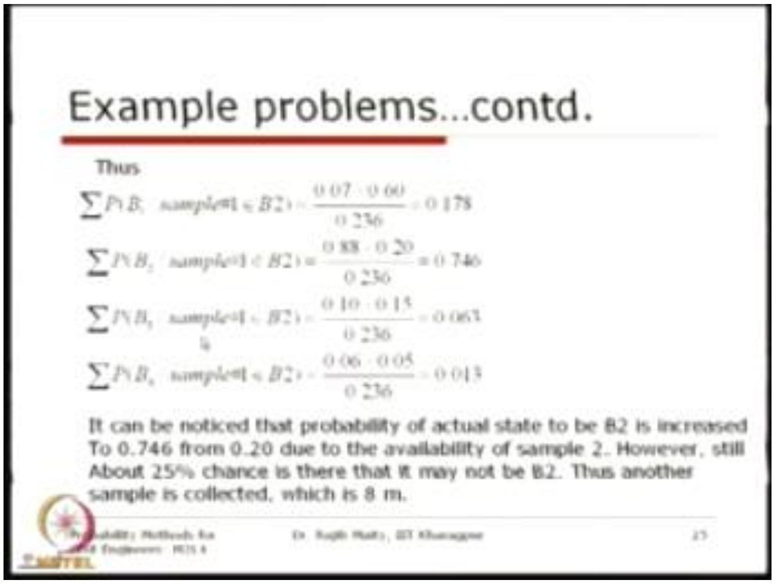


(Refer Slide Time: 50:00)



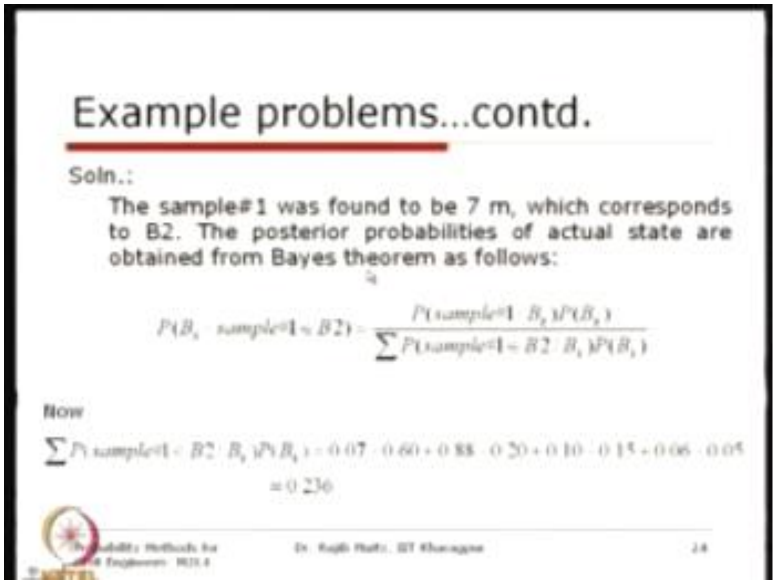
And, that is why this 0.07 and 0.6 are multiplied. Similarly, taking the other probabilities from that table and that probability 0.2, in this way, if you just add up, we get the total probability is 0.236. Now putting this one, so, we will get the updated probabilities for difference state.

(Refer Slide Time: 50:21)



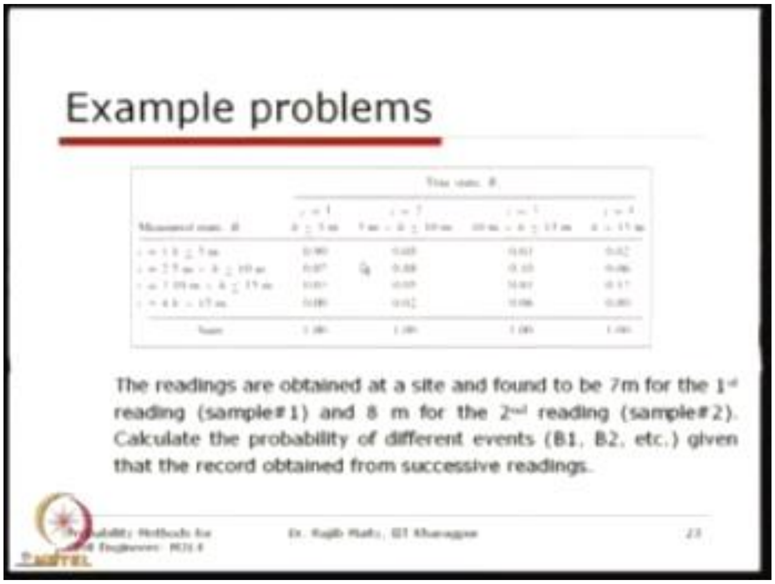
Which is as follows, so probability that it is the state one is equals to 0.07 multiplied by 0.6 is divided by the total probability 0.236.

(Refer Slide Time: 50:36)



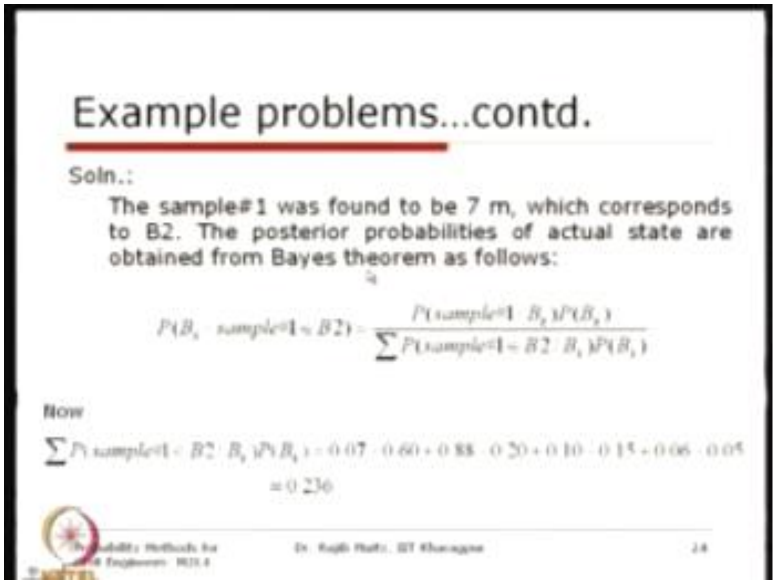
Now this 0.07 comes from that table.

(Refer Slide Time: 50:37)

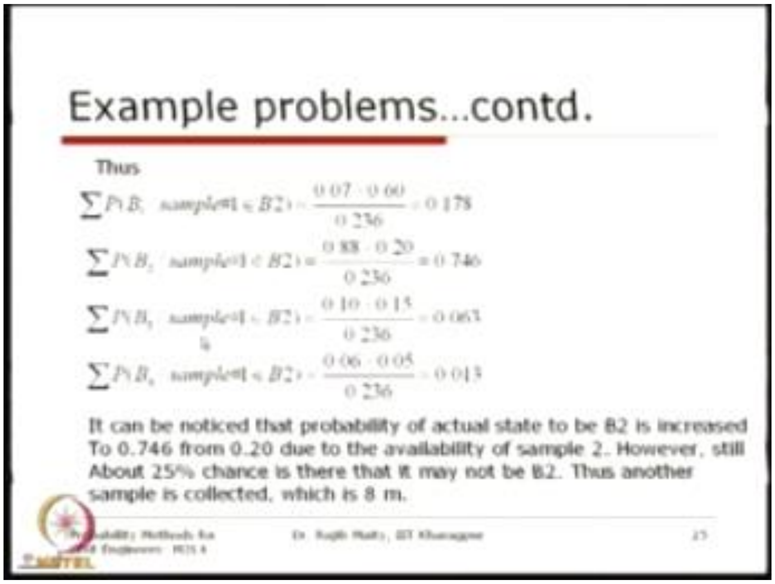


I had just now I showed from here.

(Refer Slide Time: 50:38)



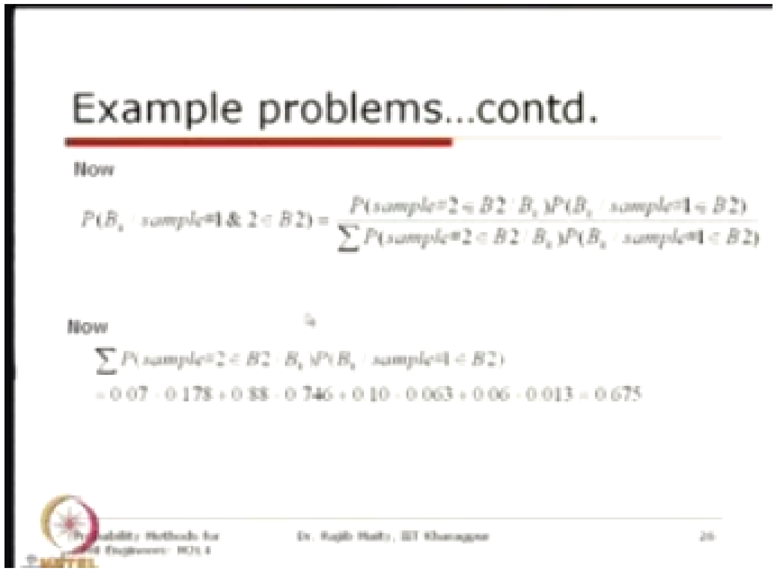
(Refer Slide Time: 50:39)



And this 0.6 comes from this prior knowledge. So this is now after the sample 1 the prior information was 0.6 it is updated to 0.178. So this, the probability of the depth of this data in one is now is now reduced from 0.6 to 0.175. Similarly the state which is in the B2 is in the 0.2 which is going to be increased from this 20 % to 74.6 % similarly all are probabilities.

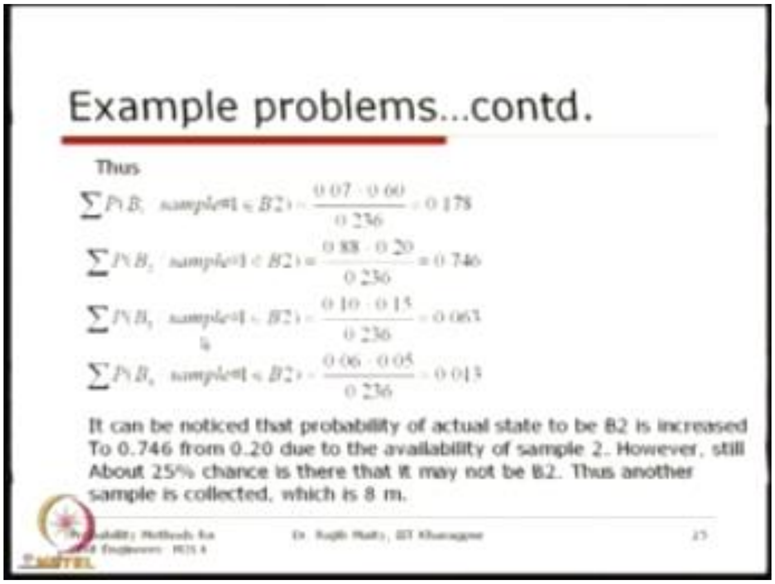
Now after taking the sample 1, these probabilities are modified and this probability of being it in the second stage has increase to the almost 75 %. But still it is not, still there is 25 % chance that this depth of this bedrock may not be in these state 2. So the another observation is collected where it is again saying that it is the depth is 8 meter that is it is again in that state two there is a second stage. So after getting the second information again this probabilities are being updated.

(Refer Slide Time: 51:52)



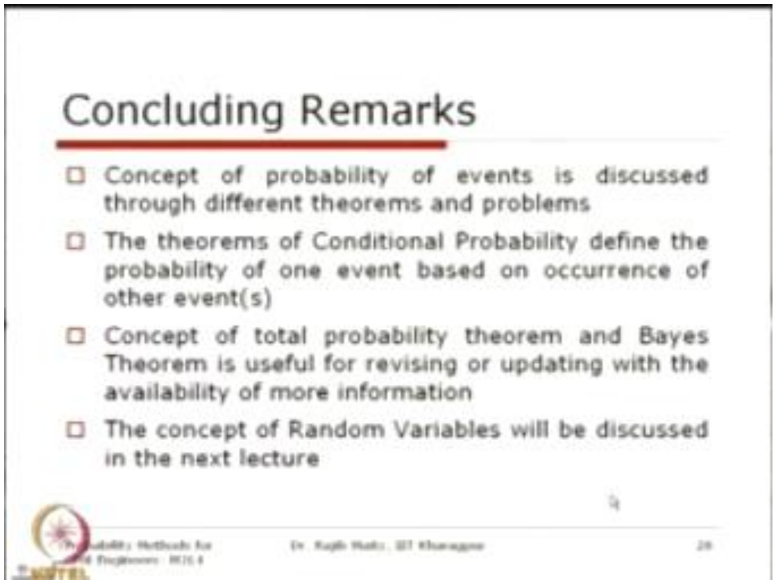
So this is now what is the probability of B_k? One condition the sample 1 and 2 both are in B2, So this can be expressed in terms of this base theorem, and here the total probability when we are calculating we are using that the performance of the, this is from that table performance of the instrument and this is from the updated information after getting sample 1. So if we add up this thing it comes to be 0.675.

(Refer Slide Time: 52:22)



Now using that 0.675 we are getting different probability for the difference state. So from 175 it is further reduced to that 1 %, 1.8 % and the probability of that it is instead 2 it is increase to the 97 % and similarity for the other states. So thus it is noticed that after obtaining sample 2 the chance of true state being B2 is very high which is 97.2 %. Thus with the help of the base theorem the probability of unknown are improved with the availability of the more information.

(Refer Slide Time: 53:01)



So in this class the concept of probability events is discussed through different theorems and problems. The theorem of conditional probability is defined, the probability of one event based on the occurrence of the other events, concept of total probability theorem and base theorem is useful for revising or updating with the availability of more information which is seen in the last example. And concept of in the next class, we will cover the concept of random variable and this we will see in the next class, thank you.

Probability Methods in Civil Engineering

End of Lecture 05

Next: "Concept and Defination of Random Variables" In Lec 06

NPTEL Video Recording Team
NPTEL Web Editing Team
Technical Superintendents

Computer Technicians

A IIT Kharagpur Production

www.nptel.iitm.ac.in